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Calculating of the Electric Transition Forces and Radii of Even-even- Nuclei of Cadmium $\left({}^{100-124}_{48}Cd\right)$ Cd Isotopes

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Abstract

In the current research, the properties of even-even nuclei with a mass number (A≥100) for $\left({}^{100-124}_{48}Cd\right)$ isotopes have been studied. This included distortion parameters (δ) derived ,from an Intrinsic, electric, quadrupole, moments (Q_0)and deformation, parameters (β_2) originate from a reduced, electric, transition probability $B(E2)\uparrow$ based on the energy of the first excited state (2+).,Roots ,mean ,square radii $< r^2 >^{1/2}$, major and minor of oval axes (a, b) in addition to the difference between them (ΔR) calculated for Cadmium isotopes. All of these parameters were calculated for cadmium isotopes by deforming the model of the shell equation in Matrix laboratory software. Major (a) and minor (b) axes were utilized to plot three-dimensional shapes (axially symmetric) and sketch two-dimensional shapes of the isotopes, allowing for the differentiation of various single-element isotopes. According to the latest research, the deformation parameters get smaller as the neutron count gets closer to the enchanted number of neutrons.

Keywords: transition probability $B(E2;0^+\rightarrow2^+)\uparrow$, electric quadrupole moment Q_o, deformation parameters (β_2, δ), mean-squared charge distribution radius $< r^2 >^{1/2}$.

1. Introduction

Previously, a hypothesis was put forward to account for the crustal structure observed in cores [1]. The closed shells evolve when big differences occur in the orbitals of the nuclei, such as at the neutron and proton "magic" numbers of 2, 8, 20, 28, 50, 82, and 126 [2]. The bonding energy for the final nucleon at shell closures is significantly higher than the value encountered in the neighboring cores [1,3]. Because they have a preferred axis, non-spherical nuclei can rotate. For deformed nuclei, this implies that they are axially symmetric, either oblate or prolate, as shown in **Figure 1** below, and that the nuclear 2 volume is constant (incompressibility) for actual solutions [1,4]. The main reason for the deformation of the nuclei is the arrangement of the valence nucleons in the unfilled nucleus shell; in other words, the deformation only occurs when the proton (Z) and neutron (N) shells are more or less packed [2, 5,6].

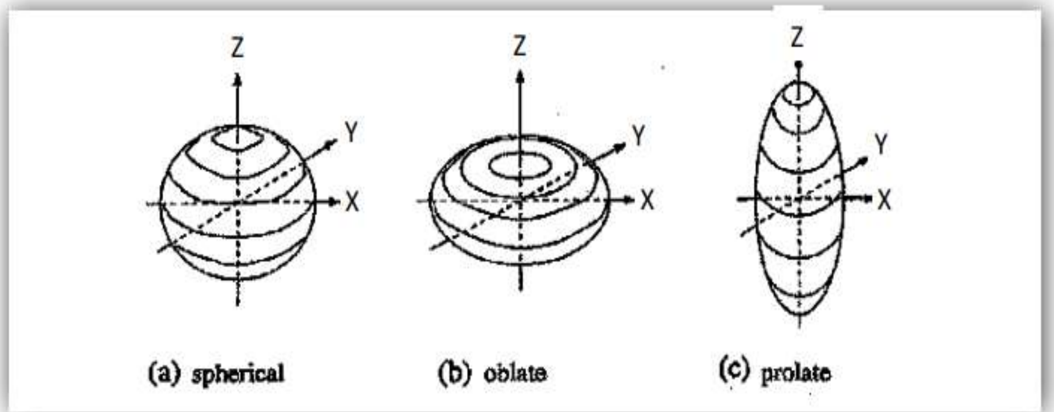

Figure 1. A graphic, representation to three nuclear: (a) spherical, (b) oblate, and (c) prolate z-axis contributes to axis of symmetry to a flattened & elongated shape [6].

The present work aims to determine the shape of the nucleus of even isotopes with an atomic mass (A) greater than or equal to 100 (A≥100) by calculating the quaternary nucleus distortion coefficient (β2) from the low transition potential B (E2) ↑ of 0+ → 2+ obtained from the energy of the first excited state of the isotopes, and comparing it to the distortion coefficient (β2) from the expected value B (E2). ↑ (SSANM) and calculate the quadruple nucleus deformation coefficients (δ) from the quadruple moment Q_o.

2. Theoretical Part

For spherically symmetric charge-distribution nuclei, an intrinsic quadrupole moment (Q0) equals zero for negative to oblate nuclei, and positive to prolate nuclei. In terms of the inherent frame of reference [7,8]. The nucleus' inherent moment (Qo) is linked to a decreased transition probability. Ɓ(E2), which is connected to deformation nuclei or departure from spherical shape [9,10].

$$Qo = [(16\pi/5)\, B(E2)e\,2b\,2/e\,2]\ ½ \text{ barn} \qquad (1)$$

Where e: denotes an electric charge to proton,
$(E2)\uparrow$ is electric quadrupole transition probability in the unit of $(e2b2)$ [11].
An electric quadrupole transition probability B(E2) of a nucleus contains information about the energy of low-lying levels of nuclei [12.13]. The even-even nuclide's initial excited states are 2+[12]. Thus, it is crucial that this state transitions to a 0+ ground state [14,15]. The fundamental experimental quantities are B(E2) values, which are unaffected by a nuclear model [16,17]. Pursuant to Global Best Fit (GLOƁAL), all that is required to estimate B(E2) is Knowledge of the energy ,E (keV) of the 2+ state,(e2 b2) [18,19].

$$B(E2)\uparrow = 2.6\, E\gamma 0 - 1Z\ 2A - 2/3 \qquad (2)$$

$E\gamma 0$: is the transition energy to gamma ray in (KeV) units.
z: atomic numbers,
A: mass number of nuclueus.

A quadrupole moment (Q0), symbolizes a homogeneous charge distribution of the nucleus, is connected to the deformation parameter (β2) utilized to determine the form of an axially, symmetrically deformed, nucleus. [15,16,20]

$$\beta_2 = \frac{Q_\circ\sqrt{5\pi}}{3ZR^2} \qquad (3)$$

Where R2: Nuclear charge, radii calculated from: [16].

$$R=R0A1/3 \qquad (4)$$

With R0 = 1.2 fm. and the atomic mass A. This equation can be used to calculate the nuclear charge radii:

R2=0.0144A2/3 barn

Deformation parameter (β2) is so related to reduce transition probability B(E2) via a formula [1].

The equation below can be used to describe the intrinsic quadruple moment of a uniformly charged ellipsoid[18,14,21]:

$$\beta 2 = 4\pi \,/\, 3ZRo\ 2\ [(E2) \uparrow e\ 2\ b\ 2/\ e\ 2\ ]\ ½ \qquad (5)$$

$$Q_0 = \frac{2Z}{5}(b^2 - a^2) \qquad (6)$$

Where a and b, the minor and major oval axes and used to express the quadrilateral distortion index δ [19,22]:

$$\delta = 0.3(b^2 - a^2) \,/\, 2(< r^2 \ >) \qquad (7)$$

Where the average of the mean-squared charge distribution radius is:

$$< r^2 >= b^2 - 2a^2/5 \qquad (8)$$

It is possible to determine the values of the nucleus distortion parameter via:

$$\delta = 0.75\ Qo \,/\, (< r2\ >\ Z) \qquad (9)$$

From (equation.5) and (equation.9) down argumentation are connected as [23,24,6].

$$\delta = 0.946\beta_2 \qquad (10)$$

Value of $< r^2 >$ was evaluated using the following expressions [25,18]:

$$< r2 > = 0.63Ro2\ (1+10/3(\pi\ a0 \,/\, Ro)\ 2\ ) \,/\, (1 + (\pi\ ao \,/\, Ro)\ 2\ )\ (A\leq 100) \qquad (11)$$

$$< r2 > = 0.63\ (1.2\ A1/3)\ 2\ (A > 100) \qquad (12)$$

which considers the impact of the surface diffusion properties on the light nuclei. The following are the potential form-factor parameters for radial Woods-Saxon [26,27]:

(Ro = 1.07 A1/3 fm) and (ao = 0.55 fm). Were derived from the fast electron scattering data.

Equations 3, 4, and 5 gave us[28,29]:

$$a = \sqrt{< r2\ > (1.66 - \frac{2\delta}{0.9})} \qquad (13)$$

$$b = \sqrt{\mathbf{5} < \boldsymbol{r2}\ > -2a^2} \qquad (14)$$

The ellipsoid axis (a,b)' major and minor variance may be written as: [30,22]:

$$\Delta R = \delta * R_o \text{ or } \Delta R = b - a$$

3. Results and Discussion

Several coefficients were calculated for multiples of an nuclei with the numbers of mass greater than or equal to 100 (A≥100) for elemental cadmium (Cd-48) and its isotopes, Which are

(^{100}Cd,^{102}Cd,^{104}Cd,^{106}Cd,^{108}Cd,^{110}Cd,^{112}Cd,^{114}Cd,^{116}Cd,^{118}Cd,^{120}Cd,^{122}Cd,^{124}Cd) in this research, these parameters are required for our study:

Table 1. The number of mass for cadmium isotopes (A), a number of neutrons (N), the energy of gamma for first level E^{γ}, average nuclear radius (R_0^2), low electrical transition potential (B (E2) ↑ in e2 b2 unit, moment Tetrapolar electrode (Q_o) in barn unit, and the parameters of deformation (β_2, δ) to Cd-48.

A	**N**	**E_γ (KeV)**	**Theoretical Value**		**Present Work**				
			B(E_2) ↑( )e^2 b^2(for SSANM)[14]	**β_2**	**δ**	**β_2**	**Q_o (b)**	**$B(E2)$ ↑ (e^2b^2)**	**R_0^2**
100	52	1004.5	0.157	0.1115	0.1118	0.1480	1.6682	0.2768	**31.02378**
102	54	776.55	257 0.	0.1407	0.1250	0.1650	1.8848	0.3534	**31.43620**
104	56	658	0.366	0.1658	0.1335	0.1758	2.0343	0.4117	**31.84570**
106	58	632.66	0.478	0.1871	0.1339	0.1759	2.0615	0.4227	**32.25217**
108	60	632.97	0.577	0.2030	0.1317	0.1726	2.0482	0.4173	**32.65665**
110	62	657.7622	0.634	0.2102	0.1371	0.1663	1.9970	0.3967	**33.05905**
112	64	617.516	0.6320	0.2073	0.1291	0.1685	2.0487	0.4175	**33.45812**
114	66	558.456	0.617	0.2025	0.1336	0.1741	2.1416	0.4562	**33.85610**
116	68	513.50	0.600	0.1974	0.1372	0.1784	2.2205	0.4905	**34.25058**
118	70	487.77	0.581	0.9120	0.1387	0.1800	2.2654	0.5105	**34.64264**
120	72	505.9	0.484	0.1733	0.1341	0.1738	2.2120	0.4867	**35.03337**
122	74	569.45	0.359	0.1476	0.1246	0.1611	2.0735	0.4277	**35.42154**
124	76	613.2	0.250	0.1219	0.1183	0.1528	1.9873	0.3929	**35.80825**

The quaternary electrical moments B(E2) of some elements are shown in **Table 1.** We can see that these values vary depending on their mass number—the number of neutrons and protons—and that when values B(E2) approach the magic number, they fall short of other elements for the same isotope. As a result, the distortion values (β_2) are also low. On the contrary, we notice from **Table 1**, that the largest value of the distortion coefficient (β2) to ($_{48}$Cd ^{118}) is equal to ($\beta 2 = 0.1800$), and the minimum value for the deformation coefficient for ($_{48}$Cd^{100}) is ($\beta 2 = 0.1480$). Residual values of β2 range between these two values, and thus we notice that the closer the number of Z&N to the magic numbers, the more stable the nuclei are. We also notice from **Table 1** that the distortion parameter of the values (δ) is as large as possible at ($_{48}$Cd ^{118}) equal to ($\delta = 0.1387$) and as low as possible at ($_{48}$Cd^{100}) equal to ($\delta = 0.1118$) and the remaining values are between these two values.

Table 2. the mass of the number (A), the number of neutrons (N), the gamma energy of the ground level, the transition probability (T) and the average half-life (τ (s)) of Cadmium (Cd) isotopes.

A	**N**	**E_i(kev)**	**E_γ(kev)**	**$t_{1/2}$(s)**	**T(s)**	**τ (s)**
100	52	1004.5	1004.5	49.1	70.8514	0.0141
102	54	776.55	776.55	(5.5m)330	476.1905	0.0021
104	56	658	658	(57.7m)3462	$4.9957*10^{3}$	$2.0017*10^{-4}$
106	58	632.64	632.66	(7.27 ps)$7.27*10^{-12}$	$1.0491*10^{-11}$	$9.5323*10^{10}$
108	60	632.986	632.97	(6.86ps)$6.86*10^{-12}$	$9.8990*10^{-12}$	$1.0102*10^{11}$
110	62	657.7638	657.7622	(5.39 ps)$5.39*10^{-12}$	$7.7778*10^{-12}$	$1.2857*10^{11}$

112	64	617.520	617.516	(6.51ps) $6.51*10^{-12}$	$9.3939*10^{-12}$	$1.0645*10^{11}$
114	66	558.456	558.456	(10.2ps) $10.2*10^{-12}$	$1.4719*10^{-11}$	$6.7941*10^{10}$
116	68	513.490	513.50	(14.1ps) $14.1*10^{-12}$	$2.0346*10^{-11}$	$4.9149*10^{10}$
118	70	487.77	487.77	(50.3m) 3018	$4.3550*10^{3}$	$2.2962*10^{-4}$
120	72	505.9	505.9	50.80	73.3045	0.0136
122	74	569.45	569.45	5.24	7.5613	0.1323
124	76	613.33	613.2	1.25	1.8038	0.5544

In **Table 2**, we note that the highest value of transition potential T(s) is equal to $4.9957 * 10^3$ at ^{104}Cd where N = 56 and the lowest value of T(s) is $7.7778 * 10^{-12}$ at ^{110}Cd and the highest value of the average half-life is $9.5323 * 10^{10}$ at ^{106}Cd. The lowest value is $2.0017*10^{-4}$ at ^{102}Cd, and the highest value for the gamma energy is 1004.5 at ^{100}Cd, which is from the even-even nuclei with O^+ spinning. Also, the gamma energy values shown in the above table were relied on for the purpose of calculating the electrical transmission probability B(E2), and through (B(E2)[23,24], it was calculated the deformation coefficients (β_2, δ) through which the shape of the nuclei was determined, depending on the deformation equation on which it was based on our current study.

It was discovered by examining the rms values of the radius $\langle r^2 \rangle^{1/2}$ in **Table 3** that these values rise with increasing mass number (A). By comparing the current values with the experimental values, it was found that a calculated value for $\langle r^2 \rangle^{1/2}$ (P.w) agrees with the experimental value of $\langle r^2 \rangle$ from references [25].

These findings confirm that when the number of nucleons approaches the magic number, the nuclei become more stable and close to spherical, and the motion of the nucleons in the sub-shell is a vibration about spherical. Figure 2 depicts the shapes of these nuclei.

Table 3. The mass of the number (A), the number of neutrons (N), the mean square root of the radius $< \mathbf{r^2} >^{\mathbf{1/2}}$, the minor and major axes (b,a) and the difference between them (ΔR) in two ways for Cadmium (Cd-48) isotopes.

A	**N**	**Theoretical Value**	**Present Work**						
		r2⟩1/2 fm [25]	**$\langle r^2\rangle$ fm**	**⟩1/2 fm**	**a (fm)**	**b (fm)**	**ΔR_1**	**ΔR_2**	**ΔR_3**
100	52	---	23.3038	4.8274	2.6165	3.2319	0.5555	0.6155	0.7799
102	54	4.4810	23.5578	4.8536	2.5964	3.2842	0.6250	0.6879	0.8754
104	56	4.5122	23.8101	4.8795	2.5855	3.3208	0.6718	0.7353	0.9388
106	58	4.5383	24.0605	4.9051	2.5915	3.3308	0.6780	0.7392	0.9453
108	60	4.5577	24.3093	4.9304	2.6029	3.3320	0.6708	0.7291	0.9334
110	62	4.5765	24.5563	4.9554	2.6191	3.3253	0.6515	0.7062	0.9045
112	64	4.5944	24.8018	4.9801	2.6214	3.3402	0.6657	0.7188	0.9223
114	66	4.6087	25.0457	5.0045	2.6182	3.3634	0.6832	0.7452	0.9585
116	68	4.6203	25.2880	5.0287	2.6169	3.3834	0.7160	0.7666	0.9881
118	70	4.6246	25.5289	5.0526	2.6200	3.3963	0.7277	0.7763	1.0023
120	72	4.6300	25.7683	5.0762	2.6358	3.3892	0.7079	0.7534	0.9732
122	74	---	26.0062	5.0996	2.6623	3.3650	0.6611	0.7027	0.9073
124	76	---	26.2428	5.1227	2.6816	3.3514	0.6313	0.6698	0.8648

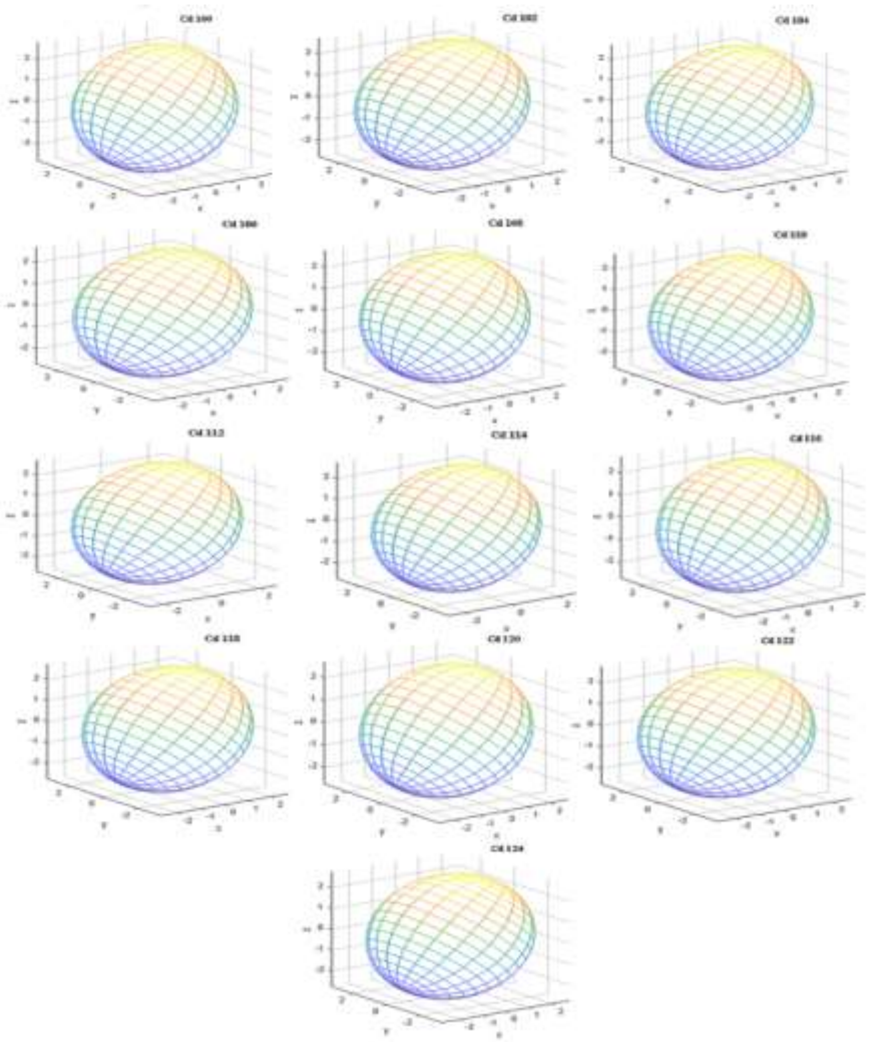

Figure 2. The 3-D Shapes for the deformation of quadrupole for $_{48}$Cd isotope from a (major) and b (minor) axes

5. Conclusion

We conclude from the results and numbers mentioned above that the first excited state energy (2+) of these nuclei starts to change progressively with the emergence of the mass numbers A, and the nucleons are outside the core. The core is in one direction, and the cores can be of stable and non-spherical shape with permanent deformation.

This means that the movement of the nuclei will be outside the closed envelope when the nuclei (protons and neutrons) are far from the magic numbers. Thus, the rotational motion and the nucleus are more deformed.

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Conflict of Interest

The authors declare that they have no conflicts of interest.

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