Ibn Al-Haitham Journal for Pure and Applied Sciences
Journal homepage: jih.uobaghdad.edu.iq
PISSN: 1609-4042, EISSN: 2521-3407
IHJPAS. 202*, ** (**)

Centralizer on Lie-ideal of Semi-prime Inverse Semi-ring

Ali JA. Abass[1*], Abdulahman H. Majeed[2], Mohammed Yasin[3] and Shrooq Bahjat Smeein[4]

[1] Department of Mathematics, College, of Science, University of Baghdad, Baghdad, Iraq.
[2] Department of Mathematic , Al-Mamoun University College, Baghdad, Iraq.
[3] Department of Mathematics, An-Najah National University, Nablus P400, Palestine.
[4] Information Department, Section Mathematics, University of Technology and Applied Science - Muscat, Sultanate of Oman.
*Corresponding Author.

Received: 10 May 2023 Accepted: 27 August 2023 Published: 20 January 2025
doi.org/10.30526/38.1.3482

Abstract

The summary purpose of this work: We extending certain results on α-centralizer of inverse semiring under specific conditions, achieve new results on lie ideal of inverse semiring with some consequent collieries, generalize assorted α-centralizer for lie ideal of inverse semiring with some collieries, investigate significant theorems on jordan α-centralizer of prime inverse semiring and we extend certain results of $\alpha -$centralizers and jordan $\alpha -$centralizers on lie-ideals of prime semi-rings to prime inverse semi-ring, we generalizing the results of Mary in to α-centralizer on semiring, Also we generalize our results on lie ideals of inverse semiring. We extending the results of Shafiq, Aslam, Javed to $\alpha -$ centralizer of Inverse semiring. $since$ R is left (right) Jordan $\alpha -$ centralizer on V, we get the output R is a left (right) $\alpha -$ centralizer on V. If it where α is an automorphism of V,$R(u) \in V$, for any $u \in V$, and $\alpha\big(Z(V)\big) = Z(V)$. We also get the following output R is a $\alpha -$ centralizer on V.

Keywords: Lie-ideal, prime inverse semi-ring, semi-prime inverse semi-ring, $\alpha -$centralizer, jordan α-centralizer.

1. Introduction

Let M be a non-empty set with binary operation $(\bullet)$ defined on M, then $(M,\bullet)$ is named semi $-$ group iff $k \bullet (s \bullet t) = (k \bullet s) \bullet t$ for any $k, s, t \in M$(1), a semi $-$ group M is named commutative semi $-$ group if $k \bullet s = s \bullet k$, holds $for\ all\ k, s \in M$ (1), A non $-$ empty set with two $-$ binary operations$(+)$ and $(\bullet)$ is named semi-ring iff the following requirements hold:

i) $(M, +)$ is commutative semi $-$ group.

ii) $(M,\bullet)$ semi − group.

iii) $a \bullet (k + s) = a \bullet k + a \bullet s$ and $(k + s) \bullet a = k \bullet a + s \bullet a$ *for all* $a, k, s \in M$ (2), $(M, +)$ is named additive commutative with neutral element 0. (i. e. *for all* $k \in M$, $k + 0 = 0 + k = k$) iff $k + s = k + n$ holds for any $k, s \in M$, and $(M,\bullet)$ is a semi − group with zero 0, *i.e.*, $0.a = a.0 = 0$ for any $a \in M$. A semi − ring $(M,+,\bullet)$ is named commutative iff $k \bullet s = s \bullet k$ holds for any $k, s \in M$ (2), Let $(M, +, \bullet)$ be an additively commutative semi-ring. Then M is named inverse semi-ring, if $(M, +)$ is an inverse semi-group (i.e) for each $k \in M$ there are a unique $k' \in M$ such that, $k = k + k' + k$ and $k' + k + k' = k'$ (2), and is called cancellative semi − ring iff for any $k, s, m \in M$, such that $k + s = k + m$, then $s = m$. A semi-ring M is named prime semi-ring if for any $k, s \in M$, $k M s = 0$ implies that either $k = 0$ *or* $s = 0$. A semi − ring M is named a semi-prime if for any $k \in M$, $k M k = 0$ mplies that $k = 0$. (3), A semi-ring M is named q − torsion free where $q \neq 0$ is an integer if whenever $qk = 0$ with $k \in M$, then $k = 0$. A commutator $[.,.]$ in inverse semi − rings defines as $[k, s] = ks + ks'$ *and*, $k \, o \, s = ks + ks$ (3). In (4) Albas presented the α − centralizer concept and the Jordan α −centralizer concept, which could be a generalization of Jordan centralizer and centralizer and tried beneath particular requirements on a 2 −torsion free semi − prime ring, each Jordan α-centralizer is α centralizer, where α could be a surjective homomorphism. Inverse semi-rings considered in different directions by numerous authors, see (5-12). In this work our aim is to consider the results of Majeed and Meften (13) in the inverse semi-ring. In this article, M will represent additive inverse semi-ring that satisfies the requirement that for any $r \in M$, $k + \acute{k}$ is located in the center Z(M) of M .

2. Preliminaries

We recalled the definitions of lie − ideal, square closed Lie − ideal of a semiring M, and some definitions, lemmas that will be used later.

Definition (2.1):(14)

An additive sub semi − group of inverse semi − ring M satisfies $[n, q] = nq + q'k \in V$ for any $k \in V$, $q \in M$, is named a Lie-ideal of M .

Definition (2.2):(14)

Let V be a lie − ideal of a ring, then V is named a squane closed Lie − ideal of M if $k^2 \in V$ *for all* $k \in V$.

Note that if V is a square closed Lie-idealof M, then $2kq \in V$ for any $k, q \in V$.

Definition (2.3):(2), (15)

Let I be a nonzero ideal of M, the set $Z(I) = \{k \in I, kq = qk,$ for any $q \in I\}$ is named the center of I.

Definition (2.4):(2), (16)

Let $q \in M$, the set $Z(M) = \{k \in M, kq = qk, for\ all\ q \in M\}$ is named the center of the semi − ring M. Clearly that $Z(M)$ is a subsemi − ring of M.

Note that if M is multiplicatively commutative then $Z(M) = M$.

Lemma (2.5):(10), (17)

Let M be an additive inverse semi-ring, for any $k, q \in M, if\ k + q = 0$ then $k = q'$. Note that in general $k + k' \neq 0$, $k + k' = 0$, iff there are some $q \epsilon M$ with $k + q = 0$ [2]

Proposition (2.6):(12),(18)

For any $r, s \in M$, the following are holds:

i. $(k + q)' = k' + q'$
ii. $(k\,q)'' = k'q = kq'$
iii. $k'' = k$
iv. $k'q' = (k'q)' = (kq)'' = kq.$

Lemma (2.7):(12),(19)

Let M be ring and $k, q, w \in M$ then

i. $[k,k] = 0$
ii. $[k + q,\ w] = [k,\ w] + [q,\ w]$
iii. $[kq,\ w] = k[q,\ w] + [k,\ w]q$
iv. $[k,qw] = q[k,w] + [k,q]w.$

Definition (2.8):(15),(20)

Let M be a semi-ring,an additive mapping $R: M \to M$ is nameda $(\alpha,\alpha)-$ derivation $if\ R(kq) = R(k)\alpha(q) + \alpha(k)R(q)$ for any $k,q \in M$, and we say that R is Jordan $(\alpha,\alpha) -$ derivation if $R(k^2) = R(k)\alpha(k) + \alpha(k)R(k)$ for any $k \in M$,where α be additive mapping on M.

Every derivation is $(\alpha,\alpha) -$ derivation is Jordan $(\alpha,\alpha) -$ derivation, but the converse in general is not true.

Definition (2.9):(3),(21)

A left (right)$\alpha -$ centralizer of a semi-ring M is an "additive mapping" $R: M \to M$ which satisfies $R(kq) + R(k)\alpha(q)' = 0, (R(kq) + \alpha(k)'R(q) = 0)$ for any $k,q \in M$. $\alpha -$centralizer of a ring M is both left and right $\alpha -$ centralizer , where α is an additive mapping on M .

Definition (2.10):(3),(22)

A left (right) Jordan $\alpha -$ centralizer of a semi-ring M is an addittive mapping $R: M \to M$ which satisfy $R(k^2) + R(k)\,\alpha(k)' = 0, (R(k^2) + \alpha(k)'R(k) = 0)$ for any $k \in M, \alpha -$ Jordan centralizer of a ring M is both left and right Jordan $\alpha -$ centralizer, where α be additive mapping on M .

3. Main Results

To verify our main results, we must utilize the following.

Lemma (3.1):(4),(23)

If $V \not\subset Z(M)$ is a Lie-ideal of a $2 -$ tortion free prime semirig $M\ and\ k,q \in M$ such that $k\,V\,q = 0$, then $k = 0$ or $m = 0$.

From this we mean by V is a square closed lie $-$ ideal of M.

Lemma (3.2)

Let M be a $2-$tortion free prime semi-ring. Suppose that $F, G : VxV \to V$ biadditive mappings. If $F(k,q)\, w\, G(k,q) = 0$ for any $k,q,w \in V$, then $F(k,q)\, w\, G(u,v) = 0$ for any $k,q,u,v,w \in V$.

Proof:

$$F(k,q)\, w\, G(k,q) = 0 \qquad for\ all\ k,q,w \in V \qquad (*)$$

Replace k with $k + u$, we have

$$F(k + u,q)\, w\, G(k + u,q) = 0 \qquad for\ all\ k,q,w,u \in V$$

By using the additive of F and G

$$F(k,q)\, w\, G(u,q) = F(u,q)'\, w\, G(k,q)$$

Replace w by $2^4\, F(k,q)\, z\, G(u,q)$

$$(F(k,q) w\, 2^4 G(u,q))\, z\, F(k,q)\, w\, G(u,q) =$$
$$F(u,q)'\, w\, 2^4 G(u,q)\, z\, F(k,q)\, w\, G(k,q) = 0$$

by (*), we get

$$2^4 F(k,q) w G(u,q) z F(k,q) w G(u,q) = 0 \qquad for\ all\ k,q,u,z \in V \qquad (**)$$

If $V \not\subset Z(M)$, by Lemma(3.1), we get

$$F(k,q)\, w\, G(u,q) = 0 \qquad for\ all\ k,q,u,w \in V$$

If $V \subset Z(M)$, multiply the relation (**) from the right by zt, where $t \in M$, we get

$$2^4 F(k,q) w\, G(u,q)\, z\, t\, F(k,q) w G(u,q) z = 0, \quad for\ all\ k,q,u,z,w \in V, t \in M$$

Since M is $2-$tortion free prime semi-ring, we have

$$F(k,q)\, w\, G(u,q)\, z = 0 \qquad for\ all\ k,q,u,z,w \in V$$

If we multiply the relation by t an element of M, which is prime, and do a right multiplication, the result is

$$F(k,q)\, w\, G(u,q) = 0 \qquad for\ all\ k,q,u,w \in V$$

We can acquire the lemma's claim by exchanging $q\ for\ q + v$, in a way analogous to the one used above.

Theorem (3.3)

Let M be $2-$tortion free prime semi-ring. If R is left (right) Jordan $\alpha-$centralizer on V, then R is a left (right) $\alpha-$ centralizer on V.

Proof:

$$R(k^2) + R(k)'\alpha(k) = 0 \qquad for\ all\ k \in V \qquad (1)$$

we replace k by $k + q$ when k,q in U, we get

$$R((k+q)^2) = R(k + q)\alpha(k + q)$$
$$R(k^2 + kq + qk + q^2) = R(k^2) + R(kq + qk) + R(q^2)$$
$$= R(k)\alpha(k) + R(kq + qk) + R(q)\alpha(q)$$
$$R(k+q)\alpha(k+q) = R(k)\alpha(k) + R(k)\alpha(q) + R(q)\alpha(k) + R(q)\alpha(q)$$

We get

$$R(kq+qk) + R(k)\alpha(q)' + R(q)\alpha(k)' = 0\ for\ all\ k,q \in V \qquad (2)$$

By replacing q with $2(kq + qk)$ and using (2), we get

$$2R(k(kq + qk) + (kq + qk)k) + 2R(k)\alpha(kq)' + 2R(k)\alpha(qk)' + R(kq + qk)\alpha(k)'$$
$$= 0$$
$$2R(k(kq + qk) + (kq + qk)k) = 2R(k)\alpha(kq) + 2R(k)\alpha(qk) + 2\, R(kq+qk)\alpha(k)$$

(3)

This can also be computed using an alternate way

$2R(k^2q + qk^2) + 4R(kqk) + 2R(k)\alpha(kq)' + 2R(q)\alpha(k^2)' = 0 \; for\ all\ k, q \in V$ (4)

From (3) and (4), we obtain

$R(kqk) + R(k)\alpha(qk)' = 0 \quad for\ all\ k, q \in V$ (5)

If we linearize (5), we get

$R(kqt + tqk) + R(k)\alpha(qt)' + R(t)\alpha(qk)' = 0 \quad for\ all\ k, q, t \in V$ (6)

Since V is a square closed Lie-ideal, we have

$$2^4(kqtqk + qktkq) \in V.$$

Now we shall compute $f = 2^4R(kqtqk + qktkq)$ in two different ways, using (5) we have

$f + 2^4R(k)\alpha(qtqk)' + R(q)\alpha(ktkq)' = 0 \quad for\ all\ k, q, t \in V$ (7)

Using (6) we have

$f + 2^4R(kq)\alpha(tqk)' + R(qk)\alpha(tkq)' = 0 \quad for\ all\ k, q, t \in V$ (8)

Comparing (7) and (8)

$R(k)\alpha(qtqk)' + R(q)\alpha(ktkq)' + R(kq)\alpha(tqk) + R(qk)\alpha(tqk) = 0$

$(R(kq) + R(k)\alpha(q)')\alpha(tqk) + (R(qk) + R(q)\alpha(k)')\,\alpha(tkq) = 0$

Introducing a additive mapping,

$$G(k, q) = R(kq) + R(k)\alpha(q)',$$

we arrive at

$$G(k, q)\alpha(tqk) + G(q, k)(tkq) = 0$$

By Lemma (2.5)

$G(k, q)\alpha(tqk) = G(q, k)'\alpha(tkq)$ (9)

We can be rewritten equality (2)in this notation as

$$G(k, q) + G(q, k)' = 0.$$

Using equality (9) and this fact, we obtain

$G(k, q)\alpha(\,t\,[k, q]) = 0 \quad for\ all\ k, q, t, z \in V$ (10)

Now using Lemma (3.2), we have

$G(k, q)\alpha(\,z\,[u, v]) = 0 \quad for\ all\ k, q, z, u, v \in V$ (11)

(i) If V is non commutative

Since α is surjective and using Lemma (3.1), we have

$$G(k, q) = 0 \quad for\ all\ k, q \in V$$

(ii) If V is commutative and $V \not\subset Z(M)$

Compute $N = 2^4\,R(kqzqk)$ in two different ways. Using (5), we have

$N + 2^4\,R(k)'\alpha(qzqk) = 0 \quad for\ all\ k, q, z \in V$ (12)

$N + 2^4\,R(km)'\alpha(zmk) = 0 \quad for\ all\ k, q, z \in V$ (13)

From (12) and (13), we arrive at

$$R(kq)\alpha(zqk) + R(k)'\alpha(qzqk) = 0$$

$$(R(kq) + R(k)'\alpha(q))\alpha(zqk) = 0$$

$G(k, q)\alpha(zqk) = 0 \quad for\ all\ k, q, z \in V$ (14)

Let $\psi\,(k, q) = \alpha(qk)$, it's clear that ψ is additive mapping, therefore

$$G(k,q)\alpha(z)\psi(k,q) = 0 \qquad for\ all\ k,q,z \in V$$

Using Lemma (3.2), we have

$$G(k,q)\alpha(z)\psi(u,v) = 0 \qquad for\ all\ k,q,z,u,v \in V$$

Implies that

$$G(k,q)\alpha(zuv) = 0 \qquad for\ all\ k,q,z,u,v \in V \qquad (15)$$

Replacing $\alpha(v)$ with $2G(k,q)\alpha(z), u$sing Lemma (3.1) and M is prime semi-ring, we have

$$G(k,q)\alpha(z) = 0 \qquad for\ all\ k,q,z \in V$$

Using Lemma (3.1)

$$G(k,q) = 0 \qquad for\ all\ k,q \in V$$

(i) If $V \subset Z(M)$

Multiplying relation (15) on the right by t, where t $\in M$ and since M is a prime, we can obtain the result.

$$G(k,q) = 0 \qquad for\ all\ k,q \in V$$

If $R(k^2) + \alpha(k)'R(k) = 0$, reaching the conclusion of the theorem with the same procedure as before completes the proof.

Lemma (3.4)

Let M be a $2-$tortion free prime semi$-$ring, $H,\alpha: M \to M$, H is $(\alpha,\alpha)-$derivation on $V\ and\ a \in V$ some fixed element, where α is automorphism of V, such that $\alpha(V) = V$ then

(ii) $H(k)H(q) = 0$ for any $k,q \in U$ implies $H = 0$ on V.

(iii)$a\alpha(k) + \alpha(k)'a \in Z(V)$ for any $k \in V$ implies $a \in Z(V)$.

Proof:

(i) $H(k)\alpha(q)H(k) = H(k)H(qk) + H(k)'H(q)\alpha(k)$

$$H(k)(H(q)\alpha(k) + \alpha(q)H(k)) + H(k)'H(q)\alpha(k) = 0$$

$$H(k)H(q)\alpha(k) + H(k)\alpha(q)H(k) + H(k)'H(q)\alpha(k) = 0$$

By hypothesis, and M is inverse semi-ring, we get

$$H(k)\alpha(q)H(k) = 0$$

Since α is automorphism of V, such that $\alpha(V) = V$, we get

$$H(k)\,V\,H(k) = 0 \qquad for\ all\ k \in V$$

If $V \not\subset Z(M)$, and α is automorphism of V, Lemma (3.2) we have $H = 0\ on\ V$.

If $V \subset$ Z(M)

$$H(k)tH(k) = 0 \qquad for\ all\ k \in V, t \in M$$

So, by primness of M, we have

$$H = 0\ on\ V$$

(ii) Define $H(k) = a\alpha(k) + \alpha(k)a'$

It is easy to see that H is a $(\alpha,\alpha)-$derivations, since $H(k) \in Z(V)$ for any $k \in V$, we have $H(q)\alpha(k) = \alpha(k)H(q)$ and also $2H(qz)\alpha(k) = 2\,\alpha(k)H(qz)$

Since M is prime, we get

$$H(q)\alpha(zk) + \alpha(q)H(z)\alpha(k)$$
$$= \alpha(k)H(q)\alpha(z) + \alpha(kq)H(z)$$
$$H(q)(\alpha(z)\alpha(k) + \alpha(k)\alpha(z)') = H(z)(\alpha(q)\alpha(k)' + \alpha(k)(q))$$
$$H(q)[\alpha(z),\alpha(k)] = H(z)[\alpha(q),\alpha(k)]$$

Since α is automorphism, take $\alpha(z) = a$. Obviously $H(a) = 0$, so, we obtain by (i)

$$H(q)H(k) = 0$$

By virtue of (i) we get $H = 0$ and hence $a \in Z(M)$.

Lemma (3.5)

Let M be a $2-$tortion free prime semi $-$ ring, R and α are additive mappings on M, and $a \in V$ some fixed element. If $R(k) = a\,\alpha(k) + \alpha(k)aandR(k\ o\ q) + R(k)o\ \alpha(q)' = 0$ and $R(k\ o\ q) + \alpha(k)'oR(q) = 0$ for any $k, q \in V$ then "$a \in Z(V)$, where α is a surjective endomorphism of V .

Proof:

By hypothesis

$$R(kq + qk) = R(k)\alpha(q) + \alpha(q)R(k) \qquad for\ all\ k, q \in V$$

$$R(kq) + R(qk) = R(k)\alpha(q) + \alpha(q)R(k) \qquad for\ all\ k, q \in V$$

$$R(kq) + R(qk) = a\alpha(kq) + \alpha(kq)a + a\alpha(qk) + \alpha(qk)a$$

$$= a\alpha(k)\alpha(q) + \alpha(k)\alpha(q)a + a\alpha(q)\alpha(k) + \alpha(q)\alpha(k)a$$

$$R(k)\alpha(q) + \alpha(q)R(k) = a\alpha(k)\alpha(q) + \alpha(k)a\alpha(q) + \alpha(q)a\alpha(k) + \alpha(q)\alpha(k)a$$

$$+ a\alpha(k)\alpha(q) + \alpha(k)\alpha(q)a + a\alpha(q)\alpha(k) + \alpha(q)\alpha(k)a$$

$$= a\alpha(k)\alpha(q) + \alpha(k)a\alpha(q) + \alpha(q)a\alpha(k) + \alpha(q)\alpha(k)a$$

$$(a + a')\alpha(k)\alpha(q) + \alpha(q)\alpha(k)(a + a') + \alpha(k)\alpha(q)a + a\alpha(q)\alpha(k) + \alpha(k)a'\alpha(q) + \alpha(q)a'\alpha(k) = 0$$

Since $a + a' \in Z(V)$

$$\alpha(k)\alpha(q)(a + a' + a) + (a + a' + a)\alpha(q)\alpha(k) + a\alpha(q)\alpha(k) + \alpha(k)a'\alpha(q) = 0$$

$$\alpha(k)\alpha(q)a + \alpha(k)a'\alpha(q) + a\alpha(q)\alpha(k) + \alpha(q)a'\alpha(k) = 0$$

$$\alpha(k)(\alpha(q)a + a'\alpha(q)) + (\alpha(q)a + a'\alpha(q))\alpha(k)' = 0$$

But α is a surjective

$$a\alpha(k) + \alpha(k)a' \in Z(V)$$

By Lemma (3.4) (ii), we get $a \in Z(V)$

$$R(k\ o\ q) + R(k)o\ \alpha(q)' = 0 \text{ and } R(k\ o\ q) + \alpha(k)'o\ R(q) = 0.$$

Lemma (3.6)

Let M be a $2-$tortion free prime semi-ring, and R, α are additive mappings on M, R satisfies $R(k \text{ o } q) + R(k)\text{o}\ \alpha(q)' = 0$ and $R(k\ o\ q) + \alpha(k)'\ o\ R(q) = 0$ for any$k, q \in V$, then $R(z) \in Z(V)$ for any $z \in Z(V)$, where α is a surjective endomorphism of V.

Proof:

$$R(kq + qk) + R(k)\alpha(q)' + \alpha(q)'R(k) = 0$$

$$R(kq + qk) + \alpha(k)'R(q) + R(q)\alpha(k)' = 0.$$

because $R(z) \in Z(V)$

Take any $t \in Z(U)$ and denote $a = R(t)$

$$2R(tk) = R(tk + kt) = R(t)\alpha(k) + \alpha(k)R(t)$$

$$= a\alpha(k) + \alpha(k)a$$

A simple check reveals that $M(k) = 2R(tk)$ is satisfies

$$M(k\ o\ q) = 2R(t(kq + qk) = 2R(tkq + qtk)$$

$$= 2R(tk)\alpha(q) + 2\,\alpha(q)R(tk)$$

$$= M(k)\alpha(q) + \alpha(q)M(k)$$
$$= M(k)o\alpha(q)$$
$$M(k\ oq) = 2R(t(kq + qk) = 2R(k(tq) + (tq)k)$$
$$= 2\alpha(k)R((tq) + 2R(tq)\alpha(k)$$
$$= \alpha(k)M(q) + M(q)\alpha(k)$$
$$= \alpha(k)\ o\ M(q)$$
$$M(k\ o\ q) = M(k)\ o\ \alpha(q) = \alpha(k)\ o\ M(q)\ \ fon\ all\ k,q \in M$$

By Lemma (3.5), we have $R(t) \in Z(M)$.

Theorem (3.7)

Let M be $2-$ tortion free prime semi $-$ ring and $R, \alpha: M \to M$ additive mappings, R satisfies $R(k\ o\ q) + R(k)o\alpha(q)' = 0\ and\ R(k\ o\ q) + \alpha(k)'oR(q) = 0\ for\ all\ k,q \in V$ then R is $a\ \alpha -$ centralizer on V, where α is an automorphism of V, $R(u) \in V$, for any $u \in V$, and $\alpha\big(Z(V)\big) = Z(V)$.

Proof :

Since U is a square closed Lie $-$ ideal of M, and by Lemma (2.5), we get

$$2R(kq + qk) = 2R(k)\alpha(q) + 2\alpha(q)R(k)$$
$$= 2\alpha(k)R(q) + 2R(q)\alpha(k)$$

If V is a commutative, we have

$$R(r^2) = R(r)\alpha(r) = \alpha(r)R(r)$$

If V is a non-commutative

Replace q by $2kq + 2qk$ in (2), we get,

$$4R(k)\alpha(kq + qk) + 4\alpha(kq + qk)R(k)$$
$$= 4\alpha(k)R(kq + qk) + 4R(kq + qk)\alpha(k)$$
$$4R(k)\alpha(k)\alpha(q) + 4R(k)\alpha(q)\alpha(k) + 4\alpha(k)\alpha(q)R(k) + 4\alpha(q)\alpha(k)R(k) =$$
$$4\alpha(k)R(k)\alpha(q) + 4\alpha(k)\alpha(q)R(k) + 4R(k)\alpha(q)\alpha(k) + 4\alpha(q)R(k)\alpha(k)$$

By using the property of $2-$ tortion free semi $-$ ring, we obtain

$$R(k)\alpha(k)\alpha(q) + \alpha(q)\alpha(k)R(k) + \alpha(k)'R(k)\alpha(q) + \alpha(q)R(k)\alpha(k)' = 0$$

Now it follows that

$$[R(k),\alpha(k)]\alpha(q) = \alpha(q)[R(k),\alpha(k)] \qquad for\ all\ k,q \in V$$

but α is surjective, then we get

$$[R(k),\alpha(k)] \in Z(V) \qquad for\ all\ k,q \in V$$

The next goal is to show that $[R(k),\alpha(k)] = 0\ \ for\ all\ k \in V$.

Take any $t \in Z(U)$

$$4R(tk) = 2R(tk + kt) = 2R(t)\alpha(k) + 2\alpha(k)R(t)$$
$$= 2R(k)\alpha(t) + 2\alpha(t)R(k)$$

Using Lemma (3.6), we get

$$R(tk) = R(k)\alpha(t) = R(t)\alpha(k) \qquad for\ all\ k,t \in V$$
$$4[R(k),\alpha(k)]\alpha(t) = 4R(k)\alpha(kt) + 4\alpha(k)'R(k)\alpha(t)$$
$$= 4R(k)\alpha(tk) + 4R(k)\alpha(t)\alpha(k)' = 0$$

Since $\alpha(Z(V)) = Z(V)$, and $[R(k),\alpha(k)]$ itself is central element, By Lemma (3.1), we get our goal.

$$2R(k^2) = R(kk + kk) = R(k)\alpha(k) + \alpha(k)R(k)$$
$$= 2R(k)\alpha(k) = 2\alpha(k)R(k).$$

By Theorem 3.3, we get our result.

4. Conclusion

In this work, we extend certain results of α-centralizers and Jordan α-centralizers on lie ideals of prime rings to prime inverse semirings.

Acknowledgment

Our researcher extends his Sincere thanks to the editor and members of the preparatory committee of the Ibn AL-Haitham Journal of Pure and Applied Sciences.

Conflict of Interest

There are no conflicts of interest.

Funding

There is no funding for the article.

References

1. Vandiver S.H. Note on a simple type of Algebra in Which the Cancellation Law of Addition Does Not Hold. Bull. Amer. Math. Soc. Nature. 1934; 40*:* 914-920.
2. Karvellas P.H.: Inversive semi-rings. J. Aust. Math. Science. 1974; 18*:* 277-288.
3. Javed M.A., Aslam M., Hussain, M. On requirement (A_2) of Bandlet, Petrich for inverse semi-rings. Int. Mathematical Forum. Nature. 2012; 59(7): 2903–2914.
4. Albas E.:On τ -Centralizers of semi-prime rings. Siberian Math. J. Science. 2007; 48: 191-196.
5. Ibraheem R.KH.; Majeed A.H.: U-S Jordan Homomorphisim of inverse semi-rings. Iraqi Journal of Science. Nature**.** 2019, 60 (8):1783-1790. https://doi.org/10.24996/ijs.2019.60.8.15
6. Rasheed M.K., Hameed F.A., Majeed A.H.: On Generalized (α, β) Derivation on Prime Semi-rings. J. Phys.: Conf. Ser. 2020, 1291-1508. https://doi.org/ 10.1088/1742-6596/1591/1/012080
7. Rasheed M.K., Majeed A.H.: Some Results of (α, β) Derivations on prime semi-rings. Iraqi Journal of Science. 2019; 60(5): 1154-1160. https://doi.org/ 10.24996/ijs.2019.60.5.23
8. Zaghir K.AD.; Majeed A.H.: (α,β)-Derivationson prime inverse semi-rings. Iraqi journal of Science. 2021; 62(11): 4081-4091. https://doi.org/10.24996/ijs.2021.62.11.28
9. Zaghir K.AD.; Majeed A.H.: (α, β) - Derivations on ideals in prime inverse semi-rings. AIP Conference Proceedings. 2023; 40(1): 4031-4110. https://doi.org/10.1063/5.0117578
10. Shafiq S.; Aslam M.; Javed, M.A.: On centralizer of inverse semiring. Discussions Mathematica General Algebra, Applications. 2016; 36: 71-84.
11. Ahmed Y.; Dudek W. A.: Left Jordan derivations on certain semi-rings. Hacet. J. Math. Stat. 2021; 50(3): 624 – 633. https://doi.org/10.15672/hujms.491343
12. Javed M.A.; Aslam M.; Hussain M.: On requirement (A_2) of Bandelt and Petrich for inverse semi-rings. Int. Math. Forum**.** Science 2012; 7(59): 2903-2914.
13. Majeed A.H.; Meften M.I.: Lie-ideals, Jordan q-Centralizers of Prime Rings. Journal of Al-Nahrain University. 2011; 14(2): 173-177. https://doi.org/ 10.22401/JNUS.14.2.22

14. Ibraheem R.KH.; Majeed A.H.: On lie Structure in semi-prime inverse semi-rings. Iraqi Journal of Science. 2019; 60(12): 2711-2718. https://doi.org/10.24996/ijs.2019.60.12.21
15. DIMITROV S.: Derivations on semirings. AIP Conference Proceedings. 2017; 1910, 060011:1-22 https://doi.org/10.1063/1.5014005
16. Golan J. S. Semirings and their applications. University of Haifa, Haifa, Palestine, 1992.
17. Golan J. S. The theory of Semirings with Applications Mathematics and Theoretical Computer science. John Wiley and Sons.Inc., New York, 1992.
18. Jonathan S. G. Semirings and their applications, University of Haifa, Haifa, Palestine, 1992.
19. Joseph H. M. Centralizing mapping of prime rings. Canned. Math.Bull. 1984; 27 (1):122-126. https://doi.org/10.4153/CMB-1984-018-2
20. Mary, D., Murugensan, R. , Namasivayam P. Centralizers on Semiprime Semirings. IOSR Jornal of Mathematics. 2016; 12(3): 86-93. www.iosrjournals.org
21. Sara A. , Aslam. M. on Li Ideal of Inverse Semirings, Italian Journal of Pure and Applied Mathematics. 2020; 44: 22-29.
22. Sultana, K.A. Some Structural Properties of Semirings, Annals of Pure and Applied Mathematics. 2014; 5(2):158-167.
23. Ibrahim, R. K. on additive mappings of inverse semirings, M.Sc. Thesis. University of Baghdad, Collage of science department, 2019.