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Common Fixed Points of Three Multivalued Nonexpansive Random Operators for One Step Iterative Scheme

Wisam Mohammed Mukhlif [1*], Sabah Hassan Malih[2]
and Shrooq bahjat Smeein[3]

[1]Department of Mathematics, College of Education for Pure Science (Ibn- ALHaitham), University of Baghdad, City Baghdad, Iraq.
[2]Department of Mathematics, College of Education for Pure Science (Ibn- ALHaitham), University of Baghdad, City Baghdad, Iraq.
[3]Information Department, Section Mathematics Sultanate of Oman.
*Corresponding Author.

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Abstract
In this paper, we introduce a new one-step iteration process in Banach space and prove the existence of a common random fixed point of three non-expansive multivalued random operators through strong and weak convergences of an iterative process. The necessary and sufficient condition for the convergence of a sequence of measurable functions to a random fixed point of non-expansive multivalued random operators in uniformly convex Banach spaces is also established. Our random iteration scheme includes new random multivalued iterations as special cases. The results obtained in this paper are an extension and refinement of previously known results. A new random iterative scheme for approximating random common fixed points of three random non-expansive multivalued random operators is defined and we have proved weak and strong convergence theorems in a uniformly convex Banach space.
Keywords: common fixed points, random operators, one-step iteration, Banach spaces.

1. Introduction
The random fixed point theories are a generalizations of the classical fixed point theories. In the 1950s the Probability School in Prague presented a study on random fixed point theory [1].On other hand,[2] obtain common random fixed point of two multivalued random operator .Recently, S. H. Khan et al.[3]introduce a new one-step iterative process to find the common random fixed point of two multivalued Non-expansive random operator. ,the nonlinear random systems have appeared in the literature (see [4-17]) . Let $\varPhi$ be a separable banach space . Let$\varUpsilon$ is subset of $\varPhi$is called proximinal if $\forall u \in \varPhi, \exists k \in \varUpsilon \ni d(u,k) = \inf\{\|u - v\| : v \in \varUpsilon\} = d(u,\varUpsilon)$. We denote the set of all subsets bounded proximinal of$\varUpsilon$ by $\varPi(\varUpsilon)$ [18] .
let $CB(\varUpsilon)$ be be all closed boundedsubsets of $\varUpsilon$. And let His Hausdorff induced by the metric space d of $\varPhi$, implies $H(\varpi, \beta) = max\{sup\, d(u,\beta)_{u\in\varpi}, sup\, d(v,\varpi)_{v\in\beta}\}$ for every $\varpi, \beta \in CB(\varUpsilon)$.
A multivalued random operator $B: \Psi \times \varUpsilon \to \varPi(\varUpsilon)$is called contraction if there is$k(\varrho) \in [0,1)$ and

for each $\varrho \in \Psi \ni$
$\mathrm{H}(B(\varrho,u),B(\varrho,v)) \leq k(\varrho)\|u-v\|$for all $u,v \in \Upsilon$
and B is said to be nonexpansive random operator if
for each $\varrho \in \Psi$
$\mathrm{H}\big(B(\varrho,u),B(\varrho,v)\big) \leq \|u-v\|$, a point $\vartheta(\varrho)$ is called random fixed point of B if $\vartheta(\varrho) \in B\big(\vartheta(\varrho)\big)$.
Study the fixed point of results paper [22-30] under multivalued non-expansive random operator.
We will give some definition …

Definition 1.1.[19] : A separable Banach space Φ is said to satisfy opials condition if the sequence $\{S_n\}$ in $\Phi, S_n \to u$ implies that $\lim\limits_{n\to\infty} sup\|S_n-u\| < \lim\limits_{n\to\infty} sup\|S_n-v\|$ for all $v \in \Phi$, $v \neq u$.

Definition 1.2. Let Υ subset of Φ, and $\mathcal{R}, \mathrm{Z}, B : \Psi \times \Upsilon \to \Pi(\Upsilon)$Three multivalued nonexpansive random operator are satisfy the condition (A') if $\exists$a non-decreasing function $\chi: [0,\infty) \to [0,\infty)$ with $\chi(0)=0, \chi(r)>0, \forall r \in (0,\infty) \ni$ either $d\big(u(\varrho), B\,u(\varrho)\big) \geq \chi\big(d(u(\varrho),F)\big)$or $d\big(u(\varrho), \mathrm{Z}u(\varrho)\big) \geq \chi\big(d(u(\varrho),F)\big)$or $d\big(u(\varrho), \mathrm{R}u(\varrho)\big) \geq \chi\big(d(u(\varrho),F)\big)$
For all $u(\varrho) \in \Upsilon$

Definition 1.3 let Υ be a non empty subset of a separable banach space Φsatisfying opials condition and $B : \Psi \times \Upsilon \to \Pi(\Upsilon)$ be amultivalued random operator is said to be demiclosed at $\zeta(\varrho)$) if $\{S_n(\varrho)\}$ and $\{\zeta_n(\varrho)\}$ are two sequences $\ni$ $\{S_n(\varrho)\}$ converges weak to $S(\varrho)$also $\{B((\varrho), S_n(\varrho)\}$ converges to $\zeta(\varrho)$ imply that $S(\varrho) \in \Upsilon$ and $\zeta(\varrho) \epsilon B(S(\varrho))$ for each $\varrho \in \Psi$, then $I-B$ is dimiclosed with respect to 0 .

Lemma 1.4[20] Let $\{S_n\}, \{\eta_n)\}$, $\{\rho_n\}$ be a sequence in uniformly convex banach space Φ. Let $\{\alpha_n\}, \{\kappa_n\}, \{\sigma_n\}$ are sequence in [0,1] with $\alpha_n+\kappa_n+\sigma_n=1$, $\lim\limits_{n\to\infty} sup\|S_n\| = q$, $\lim\limits_{n\to\infty} sup\|\eta_n\| = q$, $\lim\limits_{n\to\infty} sup\|\rho_n\| = q$ and $\lim\limits_{n\to\infty} \|\alpha_n S_n + \kappa_n \eta_n + \sigma_n \rho_n\| = q$ if $\lim\limits_{n\to\infty} \inf \alpha_n > 0$, $\lim\limits_{n\to\infty} \inf \kappa_n > 0$, $\lim\limits_{n\to\infty} \inf \sigma_n > 0$,
Then $\lim\limits_{n\to\infty} \|S_n - \eta_n\| = \lim\limits_{n\to\infty} \|S_n - \rho_n\| = \lim\limits_{n\to\infty} \|\rho_n - \eta_n\| = 0$.

2. Main Results

I**Definition***2.1 Let Φ be a* separable Banach *space and* $\Upsilon \neq \emptyset$ closed *subset and* convex .we can be written
$F = F(B) \cap F(\mathcal{R}) \cap F(\mathrm{Z})$ the set of all common random fixed point of the $\mathcal{R}, \mathrm{Z}, B : \Psi \times \Upsilon \to \Pi(\Upsilon)$ be three multivalued non-expansive random operator
with common random fixed point $\vartheta(\varrho)$. The iterations are as follows :-
$S_0(\varrho) \epsilon \Upsilon$
$$S_{n+1}(\varrho) = \alpha_n \eta_n(\varrho) + \kappa_n \rho_n(\varrho) + \sigma_n \xi_n(\varrho)\ , n \in N \tag{2.1}$$
where $\eta_n(\varrho) \in B\big(S_n(\varrho)\big)$ and $\rho_n(\varrho) \in \mathrm{Z}\big(S_n(\varrho)\big)$ such that $\|\eta_n(\varrho) - \eta_{n+1}(\varrho)\| \leq \mathrm{H}\Big(B\big(S_n(\varrho)\big), B\big(S_{n+1}(\varrho)\big)\Big) + \mu_n$and $\|\rho_n(\varrho) - \rho_{n+1}(\varrho)\| \leq \mathrm{H}\Big(\mathrm{Z}\big(S_n(\varrho)\big), \mathrm{Z}\big(S_{n+1}(\varrho)\big)\Big) + \mu_n$ and $\xi_n(\varrho) \in R\big(S_n(\varrho)\big)\|\xi_n(\varrho) - \xi_{n+1}(\varrho)\| \leq \mathrm{H}\Big(\mathcal{R}\big(S_n(\varrho)\big), \mathcal{R}\big(S_{n+1}(\varrho)\big)\Big) + \mu_n$
and $\{\alpha_n\}, \{\kappa_n\}, \{\sigma_n\}$ are sequence in (0,1) satisfying $\alpha_n + \kappa_n + \sigma_n = 1$.

Lemma 2.2 *Let Φ,$\Upsilon \neq \emptyset$,and $\mathcal{R}, \mathrm{Z}, B : \Psi \times \Upsilon \to \Pi(\Upsilon)$, Let $\{S_n(\varrho)\}$issequence defined in 2.1. If $F(B) \neq \emptyset$ and $B_{\vartheta(\varrho)} = \mathcal{R}_{\vartheta(\varrho)} = \mathrm{Z}_{\vartheta(\varrho)} = \{\vartheta(\varrho)\}$ for any $\vartheta(\varrho) \in F$
then $\lim\limits_{n\to\infty} \|x_n - \vartheta(\varrho)\|$ exists $\forall \vartheta(\varrho) \in F$.*

Proof. *let* $F \neq \emptyset$. *Let* $\vartheta(\varrho) \in F$. *Then*

$$\|S_{n+1}(\varrho) - \vartheta(\varrho)\| = \|\alpha_n\eta_n(\varrho) + \kappa_n\rho_n(\varrho) + \sigma_n\xi_n(\varrho) - \vartheta(\varrho)\|$$
$$= \|\alpha_n(\eta_n(\varrho) - \vartheta(\varrho)) + \kappa_n(\rho_n(\varrho) - \vartheta(\varrho)) + \sigma_n(\xi_n(\varrho) - \vartheta(\varrho))\|$$
$$\leqslant \alpha_n\|\eta_n(\varrho) - \vartheta(\varrho)\| + \kappa_n\|\rho_n(\varrho) - \vartheta(\varrho)\| + \sigma_n\|\xi_n(\varrho) - \vartheta(\varrho)\|$$
$$\leqslant \alpha_n d\left(\eta_n(\varrho), B(\vartheta(\varrho))\right) + \kappa_n d\left(\rho_n(\varrho), \mathrm{Z}(\vartheta(\varrho))\right) + \sigma_n d\left(\xi_n(\varrho), R(\vartheta(\varrho))\right)$$
$$\leqslant \alpha_n H(BS_n(\varrho), B\vartheta(\varrho)) + \kappa_n H\left(\mathrm{Z}\left(S_n(\varrho)\right), \mathrm{Z}(\vartheta(\varrho))\right)$$
$$+\sigma_n H\left(R(S_n(\varrho)), R(\vartheta(\varrho))\right)$$
$$\leqslant \alpha_n\|S_n(\varrho) - \vartheta(\varrho)\| + \kappa_n\|S_n(\varrho) - \vartheta(\varrho)\| + \sigma_n\|S_n(\varrho) - \vartheta(\varrho)\|$$
$$= \|S_n(\varrho) - \vartheta(\varrho)\|. \qquad (2.2)$$

Thus $\lim_{n\to\infty}\|S_n(\varrho) - \vartheta(\varrho)\|$ *exists for all* $\vartheta(\varrho) \in F$.

Definition 2.3 Let$\mathcal{R}, \mathrm{Z}, B : \Psi \times \Upsilon \to \Pi(\Upsilon)$ are said to satisfy condition (C') if $d\left((\varrho, u), v(\varrho)\right) \leq d\left(w(\varrho), v(\varrho)\right)$ for $v(\varrho) \in \mathrm{Z}(\varrho, u), w(\varrho) \in \mathcal{R}(\varrho, u)$.

Lemma2.4.*Let* Φ*and* $\Upsilon$and*Let* $\mathcal{R}, \mathrm{Z}, B : \Psi \times \Upsilon \to \Pi(\Upsilon)$*satisfying conditionn*$(C')$*and* $\{S_n(\varrho)\}$ *be sequence defined in (2.1).*

If $F \neq \emptyset$ *and* $B_{\vartheta(\varrho)} = \mathcal{R}_{\vartheta(\varrho)} = \mathrm{Z}_{\vartheta(\varrho)} = \{\vartheta(\varrho)\}$*for any* $\vartheta(\varrho) \in F$ *then*

$$\lim_{\iota\to\infty} d\left(S_n(\varrho), B(S_n(\varrho))\right) = \lim_{n\to\infty} d\left(S_n(\varrho), \mathrm{Z}(S_n(\varrho))\right) = 0 = \lim_{n\to\infty} d\left(S_n(\varrho), \mathcal{R}(S_n(\varrho))\right)$$

Proof. By Lemma (2.2), $\lim_{n\to\infty}\|S_n(\varrho) - \vartheta(\varrho)\|$exists. We suppose that $\lim_{n\to\infty}\|S_n(\varrho) - \vartheta(\varrho)\| = c$ for $c \geq 0$. Also have$B, S, \mathcal{R}$ arenonexpansive random operator and $F \neq \emptyset$, we

have$\|\eta_n(\varrho) - \vartheta(\varrho)\| = d\left(\eta_n(\varrho), \mathrm{Z}(\vartheta(\varrho))\right) \leqslant H\left(B(S_n(\varrho)), B(\vartheta(\varrho))\right) \leqslant \|S_n(\varrho) - \vartheta(\varrho)\|$. Take limsup
for both side, get

$$\limsup_{n\to\infty}\|\eta_n(\varrho) - \vartheta(\varrho)\| \leqslant c$$

Similarly,

$$\limsup_{n\to\infty}\|\rho_n(\varrho) - \vartheta(\varrho)\| \leqslant c$$

and ,

$$\limsup_{n\to\infty}\|\xi_n(\varrho) - \vartheta(\varrho)\| \leqslant c$$

as,

$$\lim_{n\to\infty}\|S_{n+1}(\varrho) - \vartheta(\varrho)\| = c$$

That mean

$$\lim_{n\to\infty}\|\alpha_n(\eta_n(\varrho) - \vartheta(\varrho)) + \kappa_n(\rho_n(\varrho) - \vartheta(\varrho)) + \sigma_n(\xi_n(\varrho) - \vartheta(\varrho))\| = c$$

Applying lemma (1.4), we get

$$\lim_{n\to\infty}\|\eta_n(\varrho) - \rho_n(\varrho)\| = 0 .$$

But from the condition (C') we obtain that $d(S_n(\varrho), \eta_n(\varrho)) \leq d(\rho_n(\varrho), \eta_n(\varrho))$,

$$\limsup_{n\to\infty} d(S_n(\varrho), \eta_n(\varrho)) \leq 0 .$$

That is,

$$\lim_{n\to\infty}\|S_n(\varrho)-\eta_n(\varrho)\|=0\,. \tag{2.3}$$

Also from lemma (1.4) and (2.2) we obtain

$$\|S_n(\varrho)-\rho_n(\varrho)\|\le\|S_n(\varrho)-\eta_n(\varrho)\|+\|\eta_n(\varrho)-\rho_n(\varrho)\|$$

implies that

$$\lim_{n\to\infty}\|S_n(\varrho)-\rho_n(\varrho)\|=0\,. \tag{2.4}$$

Also from lemma (1.4) and (2.3) we have

$$\|S_n(\varrho)-\xi_n(\varrho)\|\le\|S_n(\varrho)-\rho_n(\varrho)\|+\|\rho_n(\varrho)-\xi_n(\varrho)\|$$

implies that

$$\lim_{n\to\infty}\|S_n(\varrho)-\xi_n(\varrho)\|=0\,. \tag{2.5}$$

Now, we get

$$d\left(S_n(\varrho),B\big(S_n(\varrho)\big)\right)\le\ d(S_n(\varrho),\eta_n(\varrho)),$$

also,

$$d\left(S_n(\varrho),\mathrm{Z}\big(S_n(\varrho)\big)\right)\le\ d(S_n(\varrho),\rho_n(\varrho)),$$

and

$$d\left(S_n(\varrho),\mathcal{R}\big(S_n(\varrho)\big)\right)\le\ d(S_n(\varrho),\xi_n(\varrho)),$$

we gives $d\left(S_n(\varrho),B\big(S_n(\varrho)\big)\right)\to 0, d\left(S_n(\varrho),\mathrm{Z}\big(S_n(\varrho)\big)\right)\to 0$ *and* $d\left(S_n(\varrho),\mathcal{R}\big(S_n(\varrho)\big)\right)\to 0$ as $n\to\infty$.

Theorem 2.5. *Let* Υ *and let* Φ *satisfying the Opial'scondition. and* $\mathcal{R},\mathrm{Z}\,,B:\Psi\times\Upsilon\to\Pi(\Upsilon)$.*If* $F\neq\emptyset$ *and* $B_{\vartheta(\varrho)}=\mathcal{R}_{\vartheta(\varrho)}=\mathrm{Z}_{\vartheta(\varrho)}=\{\vartheta(\varrho)\}$*for any* $\vartheta(\varrho)\in F, I-B\,, I-\mathrm{Z}$*and* $I-\mathcal{R}$*are demi-closed to 0, then* $\{S_n(\varrho)\}$ *converges to a common*random fixed point *of B, S, R and weakly.*

Proof:- Let $\vartheta(\varrho)\in F$. by lemma (2.2), we have

$\lim_{n\to\infty}\|S_n(\varrho)-\vartheta(\varrho)\|$exists.

we prove that $\{S_n(\varrho)\}$subsequentialhave a weak unique limit in F . now, let $\omega_1(\varrho),\omega_2(\varrho)$*and* $\omega_3(\varrho)$ be weak limits of the subsequences $\{S_{ni}(\varrho)\},\{S_{nj}(\varrho)\}$ *and* $\{S_{nl}(\varrho)\}$ *of* $\{S_n(\varrho)\}$,

respect. By uselemma (2.4), there is $\eta_n(\varrho)\in B\big(S_n(\varrho)\big)$such that $\lim_{n\to\infty}\|S_n(\varrho)-\vartheta(\varrho)\|=0$ and $I-B$is demi-closedto 0 , therefore we get $\omega_1(\varrho)\in B\omega_1(\varrho)$. Similarly, $\omega_1(\varrho)\in\mathrm{Z}\omega_1(\varrho)$ *and* $\omega_1(\varrho)\in\mathcal{R}\omega_1(\varrho)$.Again in a same way, we can prove that $\omega_1(\varrho),\omega_2(\varrho),\omega_3(\varrho)\in F$.

now, we need to prove unique. For this let $\omega_1(\varrho)\neq\omega_2(\varrho)\neq\omega_3(\varrho)$. Then by the Opial's condition,

$$\begin{aligned}\lim_{n\to\infty}\|S_n(\varrho)-\omega_1(\varrho)\|&=\lim_{n_i\to\infty}\|S_{ni}(\varrho)-\omega_1(\varrho)\|\\&<\lim_{n_i\to\infty}\|S_{ni}(\varrho)-\omega_2(\varrho)\|\\&=\lim_{n\to\infty}\|S_n(\varrho)-\omega_2(\varrho)\|\\&=\lim_{n_j\to\infty}\left\|S_{nj}(\varrho)-\omega_2(\varrho)\right\|\\&<\lim_{n_j\to\infty}\left\|S_{nj}(\varrho)-\omega_3(\varrho)\right\|\\&=\lim_{n\to\infty}\|S_n(\varrho)-\omega_3(\varrho)\|\end{aligned}$$

$$= \lim_{n_l\to\infty} \|S_{nl}(\varrho) - \omega_3(\varrho)\|$$
$$< \lim_{n_l\to\infty} \|S_{nl}(\varrho) - \omega_1(\varrho)\|$$
$$= \lim_{n\to\infty} \|S_n(\varrho) - \omega_1(\varrho)\|$$

Hence , its a contradiction. Hence $\{S_n(\varrho)\}$converges and weakly in F.

Remark 2.6*Let* Υ *and let*Φ *satisfying the Opial'scondition* and ,$\mathcal{R}, \mathrm{Z}\,, B : \Psi \times \Upsilon \to \Pi(\Upsilon)$ and $\{S_n(\varrho)\}$be the sequence in (2.1),. *If* $F \neq \emptyset$ *and* $B_{\vartheta(\varrho)} = \mathcal{R}_{\vartheta(\varrho)} = \mathrm{Z}_{\vartheta(\varrho)} = \{\vartheta(\varrho)\}$then $\{S_n(\varrho)\}$converges to a common-random fixed point of B, Z,$\mathcal{R}$ and weakly.

Theorem 2.7. *Let* Φ,Υ, and $\{S_n(\varrho)\}$ and$B$, Z,,$\mathcal{R}$ *defined in the Lema(2.4). and* $F \neq \emptyset$*and*$B_{\vartheta(\varrho)} = \mathcal{R}_{\vartheta(\varrho)} = \mathrm{Z}_{\vartheta(\varrho)} = \{\vartheta(\varrho)\}$ *for any* $\vartheta(\varrho) \in F$*, then*
$\{S_n(\varrho)\}$ *converges to a common*random fixed point *of* $B, \mathcal{R}$, Z*andstronglyiff*
$\lim_{n\to\infty} inf\; d(S_n(\varrho)\,, F) = 0$

Proof. The first direction of the proof is clear. Conversely, let
$\lim_{n\to\infty} inf\; d(S_n(\varrho)\,, F) = 0$. by lemma(2.2),
$\|S_{n+1}(\varrho) - \vartheta(\varrho)\| \le \|S_n(\varrho) - \vartheta(\varrho)\|$.
This gives
$d(S_{n+1}(\varrho), F\,) \le d(S_n(\varrho), F\,)$,
so that $\lim_{n\to\infty} inf\; d(S_n(\varrho)\,, F)$exists. But, by use hypothesis,
$\lim_{n\to\infty} inf\; d(S_n(\varrho)\,, F) = 0$. Therefore we must have
$\lim_{n\to\infty} inf\; d(S_n(\varrho)\,, F) = 0$. we need toprove that $\{S_n(\varrho)\}$is a Cauchy sequence in Υ.
suppose ε >0 . we have $\lim_{n\to\infty} d(S_n(\varrho)\,, F) = 0$, thereis a constant n_0 such that $\forall n \ge n_0$, we have
$\lim_{n\to\infty} inf\; d(S_n(\varrho)\,, F) < \frac{\epsilon}{4}$
In particular, $\inf\{\|S_{n0}(\varrho) - \vartheta(\varrho)\| : \vartheta(\varrho) \in F\} < \frac{\epsilon}{4}$. There must exist a $\vartheta(\varrho)^* \in F$such that
$\|S_{n0}(\varrho) - \vartheta(\varrho)^*\| < \frac{\epsilon}{2}$
Now for $m, n \ge n_0$, we have

$$\|S_{n+m}(\varrho) - S_n(\varrho)\| \le \|S_{n+m}(\varrho) - \vartheta(\varrho)^*\| + \|S_n(\varrho) - \vartheta(\varrho)^*\|$$
$$\le 2\|S_{n0}(\varrho) - \vartheta(\varrho)^*\|$$
$$< 2\left(\frac{\epsilon}{2}\right) = \epsilon$$

Hence $\{S_n(\varrho)\}$is a Cauchy sequence in$\Upsilon$of Φ, and
therefore it must converge in Υ. Let $\lim_{n\to\infty} S_n(\varrho) = q(\varrho)$ Now

$$d\big(q(\varrho), B_{q(\varrho)}\big) \le d\big(q(\varrho), S_n(\varrho)\big) + d\big(S_n(\varrho), B_{S_n(\varrho)}\big) + H\big(B_{S_n(\varrho)}\,, B_{q(\varrho)}\big)$$
$$\le d\big(q(\varrho), S_n(\varrho)\big) + d\big(S_n(\varrho), \eta_n(\varrho)\big) + d\big(S_n(\varrho), q(\varrho)\big)$$
$$\to 0 \; as\; n \to \infty,$$

gives that $d\big(q(\varrho), B_{q(\varrho)}\big) = 0$ which implies that $q(\varrho) \in B_{q(\varrho)}$. Similarly,

$$d\big(q(\varrho), S_{q(\varrho)}\big) \le d\big(q(\varrho), S_n(\varrho)\big) + d\big(S_n(\varrho), \mathrm{Z}_{S_n(\varrho)}\big) + H\big(\mathrm{Z}_{S_n(\varrho)}, \mathrm{Z}_{q(\varrho)}\big)$$
$$\le d\big(q(\varrho), S_n(\varrho)\big) + d\big(S_n(\varrho), \rho_n(\varrho)\big) + d\big(S_n(\varrho), q(\varrho)\big)$$
$$\to 0 \; as\; n \to \infty,$$

gives that $d\big(q(\varrho), \mathrm{Z}_{q(\varrho)}\big) = 0$ which implies that $q(\varrho) \in \mathrm{Z}_{q(\varrho)}$.Similarly,

$$d\big(q(\varrho), R_{q(\varrho)}\big) \le d\big(q(\varrho), S_n(\varrho)\big) + d\left(S_n(\varrho), \mathcal{R}_{S_n(\varrho)}\right) + H\big(\mathcal{R}_{S_n(\varrho)}, \mathcal{R}_{q(\varrho)}\big)$$
$$\le d\big(q(\varrho), S_n(\varrho)\big) + d\big(S_n(\varrho), \rho_n(\varrho)\big) + d\big(S_n(\varrho), q(\varrho)\big)$$
$$\to 0 \; as \; n \to \infty,$$

implies that$q(\varrho) \in \mathcal{R}_{q(\varrho)}$. Consequently, $q(\varrho) \in F$.

Nowwe can use the conditional $(A`)$ to find the strong converge of $\{S_n(\varrho)\}$define in(2.1)., we assume that $\mathcal{R}, \mathrm{Z}, B : \Psi \times \Upsilon \to \Pi(\Upsilon)$satisfycondition $(A`)$.

Theorem 2.8. *Let* Υ, Φ *and,the sequence*$\{S_n(\varrho)\}$ *beas in Lemma (2.4). Let* $\mathcal{R}, \mathrm{Z}, B : \Psi \times \Upsilon \to \Pi(\Upsilon)$*satisfying condition* $(A`)$. *If* $F \neq \emptyset$ *and* $B_{\vartheta(\varrho)} = \mathcal{R}_{\vartheta(\varrho)} = \mathrm{Z}_{\vartheta(\varrho)} = \{\vartheta(\varrho)\}$ *for any* $\vartheta(\varrho) \in F$ *then* $\{S_n(\varrho)\}$ *converges to a common- fixed point of* B, Z,$\mathcal{R}$ *and strongly.*

Proof :since By used lemma(2.4), have $\lim_{n\to\infty} \|S_n(\varrho) - F\|$exists $\forall\vartheta(\varrho) \in F$. let $c \ge 0$. If $c = 0$, clear that. let$c > 0$. Now $\|S_{n+1}(\varrho) - \vartheta(\varrho)\| \le \|S_n(\varrho) - \vartheta(\varrho)\|$gives $\inf_{\vartheta(\varrho)\in F} \|S_{n+1}(\varrho) - \vartheta(\varrho)\| \le \inf_{\vartheta(\varrho)\in F} \|S_n(\varrho) - \vartheta(\varrho)\|$which implies that $d(S_{n+1}(\varrho), F) \le d(S_n(\varrho), F) \; and$ so $\lim_{n\to\infty} d(S_n(\varrho), F)$ exists. By condition $(A`)$either

$$\lim_{n\to\infty} \chi\big(d(S_n(\varrho), F)\big) \le \lim_{n\to\infty} d\big(S_n(\varrho), B_{S_n(\varrho)}\big) = 0$$

or

$$\lim_{n\to\infty} \chi\big(d(S_n(\varrho), F)\big) \le \lim_{n\to\infty} d\big(S_n(\varrho), \mathrm{Z}_{S_n(\varrho)}\big) = 0$$

or

$$\lim_{n\to\infty} \chi\big(d(S_n(\varrho), F)\big) \le \lim_{n\to\infty} d\big(S_n(\varrho), R_{S_n(\varrho)}\big) = 0$$

In both the cases,

$$\lim_{n\to\infty} \chi\big(d(S_n(\varrho), F)\big) = 0$$

Since χ is a non-decreasing function where$\chi(0) = 0$, $\lim_{n\to\infty} d(S_n(\varrho), F) = 0$.

5. Conclusion

The main idea of the this paper is that we have to found a new iterations by one step to approximation for common random fixed point from three multi-valued non-expansive random operator and obtain, as well as the theories of strong and weak convergence .

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Conflict of Interest

"Conflict of Interest: The authors declare that they have no conflicts of interest."

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References

1. Hans, O., Random fixed point theorems, in Transactions of the first prague Conference on Information Theory, *Statistical Decision Functions, Random Process*, **1957**, 105–125.

https://doi.org/10.1016/B978-0-12-434160-9.50009-6

2. Huang ,T.; Rhoades, B. E. A general principle for Ishikawa iterations for multi-valued mappings, *Indian J. Math. Pure Appl.* **1997**, 1091–1098.
3. Khan, S. H.; Abbas, M.; Rhoades B. E. A new one-step iterative scheme for approximating common fixed points of two multivalued nonexpansive mappings, *Rend. del Circ. Mat. di Palermo*. **2010**, 151–159. https://doi.org/10.1007/s12215-010-0012-4
4. Engl, H. Random fixed point theorems for multivalued mappings, *Pacific J. Math.* **1978**, 351–360. https://doi.org/10.2140/PJM.1978.76.351
5. Engl, H. W. Some random fixed point theorems for strict contractions and nonexpansive mappings, *Nonlinear Anal. Theory, Methods Appl.* **1978**, 619–626.
6. Itoh .S., A random fixed point theorem for a multivalued contraction mapping, *Pacific J. Math.* **1977**, 85–90. https://doi.org/10.2140/PJM.1977.68.85
7. Beg ,I. and Shahzad,N., Random fixed point theorems for nonexpansive and contractive-type random operators on Banach spaces, *Stoch J. Math.Anal. Appl* **1994**, 569–580. https://doi.org/10.1155/S1048953394000444
8. Papageorgiou, N. S. Random fixed point theorems for measurable multifunctions in Banach spaces, *Proc. Am. Math. Soc.* **1986**, 507–514. https://doi.org/10.1090/S0002-9939-1986-0840638-3
9. Sinacer, M. L.; Nieto J. J.; Ouahab, A. Random fixed point theorem in generalized Banach space and applications, *Random Oper. Stoch. Equations*. **2016**, 93–112. http://dx.doi.org/10.1515/rose-2016-0007
10. Dhage, B. C. Some basic random fixed point theorems with PPF dependence and functional random differential equations, *Differ. Equ. Appl*. **2012**, 181–195. https://doi.org/10.7153/DEA-04-11
11. El Ghabi , A. Random fixed point theorems with application to random differential equations in Banach spaces, 23 Sep., *Inaug. Days*, 54. https://doi.org/10.1155/2021/6648938
12. Beg, I. Approximation of random fixed points in normed spaces, *Nonlinear Anal. Theory, Methods Appl.* **2002** , 1363–1372. https://doi.org/10.1016/S0362-546X(01)00902-6
13. Tan, K.-K.; Yuan, X.-Z. Random fixed point theorems and approximation, *Stoch. Anal. Appl.*, 103–123. https://doi.org/10.1006/jmaa.1994.1256
14. Shahzad, N. Random fixed point theorems for various classes of 1-set-contractive maps in Banach spaces, *J. Math. Anal. Appl.* **1996**, 712–718. https://doi.org/10.1006/JMAA.1996.0407
15. Rashwan, R. A.; Albaqeri, D. M. A common random fixed point theorem and application to random integral equations, *Int. J.Math. Res. Appl.* **2014**. https://doi.org/10.14419/IJAMR.V3I1.1690
16. Plubtieng ,S.; Kumam , P.; Wangkeeree ,R. Approximation of a common random fixed point for a finite family of random operators, *Int. J. Math. Math. Sci.* **2007**. https://doi.org/10.1155/2007%2F69626
17. Beg, I.; Abbas, M. Convergence of iterative algorithms to common random fixed points of random operators, *Stoch J. Math. Anal. Appl.* **2006**. https://doi.org/10.1155/JAMSA%2F2006%2F89213
18. Nadler Jr ,S. B. Multi-valued contraction mappings., *Pacific J. Math.* **1969**, 475–488. https://doi.org/10.2140/PJM.1969.30.475
19. Opial, Z. Weak convergence of the sequence of successive approximations for nonexpansive mappings, *Bull. Am. Math. Soc.* **1967**, 591–597. https://doi.org/10.1090/S0002-9904-1967-11761-0
20. Nilsrakoo, W.; Saejung, S., A reconsideration on convergence of three-step iterations for asymptotically nonexpansive mappings, *Math .Appl. Comput.* **2007**, 1472–1478. https://doi.org/10.1016/j.amc.2007.02.026
21. Malih, S. H. Random fixed point for random Fibonacci Noor iteration scheme *J. Math.* **2021**, 775–779. https://doi.org/10.1080/09720502.2021.1884392
22. Abed , S. S.; Hasan, Z. M. Convergence comparison of two schemes for common fixed points with an application, *Ibn AL-Haitham J. Pure Appl. Sci.* **2019**, 81–92. https://doi.org/10.30526/32.2.2146
23. Abed, S. S.; Hasan, Z. M. Common fixed point of a finite-step iteration algorithm under total

asymptotically quasi-nonexpansive maps. *Baghdad Sci. J.* **2019**, 654–660. http://dx.doi.org/10.21123/bsj.2019.16.3.0654

24. Maibed , Z. H.; Thajil, A. Q., Zenali Iteration Method For Approximating Fixed Point of A Î´ ZA-Quasi Contractive mappings. *Ibn AL-Haitham J. Pure Appl. Sci.* **2021**, 78–92. https://doi.org/10.30526/34.4.2705
25. Malih, S. H. Common Fixed Point for a Pair of Asymptotically Nonexpansive and Multivalued Mapping Under Fibonacci Iteration Sequence in CAT (0) Space. in *Journal of Physics: Conference Series* **2021**. https://doi.org/10.1088/1742-6596/1897/1/012061
26. Kadhim, A. J. New Common Fixed Points for Total Asymptotically Nonexpansive Mapping in CAT (0) Space. *Baghdad Sci. J.***2021**. http://dx.doi.org/10.21123/bsj.2021.18.4.1286
27. Luaibi, H. H.; Abed,S. S. Fixed point theorems in general metric space with an application, *Baghdad Sci J*. **2021**, 812–815. https://doi.org/10.21123/BSJ.2021.18.1%28SUPPL.%29.0812
28. Ajeel, Y. J.; Kadhim, S. N. Some Common fixed points theorems of four weakly compatible mappings in metric spaces, *Baghdad Sci J*. **2021**, 543–546. https://doi.org/10.21123/BSJ.2021.18.3.0543
29. Malih, S. H., Fixed point theorems of modified Mann and Ishikawa iterations, *J. Interdiscip. Math.*, **2021**, 1093–1097. https://doi.org/10.1080/09720502.2020.1790739
30. Karahan, I.; Ozdemir, M. A general iterative method for approximation of fixed points and their applications. *Adv. Fixed Point Theory*. **2013**, 510–526.