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Generalized Heptagonal Membership Function for Fully Fuzzy Linear Fractional Programming Problems

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Abstract

Identifying the optimum solution that satisfies the restrictions and maximizes or minimizes the objective function is the aim of fully fuzzy fractional programming (FFFLP). Due to the inclusion of both fuzzy parameters and fractional variables, this problem is difficult to solve. Several approaches, including linear programming, nonlinear programming, genetic algorithms, and computational intelligence algorithms, suggested handling FFFLP. For describing the uncertainty, vagueness, or imprecision of information in the real world, triangular and trapezoidal fuzzy numbers are frequently used. Indeed, it is not always practical to limit the membership function to a triangle or trapezoid. This paper submits a new type of Heptagonal fuzzy number and a novel ranking function technique based on generalized heptagonal membership functions suggested for ordering heptagonal fuzzy numbers. With the help of the algorithm of the simplex method for fully fuzzy simplex method, obtained the optimal fuzzy solution for FFFLP.

Keywords: Fully Fuzzy Linear Fractional Programming, Heptagonal Fuzzy Number, Arithmetic Heptagonal Operations, Membership Function, Ranking function, Fully Fuzzy Simplex method.

1. Introduction

Fuzzy problem solving relies heavily on the ranking function, which provides a methodical way to evaluate solutions when choosing an option. The fractional programming problem is a decision-making challenge involving the optimization of a constrained ratio of fuzzy programming problem Modern applications of the linear fractional programming problem found in a wide range of fields, including production planning, finance, healthcare, and other areas of engineering.

Many papers use the features of fuzzy sets as a method to find the best solutions to fuzzy programming problems in which all variables are triangular or trapezoidal numbers (1,2). If a fuzzy number has seven possible membership values; we say that it is heptagonal. The range of the heptagonal fuzzy number defined by a membership function that gives each value in the range a certain level of membership. The membership function for a heptagonal fuzzy number takes the shape of a triangle or trapezoid. (3-5) discussed a form of fuzzy number named a Heptagonal fuzzy number and proposed a new version of the definition for the value and ambiguity of non-normal fuzzy .For example , Namarta and others ; proposed a ranking method for heptagon fuzzy numbers based on area. The study involves the computation of the center of centroids for ranking heptagonal fuzzy numbers.

Loganathan, T., (6, 7) proposed a method for solving FFFLP and all parameters and variables are triangular fuzzy numbers that preserves the fuzzy nature of the situation while maintaining their fuzzy characteristics. Using the close interval approximation of normalized heptagonal fuzzy numbers, which is one of the best interval approximations, Alharbi, M.G.; Khalifa, H.A. (8) attempted to solve the linear fractional programming problem with fully fuzzy normalized heptagonal fuzzy numbers. They transformed the original maximization (minimization) problem with an interval objective function into a multi-objective problem using order relations.

In order to rank triangular fuzzy numbers, Mitlif, R.J. (9,10) used a novel ranking function technique based on ordinary fuzzy numbers. The optimal solution found by first reducing the fuzzy fractional programming problem to a fractional programming problem and then solving it with the method. For solving FFLFP with trapezoidal fuzzy integers as the objective function and constraints, Gupta,D; Jain,P; Gupta,G.(11) presented a new trapezoidal fuzzy number ranking function, the proposed procedure is based on the simplex method and precise linear fractional programming. Triangular and trapezoidal fuzzy numbers and the values of the right-hand side represent the objective function and crisp numbers represents left-hand side constraints. A lots of researcher (12-18) submitted a fuzzy linear fractional programming problem and a new ranking function devised for converting the fuzzy linear fractional programming problem into an unambiguous one.

This paper proposes a new ranking function strategy based on the new generalized heptagonal membership function in order to solve an FFFLP by first transforming the FFFLP into a completely fuzzy linear problem. Finally, the optimal fuzzy solution can reached by fully fuzzy simplex method in which all the input variables are heptagonal fuzzy numbers. This paper constructed into eight sections. In section 2, the preliminary of the fuzzy set theory. Section 3 proposes a generalized heptagonal fuzzy function, (σ-cut) function, and derived the proposed ranking function. In section 4, the fuzzy mathematical operations of heptagonal fuzzy numbers. Section 5 shows the mathematical model of FFFLP. The algorithm of the fully fuzzy simplex method is in section 6; a numerical example be given in section 7, and section 8 concludes the paper.

2. Preliminary.

Definition 2.1: (19)

Let X be a real set. *The fuzzy set* $\tilde{\mathcal{A}}$ is defined by the membership function $\mathcal{M}_{\tilde{\mathcal{A}}}(x)$, $\mathcal{M}_{\tilde{\mathcal{A}}}(x)$: $\mathcal{X}\rightarrow$ [0, 1], is the degree of membership of $x \in \mathcal{X}$ in the set $\tilde{\mathcal{A}}$ and is denoted by $\tilde{\mathcal{A}}(x) = \{(x, \mathcal{M}_{\tilde{\mathcal{A}}}(x)) | \, x \in \mathcal{X}\}$.

Definition 2.2: (19,20)

A *fuzzy number* $\tilde{\mathcal{A}}$ is a set whose membership function $\mathcal{M}_{\tilde{\mathcal{A}}}(x)$ satisfies the following conditions:

- $\tilde{\mathcal{A}}$ a normal fuzzy set if there exists at least one x_0 in $\mathcal{R}$ with $\mathcal{M}_{\tilde{\mathcal{A}}}(x) = 1$.
- $\mathcal{M}_{\tilde{\mathcal{A}}}(x)$ Piecewise continuous.
- $\tilde{\mathcal{A}}(x)$ Convex if $\mathcal{M}_{\tilde{\mathcal{A}}}(x)$. $[\sigma x_1 + (1-\sigma)\, x_2] \geq$ Min ($\mathcal{M}_{\tilde{\mathcal{A}}}(x_1)$, $\mathcal{M}_{\tilde{\mathcal{A}}}\ (x_2)\}$, x_1, $x_2 \in X$, $\sigma \in$ [0, 1].

Definition 2.3: A *Heptagonal Fuzzy Number* (HEP)

Is a fuzzy number that has seven membership values on the interval [0, 1] that determine the *membership function* of a heptagonal fuzzy number. The number of these dots stands for how much of a certain element belongs to the fuzzy set.

The range of values for which the membership function is non-zero is called the *support* of a heptagonal fuzzy number**.** A heptagonal fuzzy number's *center* is the set of values where the membership function is one .The maximum value of the membership function defines the *height* of a heptagonal fuzzy number**.** The *centroid* of a heptagonal fuzzy number calculated by taking the weighted average of its seven membership values, with each value assigned a weight based on the degree to which it belongs to the heptagon.

3.Propose Generalized Heptagonal Membership Function.

Membership functions play a crucial role in finding solutions to fuzzy problems because of their capacity to accurately reflect the inherent vagueness and imprecision of real-world information. In this section, propose a nonlinear membership function $\mathcal{M}_{\widetilde{\mathcal{A}_{HEP}}}(x)$ of a Heptagonal fuzzy number.

$\widetilde{\mathcal{A}_{HEP}} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7; k_1, k_2, \omega)$, Where $\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha_4 \leq \alpha_5 \leq \alpha_6 \leq \alpha_7 \in \mathcal{R}$, $k_1, k_2 \in$ [0,1] and 0< $k_1 < k_2 < \omega$, $0 < \omega \leq 1$:

$$\mathcal{M}_{\widetilde{\mathcal{A}_{HEP}}}(x) = \begin{cases} 0 & x < \alpha_1 \\ (k_1\left(\dfrac{x-\alpha_1}{\alpha_2-\alpha_1}\right))^{1/3} & \alpha_1 \le x < \alpha_2 \\ (\, k_1 + (k_2 - k_1)\left(\dfrac{x-\alpha_2}{\alpha_3-\alpha_2}\right))^{\,1/3} & \alpha_2 \le x < \alpha_3 \\ (k_2 + (\omega - k_2)\left(\dfrac{x-\alpha_3}{\alpha_4-\alpha_3}\right))^{1/3} & \alpha_3 \le x < \alpha_4 \\ (\omega + (k_2 - \omega)\left(\dfrac{x-\alpha_4}{\alpha_5-\alpha_4}\right))^{\,1/3} & \alpha_4 \le x < \alpha_5 \\ (k_2 - (k_2 - k_1)\left(\dfrac{x-\alpha_5}{\alpha_6-\alpha_5}\right))^{1/3} & \alpha_5 \le x < \alpha_6 \\ (k_1 - k_1\left(\dfrac{x-\alpha_6}{\alpha_7-\alpha_6}\right))^{1/3} & \alpha_6 \le x \le \alpha_7 \\ 0 & x > \alpha_7 \end{cases}$$

3.1. The ($\sigma - cut$) Function

The $\sigma - cut$ function is a mathematical function used in fuzzy logic to define a subset of a fuzzy set. It is a way of defining the degree of membership of an element in a fuzzy set .in this section constructs the ($\sigma - cut$) function of a heptagonal fuzzy number as follows:

$$\widetilde{\mathcal{A}_{HEP}}_{\sigma} = \begin{cases} \alpha_1 + \dfrac{\sigma^3}{k_1}(\alpha_2 - \alpha_1) & \sigma \in [0, k_1] \\ \alpha_2 + \left(\dfrac{\sigma^3 - k_1}{k_2 - k_1}\right)(\alpha_3 - \alpha_2) & \sigma \in (k_1, k_2] \\ \alpha_3 + (\dfrac{\sigma^3 - k_2}{\omega - k_2})(\alpha_4 - \alpha_3) & \sigma \in (k_2, \omega] \\ \alpha_4 + \left(\dfrac{\sigma^3 - \omega}{k_2 - \omega}\right)(\alpha_5 - \alpha_4) & \sigma \in (k_2, \omega] \\ \alpha_5 + \left(\dfrac{\sigma^3 - k_2}{k_1 - k_2}\right)(\alpha_6 - \alpha_5) & \sigma \in (k_1, k_2] \\ \alpha_6 + \left(1 - \dfrac{\sigma^3}{k_1}\right)(\alpha_7 - \alpha_6) & \sigma \in [0, k_1] \end{cases}$$

$(inf_1\, \widetilde{\mathcal{A}_{HEP}}_{\sigma}, sup_3\, \widetilde{\mathcal{A}_{HEP}}_{\sigma}) = (\left[\alpha_1 + \frac{\sigma^3}{k_1}(\alpha_2 - \alpha_1)\right], \left[\alpha_6 + \left(1 - \frac{\sigma^3}{k_1}\right)(\alpha_7 - \alpha_6)\right]), \sigma \in [0, k_1]$

$\left(inf_2\, \widetilde{\mathcal{A}_{HEP}}_{\sigma}, sup_2\, \widetilde{\mathcal{A}_{HEP}}_{\sigma}\right) = \left(\left[\alpha_2 + \left(\frac{\sigma^3 - k_1}{k_2 - k_1}\right)(\alpha_3 - \alpha_2)\right], \left[\alpha_5 + \left(\frac{\sigma^3 - k_2}{k_1 - k_2}\right)(\alpha_6 - \alpha_5)\right]\right), \sigma \in (k_1, k_2]$

$\left(inf_3\, \widetilde{\mathcal{A}_{HEP}}_{\sigma}, sup_1\, \widetilde{\mathcal{A}_{HEP}}_{\sigma}\right) = (\left[\alpha_3 + (\frac{\sigma^3 - k_2}{\omega - k_2})(\alpha_4 - \alpha_3)\right], \left[\alpha_4 + \left(\frac{\sigma^3 - \omega}{k_2 - \omega}\right)(\alpha_5 - \alpha_4)\right], \sigma \in (k_2, \omega]$

3.2 Ranking Function

Fuzzy problem solving depends significantly on ranking functions, which help evaluate and prioritize possible solutions. The ranking function allows alternatives to be evaluated according to their level of membership in a set or category, which is particularly useful in a fuzzy environment when there is uncertainty and imprecision in the data. This section proposed a novel ranking function depending on the new nonlinear heptagonal membership function as shown:

Let $\widetilde{\mathcal{A}_{HEP}} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7; k_1, k_2, \omega)$, depending on the following function:

$$\Re\left(\widetilde{\mathcal{A}_{HEP}}\right) = \left(\tfrac{1}{2}\right) \int_0^{\omega} \left(inf_i\, \widetilde{\mathcal{A}_{HEP}}_{\sigma} + \sup_j \widetilde{\mathcal{A}_{HEP}}_{\sigma}\right) d\sigma \quad i = 1,2,3 \quad j = \begin{cases} i+2 & i=1 \\ i & i=2 \\ i-2 & i=3 \end{cases}$$

Suppose that $\Re\left(\widetilde{\mathcal{A}_{HEP}}\right) = \frac{1}{2} * (F_1 + F_2 + F_3)$ (1)

Where;

$$F_1 = \int_0^{k1} \left(\left[\alpha_1 + \left(\frac{\sigma^3}{k_1}\right)(\alpha_2 - \alpha_1)\right] + \left[\alpha_6 + \left(1 - \frac{\sigma^3}{k_1}\right)(\alpha_7 - \alpha_6)\right]\right) d\sigma$$

$$\therefore \quad F_1 = \frac{1}{4}\left[{k_1}^3(\alpha_2 - \alpha_1 + \alpha_6 - \alpha_7) + k_1(4\alpha_1 + 4\alpha_7)\right] \tag{2}$$

$$F_2 = \int_{k1}^{k2} \left(\left[\alpha_2 + \left(\frac{\sigma^3 - k_1}{k_2 - k_1}\right)(\alpha_3 - \alpha_2)\right] + \left[\alpha_5 + \left(\frac{\sigma^3 - k_2}{k_1 - k_2}\right)(\alpha_6 - \alpha_5)\right]\right) d\sigma$$

$$= \frac{{k_1}^3}{4}(\alpha_3 - \alpha_2 + \alpha_5 - a_6) ++ \frac{{k_1}^2 k_2}{4}(\alpha_3 - \alpha_2 + \alpha_5 - a_6) ++ \frac{{k_2}^2 k_1}{4}(\alpha_3 - \alpha_2 + \alpha_5 - \alpha_6) - k_1(\alpha_3 + \alpha_5) + \frac{{k_2}^3}{4}(\alpha_3 - \alpha_2 + \alpha_5 - \alpha_6) + k_2(\alpha_2 + \alpha_6) \tag{3}$$

$$F_3 = \int_{k2}^{\omega} \left(\left[\alpha_3 + \left(\frac{\sigma^3 - k_2}{\omega - k_2}\right)(\alpha_4 - \alpha_3)\right] + \left[\alpha_4 + \left(\frac{\sigma^3 - \omega}{k_2 - \omega}\right)(\alpha_5 - \alpha_4)\right]\right) d\sigma$$

$$= \frac{{k_2}^3}{4}(2\alpha_4 - \alpha_3 - \alpha_5) + \frac{{k_2}^2\omega}{4}(2\alpha_4 - \alpha_3 - \alpha_5) + \frac{k_2\omega^2}{4}(2\alpha_4 - \alpha_3 - \alpha_5) - 2\alpha_4 k_2 + \frac{\omega^3}{4}(2\alpha_4 - \alpha_3 - \alpha_5) + \omega(\alpha_3 + \alpha_5). \tag{4}$$

Now substitute equations (2), (3), (4) in eq. (1):

$$\therefore \Re\left(\widetilde{\boldsymbol{\mathcal{A}_{HEP}}}\right) = \frac{1}{8}\left[{k_1}^3(\alpha_3 - \alpha_1 + \alpha_5 - \alpha_7) + {k_1}^2 k_2(\alpha_3 - \alpha_2 + \alpha_5 - a_6) + {k_2}^2 k_1(\alpha_3 - \alpha_2 + \alpha_5 - a_6) + \frac{k_1}{4}(\alpha_1 - \alpha_3 - \alpha_5 + \alpha_7) + {k_2}^3(2\alpha_4 - \alpha_2 - a_6) + {k_2}^2\omega(2\alpha_4 - \alpha_3 - \alpha_5) + k_2\omega^2(2\alpha_4 - \alpha_3 - \alpha_5) + k_2(4\alpha_2 - 8\alpha_4 + 4a_6) + \omega^3(2\alpha_4 - \alpha_3 - \alpha_5) + \omega(4\alpha_3 + 4\alpha_5)\right]$$

$$0 < k_1 < k_2 < \omega, \; 0 < \omega \le 1$$

4. Fuzzy Mathematical Operations of Generalized Heptagonal Fuzzy Numbers (3,4)

Let $\widetilde{\mathcal{A}_{HEP}}$ *and* $\widetilde{\mathcal{B}_{HEP}}$ be two arbitrary generalized Heptagonal fuzzy numbers, such that $\widetilde{\mathcal{A}_{HEP}} = (\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_7; k_1, l_1, \omega_1)$, $\widetilde{\mathcal{B}_{HEP}} = (b_1, b_2, b_3, \dots, b_7; k_2, l_2, \omega_2)$

Define the Addition, Subtraction, and multiplication operations [10] as follows:

- $\widetilde{\mathcal{A}_{HEP}} \oplus \widetilde{\mathcal{B}_{HEP}} = (\alpha_1 + b_1, \alpha_2 + b_2, \alpha_3 + b_3, \dots, \alpha_7 + b_7; \min(k_1, k_2), \min(l_1, l_2), \min(\omega_1, \omega_2))$
- $\widetilde{\mathcal{A}_{HEP}} \ominus \widetilde{\mathcal{B}_{HEP}} = (\alpha_1 - b_7, \alpha_2 - b_6, \alpha_3 - b_5, \dots, \alpha_6 - b_2, \alpha_7 - b_1; \min(k_1, k_2), \min(l_1, l_2), \min(\omega_1, \omega_2))$
- $\widetilde{\mathcal{A}_{HEP}} \otimes \widetilde{\mathcal{B}_{HEP}} = (\alpha_1 * b_1, \alpha_2 * b_2, \dots, , \alpha_7 * b_7; \min(k_1, k_2), \min(l_1, l_2), \min(\omega_1, \omega_2))$

- $\lambda \otimes \widetilde{\mathcal{A}_{HEP}} \quad = (\lambda\alpha_1, \lambda\alpha_2, \lambda\alpha_3, \ldots, \lambda\alpha_7; k_1, l_1, \omega_1) \qquad if\ \lambda > 0$
  $\quad = (\lambda\alpha_7, \lambda\alpha_6, \lambda\alpha_5, \lambda\alpha_4, \lambda\alpha_3, \lambda\alpha_2, \lambda\alpha_1; k_1, l_1, \omega_1) \qquad if\ \lambda < 0$

5. Fully Fuzzy Linear Fractional Programming Problem (FFLFPP) (9),(20)

Consider the following FFLFP problem having m fuzzy constraints and n fuzzy variables:

$Max\ \widetilde{\omega} \cong \frac{\tilde{n}^T \tilde{x} \oplus \tilde{\beta}}{\tilde{d}^T \tilde{x} \oplus \tilde{\delta}} = \frac{\tilde{N}(x)}{\tilde{D}(x)}$

$S.to$

$\tilde{A} \otimes \tilde{x} \leq = \geq \tilde{B}\ , \qquad \tilde{x} \geq \tilde{0}$

where,

$\tilde{n}^T = \left(\tilde{n}_j\right)_{1*n}$, $\tilde{d}^T = \left(\tilde{d}_j\right)_{1*n}$, $\tilde{x} = \left(\tilde{x}_j\right)_{n*1}$, $\tilde{\beta}, \tilde{\delta} \in$ heptagonal fuzzy numbers.

$\tilde{A} = (\tilde{a}_{ij})_{m*n}$, $\tilde{B} = (\tilde{b}_i)_{m*1}$.

$\tilde{n}^T, \tilde{d}^T, \tilde{x}, \tilde{A}, \tilde{B}$ are heptagonal fuzzy numbers $\quad \forall\ 1 \leq j \leq n, \quad 1 \leq i \leq m$

6. The Algorithm of Fully Fuzzy Simplex (FFS) Method.

Step 1: Use the development complementary method (21,22) to convert the FFLFPP into FFLPP.

Step 2: Adding fuzzy slack variables $\tilde{S}_i,\ i = 1, 2, \ldots, m$ to convert all the inequalities of the constraints into equations.

Step 3: Construct the fully fuzzy simplex tableau as shown in **Table 1.**:

Table 1. FFS Method Tableau

Basic var.	$\tilde{x}_1$	$\tilde{x}_2$	… $\tilde{x}_n$	$\tilde{S}_1$	$\tilde{S}_2$	…	$\tilde{S}_n$	*R.H.S*	$\Re(R.H.S)$
$(\widetilde{\omega} \ominus \tilde{c}_j)$	$-\tilde{c}_1$	$-\tilde{c}_2$	… $-\tilde{c}_n$	(0,0,…,0)	(0,0,…,0)	…	(0,0,…,0)	$\tilde{\beta}_n$	$\Re(\tilde{\beta}_n)$
$\tilde{\boldsymbol{S}}_{\boldsymbol{1}}$	$\tilde{a}_{11}$	$\tilde{a}_{12}$	… $\tilde{a}_{1n}$	(0,0,…,1)	(0,0,…,0)	…	(0,0,…,0)	$\tilde{b}_1$	$\Re(\tilde{b}_1)$
$\tilde{\boldsymbol{S}}_{\boldsymbol{2}}$	$\tilde{a}_{21}$	$\tilde{a}_{22}$	… $\tilde{a}_{2n}$	(0,0,…,0)	(0,0,…,1)	…	(0,0,…,0)	$\tilde{b}_2$	$\Re(\tilde{b}_2)$
⋮	⋮	⋮	…	⋮	⋮	…	⋮	⋮	⋮
$\tilde{\boldsymbol{S}}_{\boldsymbol{m}}$	$\tilde{a}_{m1}$	$\tilde{a}_{m2}$	… $\tilde{a}_{mn}$	(0,0,…,0)	(0,0,…,0)	…	(0,0,…,1)	$\tilde{b}_m$	$\Re(\tilde{b}_m)$

Step 4: In the maximization problem, select the most negative value of $\Re(\widetilde{\omega} \ominus \tilde{c}_j)$ as a fuzzy entering variable, and if the problem is minimum, the most positive value of the $\Re(\widetilde{\omega} \ominus \tilde{c}_j)$ represents the entering fuzzy variable.

Step 5: Determine the fuzzy variable that leaves the basic solution by:

$$\theta = \min \left\{ \frac{\boldsymbol{\Re}(\tilde{b}_i)}{\boldsymbol{\Re}(the\ column\ elements\ of\ entering\ variable\ )} \right\}.\quad i = 1,2 \ldots, m$$

Step 6: Use the arithmetic operations of the heptagonal fuzzy number to get the new fuzzy tableau (new iteration) of a fully fuzzy simplex table.

Step 7: Repeat the steps even to get the optimal solution.

Step 8: The optimal fuzzy solution of the maximum problem is reached, when ($\Re\left(\widetilde{\omega} \ominus \tilde{c}_j\right) \geq 0$), and at the minimum problem ($\Re\left(\widetilde{\omega} \ominus \tilde{c}_j\right) \leq 0$).

7. Numerical Results

The following fractional linear programming problem paper (23):

$Max\ \omega = \frac{\tilde{4}\tilde{x}_1 + \tilde{6}\tilde{x}_2 - \tilde{2}}{\tilde{6}\tilde{x}_1 + \tilde{9}\tilde{x}_2 + \tilde{3}}$

$$s.t \qquad \tilde{x}_1+\tilde{3}\tilde{x}_2 \leq \tilde{5}$$
$$\tilde{2}\tilde{x}_1+\tilde{x}_2 \leq \tilde{2}$$
$$\tilde{x}_1, \tilde{x}_2 \geq 0$$

Now, take the above example with all the variables are heptagonal fuzzy numbers:

$$Max\ \widetilde{\omega} = \frac{(-2,0,2,4,6,8,10;\omega_1)\otimes\tilde{x}_1 \oplus(0,2,4,6,8,10,12;\omega_1)\otimes\tilde{x}_2\ominus(-4,-2,0,2,4,6,8;\omega_1)}{(0,2,4,6,8,10,12;\omega_2)\otimes\tilde{x}_1 \oplus(3,5,7,9,11,13,15;\omega_2)\otimes\tilde{x}_2\oplus(-3,-1,1,3,5,7,9;\omega_2)}$$

$$s.t$$
$$(-5,-3,-1,1,3,5,7;\ \omega_1)\otimes\tilde{x}_1\oplus(-3,-1,1,3,5,7,9;\omega_1)\otimes\tilde{x}_2 \leq (-1,1,3,5,7,9,11;\omega_1)$$
$$(-4,-2,0,2,4,6,8;\omega_2)\otimes\tilde{x}_1\oplus(-5,-3,-1,1,3,5,7;\omega_2)\otimes\tilde{x}_2 \leq (-4,-2,0,2,4,6,8;\omega_2)$$

$\tilde{x}_1, \tilde{x}_2 \geq 0$, ω_1, ω_2 are the weighted of fuzzy numbers.

Applying the proposed algorithm, the first step uses the development complementary method to convert the problem to (FFLPP) problem.

Let $Max\ \widetilde{\omega} = \frac{Min\ w_1}{Max\ w_2}$ $\qquad where,$

$$Min\ w_1 = (-2,0,2,4,6,8,10;\omega_1)\otimes\tilde{x}_1 \oplus(0,2,4,6,8,10,12;\omega_1)\otimes\tilde{x}_2 \ominus (-4,-2,0,2,4,6,8;\omega_1)$$
$$Max\ w_2 = (0,2,4,6,8,10,12;\omega_2)\otimes\tilde{x}_1 \oplus(3,5,7,9,11,13;\omega_2)\otimes\tilde{x}_2\oplus(-3,-1,1,3,5,7,9;\omega_2)$$

Since $Max\ w_1 = Min(-w_1)$

$$\therefore Max\ \ w_1 = \ominus\ [(-2,0,2,4,6,8,10;\omega_1)\otimes\tilde{x}_1 \oplus(0,2,4,6,8,10,12;\omega_1)\otimes\tilde{x}_2 \ominus (-4,-2,0,2,4,6,8;\omega_1)]$$
$$= [(-10,-8,-6,-4,-2,0,2;\omega_1)\otimes\tilde{x}_1\oplus(-12,-10,-8,-6,-4,-2,0;\omega_1)\otimes\tilde{x}_2 \oplus(-4,-2,0,2,4,6,8;\omega_1]$$

The new form of the fuzzy objective function is: $Max\ \omega' = \ Max\ w_1 + \ Max\ w_2$

$$Max\ \omega'=[(-10,-8,-6,-4,-2,0,2;\omega_1)\otimes\tilde{x}_1\oplus(-12,-10,-8,-6,-4,-2,0;\omega_1)\otimes\tilde{x}_2 \oplus(-4,-2,0,2,4,6,8;\omega_1)\oplus [(0,2,4,6,8,10,12;\omega_2)\otimes\tilde{x}_1 \oplus(3,5,7,9,11,13;\omega_2)\otimes\tilde{x}_2 \oplus(-3,-1,1,3,5,7,9;\omega_2)$$

Then the new problem becomes as follows:

$$Max\ \omega' = (-10,-6,-2,2,6,10,14;\min\ (\ \omega_1,\omega_2))\otimes\tilde{x}_1 \oplus(-9,-5,-1,3,7,11,15;\min(\ \omega_1,\omega_2))\otimes\tilde{x}_2\oplus(-7,-3,1,5,9,13,17;\min\ (\ \omega_1,\omega_2))$$

$$s.t$$
$$(-5,-3,-1,1,3,5,7;\ \omega_1)\otimes\tilde{x}_1\oplus(-3,-1,1,3,5,7,9;\omega_1)\otimes\tilde{x}_2 \leq (-1,1,3,5,7,9,11;\omega_1)$$
$$(-4,-2,0,2,4,6,8;\omega_2)\otimes\tilde{x}_1\oplus(-5,-3,-1,1,3,5,7;\omega_2)\otimes\tilde{x}_2 \leq (-4,-2,0,2,4,6,8;\omega_2)$$
$$\tilde{x}_1, \tilde{x}_2 \geq 0$$

The second step, is to convert the problem to the standard form by adding the fuzzy slack variables,

$$Max\ \widetilde{\omega'} = (-10,-6,-2,2,6,10,14;\min\ (\ \omega_1,\omega_2))\otimes\tilde{x}_1 \oplus(-9,-5,-1,3,7,11,15;\min(\ \omega_1,\omega_2))\otimes\tilde{x}_2 \oplus\tilde{0}\otimes\tilde{s}_1\oplus\tilde{0}\otimes\tilde{s}_2\oplus(-7,-3,1,5,9,13,17;\min\ (\ \omega_1,\omega_2))$$

$s.t$

$(-5,-3,-1,1,3,5,7;\ \omega_1)\otimes\tilde{x}_1\oplus(-3,-1,1,3,5,7,9;\omega_1)\otimes\tilde{x}_2\oplus\tilde{1}\otimes\tilde{s}_1\oplus \leq (-1,1,3,5,7,9,11;\omega_1)$

$(-4,-2,0,2,4,6,8;\omega_2)\otimes\tilde{x}_1\oplus(-5,-3,-1,1,3,5,7;\omega_2)\otimes\tilde{x}_2\oplus\tilde{1}\otimes\tilde{s}_2 \leq (-4,-2,0,2,4,6,8;\omega_2)$

$$\tilde{x}_1,\tilde{x}_2,\tilde{s}_1,\tilde{s}_2 \geq \tilde{0}$$

Suppose that, $\omega_1=\omega_2=1$, The first tableau of the fully fuzzy simplex is shown in **Table 2** below:

Table 2. Primary Table of the FFS Method

B.V		$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$R.H.S$	θ
A_0	$\widetilde{\omega}'\ominus\tilde{c}_j$	$(-14,-10,-6,-2,2,6,10;1)$	$(-15,-11,-7,-3,1,5,9;1)$	$(\tilde{0};1)$	$(\tilde{0};1)$	$(-7,-3,1,5,9,13,17;1)$	
A_1	$\tilde{s}_1$	$(-5,-3,-1,1,3,5,7;1)$	$(-3,-1,1,3,5,7,9;1)$	$(\tilde{1};1)$	$(\tilde{0};1)$	$(-1,1,3,5,7,9,11;1)$	5/3
A_2	$\tilde{s}_2$	$(-4,-2,0,2,4,6,8;1)$	$(-5,-3,-1,1,3,5,7;1)$	$(\tilde{0};1)$	$(\tilde{1};1)$	$(-4,-2,0,2,4,6,8;1)$	2/1

By the ranking, function techniques,

The entering variable $= min\{\Re(\widetilde{\omega}'\ominus\tilde{c}_1),\Re(\widetilde{\omega}'\ominus\tilde{c}_2)\} = min\{-2,-3\} = -3$

The leaving variable depends on θ

$$\theta = min\left\{\frac{\Re(-1,1,3,5,7,9,11;1)}{\Re(-5,-3,-1,1,3,5,7;1)},\frac{\Re(-4,-2,0,2,4,6,8;1)}{\Re(-5,-3,-1,1,3,5,7;1)}\right\} = \min\{\frac{5}{3},\frac{2}{1}\} = \frac{5}{3}$$

Therefore, the entering variable is $\tilde{x}_2$ and the leaving variable is $\tilde{s}_1$.

The pivot element of heptagonal fuzzy numbers is: $(-5,-3,-1,1,3,5,7;1)$

Now, use the operation of the heptagonal fuzzy numbers to find the new table as follows:

The pivot row $A_1{}' = \left(\frac{1}{3}\right)\otimes A_1$, and the other rows: $A_0{}' = A_0\oplus A_1$,

$A_2{}' = (-1\otimes A_1{}')\oplus A_2$

The new iteration of the fully fuzzy simplex method, as shown in **Table 3**:

Table 3. New Iteration for the FFS method

B.V		$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$R.H.S$
$A_0{}'$	$\widetilde{\omega}'\ominus\tilde{c}_j$	$(-19,-13,-7,-1,5,11,17;1)$	$(-18,-12,-6,0,6,12,18;1)$	$(0,0,\ldots,1;1)$	$(0,\ldots,0;1)$	$(-7.3,-2.6,2,6.6,11.3,16,20.6;1)$
$A_1{}'$	$\tilde{x}_2$	$(-1.6,-1,-0.3,0.3,1,1.6,2.3;1)$	$(-1,-0.3,0.3,1,1.6,2.3,3;1)$	$(0,0,\ldots,0.3)$	$(0,\ldots,0;1)$	$(-0.3,0.3,1,1.6,2.3,3,3.6;1)$
$A_2{}'$	$\tilde{s}_2$	$(-6.3,-3.6,-1,1.6,4.3,7,9.6;1)$	$(-8,-5.3,-2.6,0,2.7,5.3,8;1)$	$(-0.3,0,0,\ldots,0;1)$	$(0,0,\ldots,1;1)$	$(-7.6,-5,-2.3,0.3,3,5.6,8.3;1)$

Since, $\Re(\widetilde{\omega}\ominus\tilde{c}_1) < 0$,then repeated the steps to find the optimal solution as follows in **Table 4**.

Table 4. The Optimal Table Solution for the FFS method

B.V	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$R.H.S$
$\tilde{\omega}' \ominus \tilde{c}_j$	$\binom{-22.7,-15.1,-7.6,0}{,6.5,15.2,22.7;1}$	$\binom{-22.8,-15.1,-7.5,0}{,7.6,15.18,22.8\ ;1}$	(-0.2,0,0,0 0,0,0.2;1)	(0,…,0.6;1)	$\binom{-14.9,-7.6,-0.3,6.9,}{14.3,21.6,28.9;1}$
$\tilde{x}_2$	$\binom{-3.5,-2.4,-1.13,0,}{1.2,2.3,3.5;1}$	$\binom{-2.6,-1.36,-0.2,1,}{2.1,3.3,4.6;1;1}$	$\binom{0,0,\dots,}{0.06;1}$	$(-0.2,\dots,0;1)$	$\binom{-1.9,-0.8,0.4,1.5,}{2.7,4,5.1;1}$
$\tilde{x}_1$	$\binom{-3.7,-2.1,-0.6,1,}{2.5,4.2,5.7;1}$	$\binom{-4.8,-3.1,-1.5,0,}{1.6,3.18,4.8;1}$	$\binom{-0.2,0,0,}{\dots,0;1}$	$(0,0,\dots,0.6;1)$	$\binom{-4.5,-3.0,-1.38,0.18,}{1.8,3.36,4.98;1}$

Finally, $\Re(\tilde{x}_1) = 0.204$, $\Re(\tilde{x}_2) = 1.5$ and $\Re(\tilde{\omega} \ominus \tilde{c}_j) \geq 0$ .So , the optimal solution is reached.

$$Max\ \tilde{\omega} = \frac{(-2,0,2,4,6,8,10;1)\otimes\binom{-4.5,-3.0,-1.38,0.18,}{1.8,3.36,4.98;1}\oplus(0,2,4,6,8,10,12;1)\otimes\binom{-1.9,-0.8,0.4,1.5,}{2.7,4,5.1;10;1}\ominus(-4,-2,0,2,4,6,8;1)}{(0,2,4,6,8,10,12;1)\otimes\binom{-4.5,-3.0,-1.38,0.18,}{1.8,3.36,4.98;1}\oplus(3,5,7,9,11,13,15;1)\otimes\binom{-1.9,-0.8,0.4,1.5,}{2.7,4,5.1;10;1}\oplus(-3,-1,1,3,5,7,9;1)}$$

$$Max\ \tilde{\omega} = \frac{\Re(\ 1,-7.6,-5.16,7.72,32.4,68.88,115;1)}{\Re(\ -8.7,-11,-1.72,17.58,49.1,92.6,145.2;1)} = \frac{37.5252}{47.7252} = 0.7863.$$

While, the crisp optimal solution of the problem is $\tilde{x}_1 = 0.2$, $\tilde{x}_2 = 1.6$, $\omega = 0.5$.

8. Conclusion

The paper proposes a novel ranking function for heptagonal fuzzy numbers based on the proposed generalized heptagonal membership function. The algorithm of the fully fuzzy simplex method with the help of the proposed ranking function is suitable for finding the optimal fuzzy solution of a fully fuzzy fractional linear programming problem. Through a numerical example, it proved that the optimal solution obtained for a fully fuzzy fractional linear programming problem using the arithmetic operations of heptagonal fuzzy numbers is more efficient according to the crisp solution of the problem.

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Conflict of Interest

The authors declare that they have no conflicts of interest.

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