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On Nano Topological Spaces with Ideal Semi-p-open Sets

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Abstract
The purpose of this paper is to introduce a new type of spaces called nano topological spaces and investigate the relation between nano topological space and nano ideal semi preopen set. The objective of this study is to develop a nano topological model. We study this model from three aspects, which are lower, upper approximation and their properties. Basic properties and characterizations related to these sets are given. And define the concept Nano ideal semi preopen set by using nano topological space and some properties of this set. The main aim of rough sets is to increase the accuracy measure and reduce the boundary region of sets by increasing the lower approximations and decreasing the upper approximations.
Keywords: Nano pre-open sets, Nano pre-closed sets, Nano ideal semi-p-open sets, Nano ideal semi-p-closed sets.

1. Introduction
An ideal I on a non-empty set M is a nonempty is defined as a collection of subsets that satisfies (i)A ∈ I and B ⊆ A implies B ∈ I. (heredity property). (ii) A ∈ I and B ∈ I implies A ∪ B ∈ I. (additively property) (10-11, 20-29, 31). Given a topological space (X, T) with an ideal I on X and if P(M) is the set of all subsets of M, a set operator $(\ )^*$: P(M) → P(M), called a local function of A with respect to T and I is defined as follows: for A ⊂ M, A^*(I, T) = {x ∈ X : U ∩ A ∈ I for every U ∈ T(m)}, where T (m) = {U ∈ T: m ∈ U}. (21- 22, 9).
A mapping ψ: ℙ(M) → ℙ(M) defined as ψ(A)= A ∪ A^* for all A ∈ ℙ(M), where ℙ(M) denotes the power set of M.
The map ψ satisfies the Kuratowski closure axioms (15-16, 8):
I. ψ (ϕ)= ϕ.
II. If A ⊆ B, then ψ(A) ⊆ ψ(B).
III. If A ⊆ M, then ψ (ψ (A)) = ψ(A).
IV. If A and B ⊆ M, ψ (A ⋃ B) = ψ (A) ⋃ ψ (B).

Furthermore, the Kuratowski (8) closure operator represented the closure of a set in topology T^*(I, T), called T^*- topology, finer than T, which is defined and denoted by CL^*(A) = A ∪ A^*, and then T^*(I, T) = {A ⊆ M: CL^*(M – A) = (M – A)}. The collection 𝔹 (I, T) = {A – B; A ∈ T and B ∈ I} is a basis for T^*(T, I).

When there is no chance for confusion, the simple A^* write for A^*(I, T) and T^* for T^*(I, T). Then the notion (M, T, I) will denoted to a topological space (M, T) with an ideal I on M, with no separation properties assumed and called an ideal topological space or an ideal topological space for short.

The elements of T^* are called T^*-open sets. If A^c is a T^*-open set, then A is called T^*-closed and so it is closed in the topological space (M, T^*). A subset A of an ideal topological space (M, T, I) is a T^*-closed if and only if $A^* \subseteq$ A. A subset A of an ideal topological space (M, T, I) is said to be T^* dense if CL^* (A)= M. It is clear that, in (M, T, I), If I= $\{\emptyset\}$ in (M, T, I), then T= T^*(I, T). If A $\subseteq$ M, the interior of A denoted by int^* (A). (1-7,14)

By using the nano topological space, a new kind of nano open set is presented which is nano-I-SPO set with remarks explanatory table for this kind of set. The notion of a nano topology was introduced by (12-17) in terms of upper (lower) approximations and their boundary regions. They also defined nano closed sets and some nano topological operators such as nano interior and nano closure operators. Parimala et al. used topological ideals to study nano ideal topological spaces. Some properties of nano ideal spaces are studied in (2-13, 24)

2. Materials and Methods

Definition 2.1: Consider a mathematical structure denoted by K = (M, R), M a set, R is a broad binary relation. This structure is known as an approximation space. Additionally, let T_R denote the topology associated with K. The triple K = (M, R, T_R) is commonly referred to as a topologized approximation space in academic literature (5-24).

Definition 2.2: Let M denote a nonempty set and R denote an equivalence relation on M. Let A be a subset of M. subsequently (5-18, 26):

- Lower approximation of set A with t relation R is represented as L_R(A). In other words, set L_R(A) is present the union of all equivalence classes R(m) such that R(m) a subset of A and x belongs to M, where R(m) represents the equivalence class specified by m belonging to M.
- Upper approximation of set A with respect to relation R is represented as U_R(A). The equation be expressed as follows: U_R(A) is equal to the union of any R(m) such that intersection of R(m) and A is not empty, where m belongs to the set M.
- Boundary area of set A with relation to set R is represented as B_R(A). In other words, set of boundary points B_R(A) equal to upper limit set U_R(A) minus the lower limit set L_R(A).

Property 2.3: If (M, R) is an approximation space and A, B are subsets of M, then (5-25):

- L_R(A) $\subseteq$ A $\subseteq$ U_R(A).
- $L_R(\varphi)$ = $U_R(\varphi)$= φ
- L_R(M)=U_R(M)=M
- U_R(A∪B) =U_R(A) ∪ U_R(B).
- U_R(A∩ B)⊆ U_R(A)∩U_R(B).
- L_R(A∪B) ⊇L_R(A)∪ L_R(B).
- L_R(A∩B) = L_R(A)∩ L_R(B).
- L_R(A)⊆ L_R(B) and U_R(A)⊆U_R(B) whenever A⊆B.
- $U_R(A^c)$ = $[L_R(A)]^c$ and $L_R(A^c)$ = $[U_R(A)]^c$
- $U_R[U_R$(A)] = $L_R[L_R$(A)] = U_R(A).
- $L_R[L_R$(A)] = $U_R[L_R$(A)] = L_R(A).

Definition 2.4: Let M represent universe, R denote an equivalence on M, and T_R(A) be the set containing M, φ, L_R(A), U_R (A), and B_R(A), where A is a subset of M. According to property 2.2, it may be concluded that T_R(A) adheres as the following (6-23, 27-29):

- M, $\varphi \in T_R$(A).
- The union of the elements of any sub-collection of T_R(A) is in T_R(A).
- The intersection of the elements of any finite subcollection of T_R(A) is in T_R (A).

This means that T_R (A) is a topology on X called the nano topology on X with respect to A and (X, T_R (A)) as a nano topological space. The elements of T_R(A) are called nano-open sets (briefly N-open sets). The complement of N-open set is called nano-closed denoted by N-closed set. The nano-interior of a subset A of X denoted by N-int(A) and nano-closure of a subset A of X denoted by N-CL(A).

Example 2.5:

Let M= {m, n, z}. R a relation defined on M, where R= {(m, m), (n, n), (z, z), (m, z), (z, m)}. Additionally, R_m = R_z= {m, z}. The equivalence class R_n= {n}. So, M/R= {{n}, {m, z}}, From **Table (1)** can be calculated U_R(A), L_R(A), B_R(A), and from these can be calculated T_R (A) (**Table 2**).

Table 1. Nano topological space of A.

P(M)	U_R(A)	L_R(A)	B_R(A)	T_R(A)
M	M	M	ϕ	{M,ϕ}
ϕ	ϕ	ϕ	ϕ	{M,ϕ}
{m}	{m, x}	ϕ	{m, z}	{M,ϕ,{m, z}}
{n}	{n}	{n}	ϕ	{M,ϕ,{n}}
{z}	{m, z}	ϕ	{m, z}	{M,ϕ,{m., z}}
{m, n}	M	{n}	{m, z}	{M,ϕ,{n}, {m, x}}
{m., z}	{m, z}	{m, x}	ϕ	{M,ϕ,{m, z}}
{n, z}	M	{n}	{z, z}	{X,ϕ,{n}, {z, z}}

Table 2. Nano Pre-Open Set.

P(M)	T_R(A)	N-PO(A)	N-PC(A)
M	{M,ϕ}	P(M)	P(M)
ϕ	{M,ϕ}	{M,ϕ}	P(M)
{m}	{M,ϕ,{m, z}}	{M,ϕ,{m}, {z}, {m, n}, {m, z}, {n, z}}	{M,ϕ,{m}, {n}, {x}, {m, n}, {n, z}}
{n}	{M,ϕ,{n}}	{M,ϕ,{n}, {m, n}, {n, z}}	{M,ϕ,{m}, {z}, {m, z}}
{z}	{M,ϕ,{m, z}}	{M,ϕ,{m}, {z}, {m, n}, {m, z}, {n, z}}	{M,ϕ,{m}, {n}, {x}, {m, n}, {n, z}}
{m, n}	{M,ϕ,{n}, {m, x}}	P(M)	P(M)
{m, z}	{M,ϕ,{m, x}	{M,ϕ,{m}, {z}, {m, n}, {m, z}, {n, z}}	{M,ϕ, ,{m}, {n}, {z}, {m, n}, {n, z}}
{n, z}	{M,ϕ,{n}, {m, x}}	P(M)	P(M)

Definition 2.6: Let (M, T_R(A) be a nano topological space and A ⊆ M. Then A is said to be Nano p-open if A ⊆ N-int(N-cl(A)) The complement of A is called nano p-closed set. The shortcuts N-PO(M) (respectively, N-PC(M)) it is the family of all Nano p-open (respectively, Nano p-closed) sets (4-19, 30).

In a nano topological space (M, T_R(A)), such that A ⊆ M and from the next table we can be calculated N-PO(M) and N-PC(M).

Definition 2.7: Let M be a non-empty set and R be an equivalence relation on M. Suppose A is a subset of M, so R is defining the accuracy M by using the concepts of lower and upper approximation (3).

$Y(A)=\frac{|L_R(A)|}{|U_R(A)|}$; $|U_R(A)| \neq \emptyset$

The second accuracy measure of A is referred to as the semi-accuracy measure of approximation.

Y_s(A)=$\frac{|U_R(L_R(A))|}{|U_R(A)|}$; $|U_R(A)| \neq \emptyset$

The third measure of accuracy for A is referred to as the pre-accuracy measure of approximation.

Y_P(A)=$\frac{|L_R(U_R(A))|}{|U_R(A)|}$; $|U_R(A)| \neq \emptyset$

Example: Let M= {m, n, z}. If we say R a relation defined for M, where R= {(m, m), (n, n), (z, z), (m, n) (n, m)} The equivalence relation R_m =R_n= {m, n}. Additionally, the equivalence relation R_z ={z}. Therefore, the quotient set M/R= {{m, n}, {z}}. From **Table (3)** can be calculated U_R(A), L_R(A), and B_R(A). Subsequently, from these can be calculated accuracy (**Table 4**).

Table 3. Accuracy for A

P(M)	U_R(A)	L_R(A)	B_R(A)	$L_R(U_R$(A))	$U_R(L_R$(A))
M	M	M	ϕ	M	M
ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
{m}	{m, n}	ϕ	{m, n}	{m, n}	ϕ
{n}	{m, n}	ϕ	{m, n}	{m, n}	ϕ
{z}	{z}	{z}	ϕ	{z}	{z}
{m, n}	{m, n}	{m, n}	ϕ	{m, n}	{m, n}
{m, z}	M	{z}	{m, n}	M	{z}
{n, z}	M	{z}	{m, n}	M	{z}

Table 4. Accuracy measure for A

P(M)	Y(A)	Y_s(A)	Y_p(A)
M	1	1	1
{m}	0	0	1
{n}	0	0	1
{z}	1	1	1
{m, n}	1	1	1
{m, z}	$1/3$	$1/3$	1
{n, z}	$1/3$	$1/3$	1

3. Nano Ideal Semi-P-Open Set

This section and using the notion ideal semi pre-open set and N-pre-open set

Definition 3.1: In nano ideal topological space (M,T_R(A), I), A is called nano ideal semi-P-open set dented by NI-SPO if U- A ∈ I and A- (p- cl (U)) ∈ I such that U is a nano p-open subset of M. The family of all nano ideal semi-p-open sets in (M,T_R(A), I) dented by NI-SPO(M), and the family of all nano ideal semi-p- closed sets is called NI-SPC(M).

From **Table (5)** can be calculated and note all nano I-SPO set from the nano ideal topological space (M, T_R(A), I), where;

M= {m, n, z}, R= {(m, m), (n, n), (z, z), (m, z), (z, m)}, I={{M},∅}, and $R_{(m)}$=$R_{(z)}$= {m, z}, $R_{(n)}$={n}, so, M /R= {{m, z}, {n}}.

So, we introduce and investigate the concepts of lower and upper approximations for nano ideal topological space and will be studied and show that the accuracy for NI-SPO set is the best approximations.

So, we can take the sets: A= {m, n}, NI-SPO(M) =P(M), R= {(m, m) (n, n), (z, z)}, M/R={{m}, {n}, {z}}, then **Tables (6** and **7**):

Table 5. Nano I-SPO set

P(M)	U_R(A)	L_R(A)	B_R(A)	T_R(A)	NI-SPO(M)
M	M	M	ϕ	{M,ϕ}	P(M)
ϕ	ϕ	ϕ	ϕ	{M,ϕ}	P(M)
{m}	{m, x}	ϕ	{m, z}	{m,ϕ,{m, z}}	{M,ϕ,{m}, {z}, {m, n}, {m, z}, {n, z}}
{n}	{n}	{n}	ϕ	{M,ϕ,{n}}	{M,ϕ,{n}, {m, n}, {n, z}}
{z}	{m, z}	ϕ	{m, z}	{M,ϕ,{m, z}}	{M,ϕ,{m}, {z}, {m, n}, {m, z}, {n, z}}
{m, n}	M	{n}	{m, z}	{M,ϕ,{n}, {m, z}}	P(M)
{m, z}	{m, z}	{m, z}	ϕ	{M,ϕ,{m, z}}	{M,ϕ,{m}, {z}, {m, n}, {m, z}, {n, z}}
{n, z}	M	{n}	{m, z}	{M,ϕ,{n}, {m, z}}	P(M)

Table 6. Accuracy for NI-SPO sets

NISPO(M)	U_R(NISPO(M))	L_R(NISPO(M)	B_R(NISPO(M))	$L_R(U_R$(NISPO(M)))	$U_R(L_R$(NISPO(M)))
M	M	M	ϕ	M	M
ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
{m}	{m}	{m}	ϕ	{m}	{m}
{n}	{n}	{n}	ϕ	{n}	{n}
{z}	{z}	{z}	ϕ	{z}	{z}
{m, n}	{m, n}	{m, n}	ϕ	{m, n}	{m, n}
{m, z}	{m, z}	{m, z}	ϕ	{m, z}	{m, z}
{n, z}	{n, z}	{n, z}	ϕ	{n, z}	{n, z}

Table 7. Accuracy measure for NI-SPO sets

NI-SPO(M)	M(NI-SPO(M))	M_s(NI-SPO(M))	M_p(NI-SPO(M))
M	1	1	1
{m}	1	1	1
{n}	1	1	1
{z}	1	1	1
{m, n}	1	1	1
{m, z}	1	1	1
{m, z}	1	1	1

We notice through the previous example that the accuracy of the sets under study is (1), and this example the reason for generalizing the concept of nano semi-open set using ideal

4. The Application in Nano Ideal Topological Space

A middle ear infection, also known as otitis media, is characterized by an infection that specifically targets the anatomical region situated posterior to the eardrum. The eardrum, a delicate membrane, serves as a barrier between the middle ear and the external auditory canal. The middle ear is a hollow chamber situated within the cranial bones and is characterized by a mucous membrane that resembles the lining of the nasal and oral cavities. This chamber is connected to the posterior region of the nasal cavity through a narrow passage called the "Eustachian tube," which typically remains closed until it opens during the act of swallowing. The channel facilitates the ingress of fresh air into the middle ear, for the purpose of replenishing oxygen levels depleted by the lining of the middle ear. Additionally, this process is essential for equalizing the pressure in the middle ear with the atmospheric pressure outside the cranial cavity. Otitis media is characterized by a cluster of signs and symptoms that often manifest a few days after the onset of a cold or upper respiratory infection. These include but are not limited to: impaired equilibrium and the presence of aural secretions. The medical conditions under consideration include anorexia nervosa, otalgia, and auditory impairment.

The following **Table (8)** shows input about four injured {x, y, z, r}, we will mark to the sign (yes) if the symptoms are clear to the person and refer the sign (no) if the symptoms do not show.

Table 8. Information of Middle ear infection

Patient person	Loss of balance (L)	Secretions from the ear (S)	$\mathcal{A}$norexia ($\mathcal{A}$)	Pain in the ear (P)	Hearing impairment (H)	Middle ear infection (M)
m	yes	yes	yes	yes	yes	yes
n	yes	no	yes	yes	no	yes
z	no	no	yes	yes	no	no
r	no	no	No	yes	no	no

Let M, indicated as M= {m, n, z, r} in the table below, represent the group of patients with high blood urea. Assume that A= {m, z} is a subset of M. Let R be the equivalency relation specified on M as well. A relation R is the set M if and only if R = {(m_i, m_j); m_i, m_j∈ M}, where m_i and m_j share the same set of symptoms. The following determines the set of equivalency classes that match to R: M/R = {m, n, z, r} take into account the T_R(A) set with recognize to A, and in this context:

T_R(A) = {M, ∅, {m, z}}

We outline I as follows:

I = {∅, {n}, NPO(M) = {M, ∅, m, n, {m, n}, {m, z}, {m, r}, {n, z}, {z, r}, {m, n, z}, {m, n, r}, {m, z, r}, {n, z, r}}

Furthermore, NI-SPO(M) is described as:

NI-SPO(M) = {M, ∅, m, z, {m, n}, {m, z}, {m, r}, {n, z}, {z, r}, {m, n, z}, {m, n, r}, {m, z, r}, {n, z, r}}

If we were to cast off the column representing Loss of balance (L), the ensuing equation M/R-(L) might be:

M/R-(L) = {m, r, {n, z}}

And the equation T_R(A)-(L) might be:

T_R(A)-(L) = {m, {n, z}, {m, n, z}, M, ∅}

In this context, NPO(M) is:

NPO(M) = {M, ∅, m, n, z, {m, n}, {m, z}, {n, z}, {m, n, z}, {m, n, r}, {m, z, r}}

And NI-SPO(M) is:

NI-SPO(M) = {M, ∅, m, n, z, {m, n}, {m, z}, {n, z}, {m, n, z}, {m, n, r}, {m, z, r}, {n, z, r}}

If we have been to put off the Secretions from the ear (S) column, then:

M/R-(S) = m, n, z, r

And T_R(A)-(S) would be:

T_R(A)-(S) = {M, ∅, {m, z}}

In this context, NPO(M) is:

NPO(M) = {M, ∅, m, z, {m, n}, {m, z}, {m, r}, {n, z}, {z, r}, {m, n, z}, {m, n, r}, {m, z, r}, {n, z, r}

NI-SPO(M) is:

NI-SPO(M) = {M, ∅, m, z, {m, n}, {m, z}, {m, r}, {n, z}, {z, r}, {m, n, z}, {m, n, r}, {m, z, r}, {n, z, r}.

5. Results

From all, core(R) = {L, $\mathcal{A}$}. That is mean the Loss of balance. The presence of anorexia, akin to that of ammonia, can be considered both essential and sufficient to infer the presence of a middle ear infection in an individual who is damaged. The preceding data might be presented in the following **Table (9).**

Table 9. Effective symptoms

Equivalent classes	Nano-topology	Nano I-SPO(M)	Nano I-SPC(M)
M/R={{m}, {n}, {z}, {r}}	T_R(A)= {M, ∅, {m, z}}	{M,ϕ,{m}, {z}, {m, n}, {m, z}, {m, r}, {n, z}, {z, r}, {m, n, z}, {m, n, r}, {m, z, r}, {n, z, r}}	{M,ϕ,{m}, {n}, {z}, {r}, {m, n}, {m, r}, {n, z}, {n, r}, {z, r}, {m, n, r}, {n, z, r}
M/R-(L) = {{m}, {r}, {n, z}},	T_R(A)-(L)= {{m}, {n, z}, {m, n, z}, M,ϕ},	{M,ϕ,{m}, {n}, {z}, {m, n}, {m, z}, {n, z}, {m, n, z}, {m, n, r}, {m, z, r}},	{M,ϕ,{n}, {z}, {r}, {m, r}, {n, r}, {z, r}, {m, n, r}, {m, z, r}, {n, z, r}}
M/R-(S)={{m}, {n}, {z}, {r}}	T_R(A)-(S)= {M,ϕ,{m, z}}	{M,ϕ,{m}, {z}, {m, n}, {m, z}, {m, r}, {n, z}, {z, r}, {m, n, z}, {m, n, r}, {m, z, r}, {n, z, r}}	{M,ϕ,{m}, {n}, {z}, {r}, {m, n}, {m, r}, {n, z}, {n, r}, {z, r}, {m, n, r}, {n, z, r}
M/R-($\mathcal{A}$)={{m}, {n}, {z, r}}	T_R(A)($\mathcal{A}$)={M,ϕ,{m, n}}	P(M)	P(M)
M/R-(P)={{m}, {n}, {z}, {r}}	T_R(A)(P) = {M,ϕ,{m, z}}	{M,ϕ,{m}, {z}, {r}, {m, n}, {m, z}, {m, r}, {n, z}, {m, r}, {z, r}, {m, n, z}, {m, n, r}, {m, z, r}, {y, z, r}}	{M,ϕ,{m}, {n}, {z}, {r}, {m, n}, {m, z}, {m, r}, {n, z}, {n, r}, {z, r}, {m, n, z}, {m, n, r}, {n, z, r}
M/R-(H)={{m}, {n}, {z}, {r}}	T_R(A)(H) = {M,ϕ,{m, z}}	{M,ϕ,{m}, {z}, {m, n}, {m, z}, {m, r}, {n, z}, {z, r}, {m, n, z}, {m, n, r}, {m, z, r}, {n, z, r}}	{M,ϕ,{m}, {n}, {z}, {r}, {m, n}, {m, r}, {n, z}, {n, r}, {z, r}, {m, n, r}, {n, z, r}

6. Conclusion

Rough set theory has been widely recognized as a form of generalization of classical set theory in a particular manner. Moreover, this mathematical tool is assent ill for addressing the issue of ambiguity, specifically in relation to uncertainty. The boundary area technique is commonly associated with the concept of vagueness, which refers to the presence of elements that cannot be definitively classified as belonging to either the set or its complement. This paper is divided into three pieces. The initial section of the study presents an introduction to and examination of the notion of ideals within the context of topological spaces. The second section presents the definition of the partition that characterizes a topological space, referred to as an approximation space. Additionally, certain aspects of this approximation space are introduced. In the third portion, the concept of ideal semi p-open set and N-p-open set is utilized to determine and record all nano I-SPO sets within the nano ideal topological space. An applied case was shown, which elucidated the process of generating interest from a nano ideal semi p-open set.

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Conflict of Interest

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