Derivation of Mathematical Equations to Calculate the Geographical Coordinates of Unknown Position Situated at a Distance from the Observer Position Using GPS Data

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Abstract

This research introduced the derivation of mathematical equations to calculate the Cartesian and geographical coordinates of a site situated at a far distance from the observer position by using GPS data. The geographical coordinates ($\varphi_{obs.}$, $\lambda_{obs.}$, $h_{obs.}$) for observer position were transformed to Cartesian coordinates ($X_{obs.}$, $Y_{obs.}$, $Z_{obs.}$) of observer position itself. Then the Cartesian coordinates of unknown position mathematically were calculated from these calculated equations, and its transformed to geographical coordinates of ($\varphi_{unk.}$, $\lambda_{unk.}$) position.

Keywords:Global Positioning System (GPS), Geographical & Cartesian Coordinates, Geodetic science, World Geodetic System (WGS-84).

* (Amaal, J. H. (2006), Position Evaluation for a Distance Point Using GPS and LRF Techniques, PhD)

Introduction

The spheroid or geodetic latitude of a point on a spheroid is defined to be the angle between the normal at the point and the plane of the spheroid equator [1]. The geodetic longitude of a point on a spheroid is the angles between the meridians plane through this point and an arbitrarily defined zero meridian planes. This zero meridian plane may be defined by a point on the spheroid, or (actually) otherwise. The position of points being defined by latitude and longitude on a prescribed spheroid, the rational definition of the height of any point is its distance above the spheroid measured along the spheroid normal. Such heights are called "***spheroid heights***". Spheroid heights are required for some purposes, but the geoid or mean sea-level surface has great significance, and a more generally useful height is the distance (whether it is measured along the vertical or the normal is immaterial) above the geoid. Such heights are called "***geoidal heights***" and, unless otherwise stated, the height of a point implies its geoidal height. For common use it is essential that the zero height contours should lie close to mean sea-level, and spheroid heights are not an acceptable basis for topographical contouring [2].Definitions of heights or levels requires consideration of further parameters and reference levels or surface, and are relevant to some of the vertical coordinate systems of epicenter. Additionally they become relevant when considering three dimensional Cartesian coordinates [3].

Geographic Coordinate

Geographic coordinates of positions are always expressed as a latitude and longitude. The latitude and longitude are related to a particular earth figure, which may be a sphere for most atlas maps and imprecise work, but for rigorous purposes is a mathematical figure which much more closely approximates the shape of the real earth. The earth's shape is assumed to be an ellipsoid of revolution, (the figure defined by an ellipse rotated round its minor axis), sometimes (though not in ***epicenter***) termed a spheroid.

For historical reasons there are several such ellipsoids in use for mapping different countries of the world , each is defined by the length of its semi–major axis , a bout (6378.137 km) and either its semi–minor axis or , more usually , by the reciprocal of its flattening, a figure about (298 km) .

Each of these ellipsoids is generally matched to one or several countries. An ellipsoid appropriate for one region of the earth is not always appropriate to other regions [3].

Another property is needed to uniquely specify geographical positions. This is the position and orientation of the ellipsoid relative to the earth. The term used to describe this fitting of an ellipsoid to the earth is the geodetic datum. Many geodetic datum's exist throughout the world, each usually associated with the national survey of particular country or continent. More recently several new geodetic datum's have been successively derived, from steadily accumulating satellite and other data, to provide for a best worldwide fit [2].

All precisely surveyed and mapped points and features on the earth's surface will be uniquely defined in position by stating their coordinates, (grid or geographical), the projection if grid, and the geodetic datum, including its earth ellipsoid. Therefore, in order to be unambiguous and precise in the definition of position it is essential that as well as quoting latitude , longitude and height it is also necessary to specify the geodetic datum , including ellipsoid , and the level to which the height coordinate value are related [3].

Cartesian Coordinates

The earth being supposed to be rigid, the reference system must be fixed to its center. The Z-axis is taken parallel to the mean axis of rotation. The X-axis is parallel to the corresponding equator, in the meridian of Greenwich, and the Y-axis is perpendicular to them, towards (90º) east. These axes are parallel to those conventionally adopted for geodetic reference spheroids. The origin can be taken at the center of any convenient reference spheroid on which one of the astro–triangulation stations can be located. *(Note: It is not essential that the coordinates accepted for the opening station should be associated with any spheroid and datum which may be in current use, but it may be convenient to have it so)*[2].

Global Positioning system (GPS)

The Global Positioning System is a constellation of satellites which orbit the earth twice a day, transmitting precise information of time and position with a GPS receiver; users can determine their location anywhere on earth. Position and navigation information is vital to a broad range of professional and personal activities, including boating, surveying, aviation, national defense, vehicle tracking , navigation, more [4].

GPS is a satellite–based system that uses a constellation of (24 satellites) to give a user an accurate position [5, 6].The GPS is a network of satellites that continuously transmit coded information, which makes it possible to precisely identify locations on earth by measuring distance from the satellites [7].

The determination of position may be described as the process of triangulation using the measured range between the user and many satellites. The ranges are inferred from the time of propagation of the satellite signals. Four satellites are required to determine the three coordinates of position and time. The time is involved in the correction to the receiver clock and is ultimately eliminated from the measurement of position. The design of the GPS constellation had the fundamental requirement that at least four satellites must be visible at all times from any point on earth[8].

GPS Technical Overview

The technical and operational characteristics of GPS are organized into three distinct segments:

1. The space segment.
2. The operational control segment.
3. The user equipment segment.

The GPS signals, which are broadcast by each satellite and carry data to both user equipment and the ground control facilities, link the segments together into one system. The GPS system is shown in fig. (1) [9].

GPS Receiver Technology

The basis of GPS technology is precise time and position information. A GPS receiver receives the signals, listening to three or more satellites at once, to determine the user's position on earth [4].

A GPS receiver cannot work inside building which shield, the high- frequency satellite transmissions. The user must therefore first find the right place to stand, preferably in a fairly level area with a full horizon, before switching on the GPS receiver [10].

Next it begins to acquire the satellite signals and may tell the user when it has a good connection by displaying the signal strength. When signals from three satellites are being received, it may perform a first, rough calculation of the geographical position and inform about the longitude and latitude. When more satellites are acquired, it will add the altitude above the sea level [6].

World Geodetic System (WGS-84)

The standard physical model of the earth which is used for GPS applications is the ***World Geodetic System-1984(WGS-84)***[11].

One part of WGS-84 is a detailed model of the Earth's gravitational irregularities. Such information is necessary to derive accurate satellite ephemeris information;however, the estimating of latitude, longitude, and height of a GPS receiver are concerned. For this purpose, WGS-84 provides an ellipsoidal model of the earth's shape, as shown in fig. (2). In this model, cross–sections of the earth parallel to the equatorial plane are circular. The equatorial cross section of the earth has a radius of (6378.137 km.), which is the mean equatorial radius of the earth. In the WGS-84 earth model, cross-sections of the earth normal to the equatorial plane are ellipsoidal. In an ellipsoidal cross-section containing the Z-axis, the major axis coincides with the equatorial diameter of the earth. Therefore, the semi-major axis (a) has the same value as the mean equatorial radius given above. The minor axis of the ellipsoidal cross-section shown in fig.(2) corresponds to the polar diameter of the earth; the semi-minor axis (b) in WGS-84 is taken to be (**6356.752** km.) Thus, the eccentricity of the earth ellipsoid (e) is: [12]

$$\mathbf{e}=\sqrt{\frac{\left(\mathbf{a}^2-\mathbf{b}^2\right)}{\mathbf{a}^2}} \quad \text{------------------ (1)}$$

Geographic Coordinate System Transformation Formula

The latitude (φ)is the angle measured between the equatorial plane and normal line of the ellipsoid and longitude **(**λ**)** is the angle measured from the ***Greenwich meridian***[13].
Latitude (φ) and longitude (λ), in terms of geographic coordinate system may be expressed in terms of a geocentric (earth–centered) Cartesian coordinate system(X,Y, Z**)** with the Z-axis corresponding with the polar axis positive northwards, the X-axis through the intersection of the Greenwich meridian and equator, and the Y-axis through the intersection of the equator with longitude (90º) E.

Geocentric coordinate systems are conventionally taken to be defined with X-axis through the intersection of the Greenwich meridian and equator. This requires that the equivalent geographic coordinate system is based on the ***Greenwich meridian***.
In application of the formulas below, geographic coordinate system based on a non-Greenwich prime meridian should first be transformed to their Greenwich equivalent.
If the earth's spheroid semi-major axis is (a) equal (6378.137 km.), and semi-minor axis is (b) equal (6356.852 km.), the Cartesian coordinate system(X, Y, Z) equationsare[14].

$$X=(N+h).\cos\varphi.\cos\lambda......(2)$$

$$Y=(N+h).\cos\varphi.\sin\lambda........(3)$$

$$Z=\left(\frac{b^2}{a^2}.N+h\right).\sin\varphi..........(4)$$

Where **N** is the prime vertical radius of curvature at latitude **φ** and is equal to:

$$N=\frac{a}{\sqrt{\left(1-e^2.\sin^2\varphi\right)}} \quad \text{-------------- (5)}$$

φ and **λ** are respectively the latitude and longitude (related to the prime meridian) of the point, **h** is height above the ellipsoid [15].

Methodology

The GPS receiver was installed on WGS-84, and the geographical coordinates (latitude, longitude and height) of the site were determined also the units of the coordinates (deg., meter) were determined. The following steps are then done:

1. After selecting of appropriate position of the observer according to the unknown site. Four satellites were considered to determine the geographical coordinates of the observer position.
2. An approximate value of the distances between the observer position and the unknown position of the site (i.e. the range(R)) was measured by using the laser rangefinder.
3. Using equation (2, 3 and 4) the geographical coordinates (φ_i, λ_i, h_i) of the observer were transformed to Cartesian coordinates (X_i, Y_i, Z_i), where (i = 1, 2, 3, 4…).
4. Derivation of the mathematical equations to calculatethe Cartesian coordinates of the unknown position in three dimensions ($X_{unk.}$, $Y_{unk.}$, $Z_{unk.}$).
5. These Cartesian coordinates of the unknown position transform to the geographic coordinates by using the equations (6 and 7):[2, 16].

$$\varphi_{unk.} = \tan^{-1}\left(\frac{Z_{unk.} + N_i.e^2.\sin\varphi_1}{\sqrt{X_{unk.}^2 + Y_{unk.}^2}}\right) \text{---------} (6)$$

$$\lambda_{unk.} = \tan^{-1}\left(\frac{Y_{unk.}}{X_{unk.}}\right) \text{-------------------------} (7)$$

Results and Discussions

The importance of this research is to solve the problem of determining the unknown position coordinates for a point or site from a long distance. In derivation these equations in order to find the unknown position, four measuring positions of the observer at a distance from the unknown position using the GPS receiver were required. Then the distances between the observer positions and unknown position are measured using the laser rangefinder (LRF). Using thisderivation, the final form of the three equations necessary to determine the Cartesian coordinates of the unknown positionwere found.

The derivation of a mathematical equations to measure the Cartesian coordinates of the unknown position in three dimensions (X_{unk}, $Y_{unk.}$, $Z_{unk.}$) situated at a distance from theobserver position. By useing four observer positions (X_i, Y_i, Z_i) and the range (R_i) between the observer and unknown positions, the resulted equations werefoundations:

The X-axis of the Cartesian coordinates:

The Y-axis of the Cartesian coordinates:

$$Y_{unk.} = -\frac{1}{D_6}[D_4 + D_5\, X_{unk.}] \text{...............}(9)$$

The Z-axis of the Cartesian coordinates:

$$Z_{unk.} = \frac{1}{(Z_2 - Z_1)}\left[C_1 + (X_1 - X_2)\left(\frac{D_3D_4 - D_1D_6}{D_2D_6 - D_3D_5}\right) + \frac{D_2}{D_3}(Y_1 - Y_2)\left(\frac{D_1D_6 - D_3D_4}{D_2D_6 - D_3D_5}\right) - \frac{D_1}{D_3}(Y_1 - Y_2)\right] \text{......}(10)$$

After that the calculation of the unknown position in three dimensions (X_{unk}, $Y_{unk.}$, Z_{unk}) by the deviated equations could be done. Then, this research could perform another transformation from the Cartesian coordinates of unknown position ($X_{unk.}$, $Y_{unk.}$, $Z_{unk.}$) to the geographic coordinates($\varphi_{unk.}$, $\lambda_{unk.}$).

Conclusions and Suggestions

From the obtained result, the following conclusions and suggestions can be derived:

1. 1.From the final deviated equations (8, 9, and 10) which are used to measure the unknown position that is situated at a distance from the observer position must be found four observers to hold the GPS receiver.
2. LRF must be found to give the value of range between the observer and the unknown position.
3. These mathematical equations are successful in the practical applications.

References

1. Ibadi, A. G. (1980), Map projections, Baghdad University, Department of Engineering Surveying.
2. Bomford, G. (1977), Geodesy, 3rd ed., Oxford University.
3. Htm, Epicentre User Guide, POSC Specifications(1997),Coordinate Systems-A Deeper Understanding, Geography Department, University Of Texas.
4. Kumar, B. H. ;Reddy, K. C. and Namassivaya, N. (2008), Determination of GPS Receiver Position Using Multivariate Newton - Raphson Technique for Over Specified Cases, International Journal of Applied Engineering Research (IJAER), 13, 1457-1460.
5. Leica Geosystem (1999), Introduction to GPS (Global Positioning System), AG, Heerbrugg , Switzerland.
6. Al-Delami, O. M. (2002), Accuracy Measurement of Geodesy and GPS, Tech. University.
7. German Company (2000), GPS Guide for Beginners, USA.
8. Nelson, R. A. (1999), The Global Positioning System a National Resource, Via Satellite.
9. Edu / Openbook (2000), The Global Positioning System; A Shared National Asset, The National Academy Of Sciences.
10. Grejner, B.; Dorota, A. and Wang, J. (1998), Gravity Modeling for high accuracy GPS/INS integration, navigation, 45, 209-220.
11. Defense Mapping Agency (1992), world geodetic 1984 (WGS-84), DMA TR 8350.2, 2nd.ed., Fairfax, UA.
12. Kaplan, E.D. and lava, J.I. (1993), Fundamentals of Satellite Navigation, Hughes space and communication.
13. Staff Members of Surveying and Faculty of Engineering (1994), Geodetic surveying, Dar El - Rateb Alexandria- Beirut Arab University.
14. Hofman, Wellenhof, B. Lichtenegger, H. and Ellins, J. (1993), GPS Theory and Practice, New York.
15. Epicentrevsage guide, (2000), Geographic Coordinates System Transformations, POSC Specifications Version 2.2,
16. Ash, T. ;Coatsee, J. ;Silva, R. and Brown, A. (1995),A Low Cost GPS/Inertial Mapping (GIM) System,Mobile Mapping Symposium, Columbus, OH.

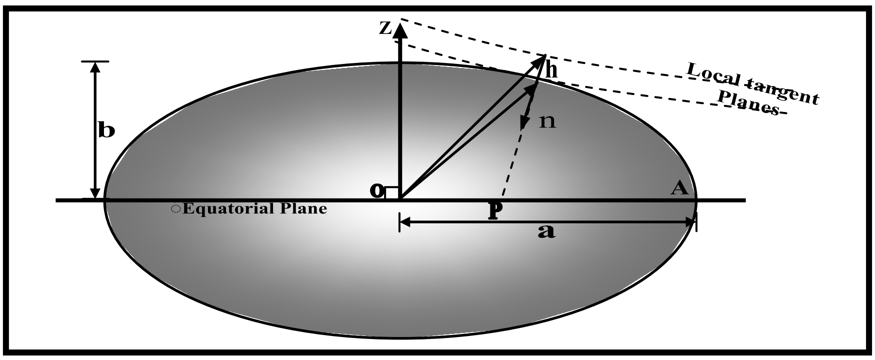

Figure No.(1) Ellipsoidal model of earth (cross-section normal to equatorial plane) [13]

اشتقاق معادلات رياضية لحساب الإحداثيات الجغرافية لموقع مجهول موضوع بمسافة عن موقع الراصد باستعمال بيانات GPS

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الخلاصة

قدم البحث اشتقاق معادلات رياضية لحساب الإحداثيات الديكارتية والجغرافية لموقع مجهول الإحداثيات بعيد عن موقع الراصد وذلك باستعمال بيانات GPS. اذ حولت الإحداثيات الجغرافية **(**$h_{obs.}$, $\varphi_{obs.}$, λ_{obs}**)** لموقع الراصد إلى إحداثيات ديكارتية (X_{obs}، Y_{obs}, $Z_{obs.}$) لموقع الراصد نفسه.بعد ذلك حسبت الإحداثيات الديكارتية للموقع المجهول **(**$X_{unk.}$,.$Y_{unk.}$, Z_{unk}**)** مـن خـلال المعـادلات المحسوبة،ومن ثم تحويلها إلى إحداثيات جغرافية للموقع($\varphi_{unk.}$, $\lambda_{unk.}$).

الكلمات المفتاحية : نظام التموضع العالمي (GPS)، الاحداثيات الجغرافية والديكارتية، عِلْم الجيوئيد، نظام الجيوئيد العالمي (WGS-84).

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