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Analysis of Loaded Beam, Cantilever, and Elongated Vertical Column via Integral Rohit Transform

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Abstract

Structural analysis is a branch of solid mechanics which utilizes straight forward models for solids. The main objective of Structural analysis is to find out the effect of loads on the physical structures and their components. A beam is a structure with a constant cross-section and is described by its significant length in comparison to its thickness and width. A cantilever, on the other hand, is a slender beam with a uniform cross-sectional shape that is fixed horizontally at one end and subjected to a load at the other end. Columns, which serve as vertical compression members in building frames, are susceptible to buckling and failure when subjected to relatively small axial loads. The analysis of loaded beams, cantilevers, and elongated vertical columns is typically carried out using the principles of calculus. However, this paper introduces the integral Rohit transform for the analysis of loaded beam supported at ends, cantilever, and elongated columns with low buckling axial loads. It is found that the depression grows as the cantilever and beam lengths that are loaded in the middle and supported at both ends rise. An attempt has been made to analyze the elongated column with low axial buckling loads and derive the Euler's formula for buckling load. The obtained solutions are graphically represented, and the results demonstrate accuracy, capability and effectiveness of the integral Rohit transform technique when compared to existing methods in the literature. The Rohit transform involves simple formulation and less computational work compared to other methods available in the literature.

Keywords: Loaded Beam, Cantilever, Elongated Columns, Integral Rohit Transform

1. Introduction

A beam is a structural element with a uniform cross-section. It is described by its significant length compared to its width and thickness. In such structures, the shearing stress across any cross-section is considered to be negligibly small [1]. A cantilever, on the other hand, refers to a slender and uniform beam that is horizontally fixed at one end and subjected to loading at the opposite end. Beams are commonly employed in the construction of bridges or for the purpose of supporting heavy loads, often found in the structure of multistoried buildings [2]. In engineering design, the elastic behavior of materials assumes a critical role in various applications such as the construction of buildings, bridges, automobiles, and rope-ways. The property of elasticity in beam materials leads to the generation of a restoring couple when subjected to deforming forces, which acts in equilibrium and is equal in magnitude but opposite in direction to the bending couple. This restoring moment is known as the bending moment [3]. Building frames employ columns as one of their vertical compression components, and they are susceptible to buckling and failing at mild axial stresses. These buckles when the axial load reaches a threshold value known as the critical buckling load because they are significantly longer than their lateral dimensions [4]. One of the failures of a structure supporting a load is buckling. Because they are thin, columns buckle when the axial load reaches a threshold amount called the critical buckling load. They also deflect laterally when compressed. It has been shown that low buckling axial loads cause the columns to fail. According to Euler's Theory of Columns, a column behaves to resist buckling. Buckling is influenced by the end condition of the columns and flexural rigidity. Commonly, standard methods like the calculus technique are used to analyze loaded beams supported at their ends, cantilevers, and elongated columns with low buckling axial loads [5–8]. Moreover, Euler's Theory of Columns is used to find out the buckling load of the column. This study presents the analysis of the loaded beam supported at its ends, the cantilever, and the elongated columns with modest buckling axial loads via the integral Rohit transform. This integral transform [9] has been proposed by the author Rohit Gupta in the year 2020. It has been applied to solve initial value problems in science and engineering [10-12]. In contrast to the calculus method, the suggested method presents an alternate approach for the analysis of loaded beams supported at their ends, cantilevers, and elongated columns with low buckling axial loads.

2. Rohit Transform and Its Properties

The integral Rohit transform, also written as integral RT, [9] is defined for a function of exponential order by the integral Equations as

$\mathrm{R}\{\mathrm{h}(\mathrm{t})\} = q^3 \int_0^\infty e^{-qt}\, h(t)dt,\ t \geq 0\ , q_1 \leq q \leq q_2.$

The variable q is used to factor the variable t in the argument of the function h.

The Rohit transforms of unidentified functions [10] are given by

- $R\,\{t^n\} = \frac{n!}{q^{n-2}}$
- $R\,\{sinbt\} = \frac{b\,q^3}{q^2+b^2}$
- $R\,\{cosbt\} = \frac{q^4}{q^2+b^2}$
- $R\,\{e^{bt}\} = \frac{q^3}{q-b}$

The Rohit transforms (RT) of some derivatives are [11] given by

$R\,\{g'(t)\} = qG(q) - \; q^3 g(0)$,

$R\{g''(t)\} = q^2 G(q) - \; q^4 g(0) - \; q^3 g'(0)$,

$R\{g'''(t)\} = q^3 G(q) - q^5 g(0) \; - \; q^4 g'(0) - \; q^3 g''(0)$.

In general, $R\{g^{\mathrm{n}}(t)\} = q^{\mathrm{n}} R\{g(t)\} - \sum_{k=1}^{n} q^{n-k+3}\, g^{\mathrm{k}-1}(0)$.

A unit step function is written as $U(t-a) = 0 \; for \; t < a \; and \; 1 \; for \; t \geq$ a.

The Rohit transform of a unit step function is given by

$$\mathrm{R}\{U(t-a)\} = \; q^3 \int_0^{\infty} e^{-qt}\, U(t-a) dt,$$

$$\mathrm{R}\{U(t-a)\} = \; q^3 \int_a^{\infty} e^{-qt}\, dt,$$

$$\mathrm{R}\{U(t-a)\} = \; q^2 e^{-qa}.$$

Shifting property of Rohit transform

Let R{g(t)} $= G(q), \; then \; R[g(t-a)U(t-a)] = e^{-qa} G(q)$.

Proof: The Rohit transform of $[g(t-a)U(t-a)]$ is given by

$$R[g(t-a)U(t-a)] = \; q^3 \int_0^{\infty} e^{-qt}\, g(t-a)U(t-a)dt,$$

$$R[g(t-a)U(t-a]) = \; q^3 \int_a^{\infty} e^{-qt}\, g(t-a)dt,$$

$$R[g(t-a)U(t-a)] = \; q^3 \int_0^{\infty} e^{-q(v+a)}\, g(v)dv, \quad where \; v = t-a,$$

$$R[g(t-a)U(t-a)] = e^{-q(a)}\, q^3 \int_0^{\infty} e^{-q(v)}\, g(v)dv,$$

$$R[g(t-a)U(t-a)] = e^{-q(a)}\, q^3 \int_0^{\infty} e^{-q(t)}\, g(t)dt,$$

$$R[g(t-a)U(t-a)] = e^{-q(a)} G(q).$$

3. Algorithm for Proposed Method

The algorithm for the proposed method is as follows:

Firstly, brief information of the integral Rohit transform and its attributes is provided.

Secondly, the analysis of loaded beam supported at ends, cantilever, and elongated columns with low buckling axial loads is done via the integral Rohit transform.

Thirdly, the obtained solutions are graphically represented, and the results obtained are compared to existing methods in the literature.

Finally, the conclusions of the study are presented.

4. Material and Method

In this section, the analysis of loaded beam supported at its ends, cantilevers, and elongated columns with low buckling axial loads is carried out via the integral Rohit transform.

4.1 *Analysis of Loaded Beam Supported at Its Ends*

In this study, consider a beam supported on the two knife edges A and B and loaded in the middle with a load W vertically downwards. Let L be the length of the beam between the points A (at x = 0) and B (at x = L). The bending moment [1], [12] at the section X is given by the differential Equation:

$$\ddot{y}(\mathrm{x}) + \frac{\mathrm{W}}{2\mathrm{YI}} x = 0 \quad (1)$$

where 'I' is the geometrical moment of inertia and Y is the Young's modulus. Here y is the depression of the beam at the section X at the distance x from the end A.

On taking the Rohit transform of Equation (1), we have

$$R\{\ddot{y}(\mathrm{x})\} + \frac{\mathrm{W}}{2\mathrm{YI}} R\{x\} = 0,$$

$$q^3 \int_0^\infty e^{-qx}\, \ddot{y}(\mathrm{x})\, dx + \frac{\mathrm{W}}{2\mathrm{YI}} q = 0,$$

$$q^3 \left[\int_0^L e^{-qx}\, \ddot{y}(\mathrm{x})\, dx + \int_L^\infty e^{-qx}\, \ddot{y}(\mathrm{x})\, dx\right] + \frac{\mathrm{W}}{2\mathrm{YI}} q = 0.$$

As 0 < x < L, therefore, $\int_L^\infty e^{-qx}\, \ddot{y}(\mathrm{x})\, dx = 0.$

Thus,

$$q^3 \int_0^L e^{-qx}\, \ddot{y}(\mathrm{x})\, dx + \frac{\mathrm{W}}{2\mathrm{YI}} q = 0,$$

$$q^3 \left[e^{-qL} y'(L) - y'(0) + q \int_0^L e^{-qx}\, \mathrm{y}'(x) dx\right] + \frac{\mathrm{W}}{2\mathrm{YI}} q = 0,$$

$$q^3 \left[e^{-qL} y'(L) - y'(0) + q\, e^{-qL} y(L) - q\, y(0) + q^2 \int_0^L e^{-qx}\, \mathrm{y}(\mathrm{x})\, dx\right] + \frac{\mathrm{W}}{2\mathrm{YI}} q = 0,$$

$$q^3 e^{-qL} y'(L) - q^3 y'(0) + q^4\, e^{-qL} y(L) - q^4\, y(0) + q^2 R\{y(x)\} + \frac{\mathrm{W}}{2\mathrm{YI}} q = 0 \quad (2)$$

Applying initial conditions: y (0) = y(L) = 0, $\dot{\mathrm{y}}(0) = C\ and\ \dot{\mathrm{y}}(\mathrm{L}) = D$. Equation (2) becomes

$$q^3 e^{-qL} D - q^3 C + q^2 R\{y(x) + \frac{\mathrm{W}}{2\mathrm{YI}} q = 0,$$

$$R\{y(x)\} = qC - qe^{-qL} D + \frac{\mathrm{W}}{2\mathrm{YI}} \frac{1}{q}.$$

Taking inverse Rohit transform, we have

$$\mathrm{y}\ (\mathrm{x}) = Cx - D(x - L)\, U(t - L) - \frac{\mathrm{W}}{2\mathrm{YI}} \frac{x^3}{3!}. \quad (3)$$

Now, for $\mathrm{x} < \mathrm{L}, U(x - L) = 0.$

Thus, $\mathrm{y}\ (\mathrm{x}) = Cx - \frac{\mathrm{W}}{2\mathrm{YI}} \frac{x^3}{3!}$ (4)

At $\mathrm{x} = \frac{\mathrm{L}}{2}$ and $\dot{\mathrm{y}}\left(\frac{\mathrm{L}}{2}\right) = 0.$ Therefore, using Equation (4) and solving for C, we get

$$C = \frac{3\mathrm{W}L^2}{32\mathrm{YI}} \quad (5)$$

Using Equation (5) in Equation (4), we have

$$y\ (x) = (\frac{3WL^2}{32YI}x - \frac{W}{2YI}\frac{x^3}{3!}),\ \text{where } 0 < x < L \tag{6}$$

Taking, for example, $\frac{W}{YI} = 32\ and\ L = 10,$ the graph of y(x) is shown in the Figure 1.

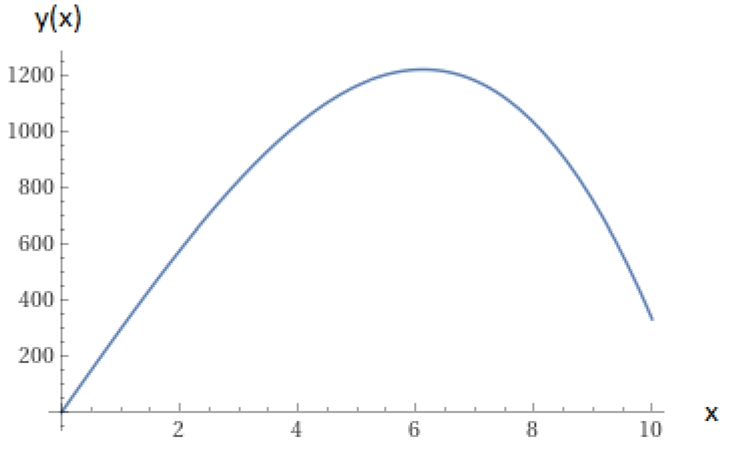

Figure 1. Numerical solution of Equation (1).

At the middle, the total depression is given by

$$y\ (L/2) = \frac{1}{3!}[\frac{3WL^2}{8YI}L/2 - \frac{W}{2YI}(L/2)^3],$$

$$\Rightarrow y\ (L/2) = \frac{WL^3}{48YI} \tag{7}$$

For a beam of circular cross-section [4], [13], we have

$I = \pi r^4/4$

Hence, from Equation (7), we have

$$y(L/2) = \frac{WL^3}{12Y\pi r^4}$$

4.2 Analysis of Cantilever Beam

In this study, consider a horizontal beam AB of length L attached at end A (at x = 0) and loaded with a load W vertically downward from the free end B (at x = L). The bending moment [3], [4] at point X is obtained from the differential Equation:

$$\ddot{y}(x) + \frac{W}{YI}(L - x) = 0 \tag{8}$$

where I is the geometrical moment of inertia and Y is the Young's modulus. Here, y is the depression of the beam at the section X at the distance x from fixed end A.

On taking the Rohit transform [10], [11] of Equation (8), we get

$$q^3\int_0^\infty e^{-qx}\,\ddot{y}(x)\,dx + \frac{W}{YI}(Lq^2 - q) = 0,$$

$$q^3\left[\int_0^L e^{-qx}\,\ddot{y}(x)\,dx + \int_L^\infty e^{-qx}\,\ddot{y}(x)\,dx\right] + \frac{W}{YI}(Lq^2 - q) = 0.$$

As $0 < x < L$, therefore, $\int_L^\infty e^{-qx}\,\ddot{y}(x)\,dx = 0.$

Thus,

$$q^3\int_0^L e^{-qx}\,\ddot{y}(x)\,dx + \frac{W}{YI}(Lq^2 - q) = 0,$$

$$q^3\left[e^{-qL}y'(L) - y'(0) + q\int_0^L e^{-qx}\,y'(x)dx\right] + \frac{W}{YI}(Lq^2 - q) = 0,$$

$$q^3\left[e^{-qL}y'(L)-y'(0)+q\,e^{-qL}y(L)-q\,y(0)+q^2\int_0^L e^{-qx}\,\mathrm{y(x)}\,dx\right]+\frac{\mathrm{W}}{\mathrm{YI}}(Lq^2-q)=0,$$

$$q^3e^{-qL}y'(L)-\ q^3y'(0)+\ q^4\,e^{-qL}y(L)-\ q^4\,y(0)+q^2R\{y(x)\}+\frac{\mathrm{W}}{\mathrm{YI}}(Lq^2-q)=0 \quad (9)$$

Applying initial conditions: $y\,(0) = 0$, $\dot{y}(0)=0, y'(L)=D\ and\ y(L)=C$. Equation (9) becomes,

$$q^3e^{-qL}D+\ q^4\,e^{-qL}C+q^2R\{y(x)\}+\frac{\mathrm{W}}{\mathrm{YI}}(Lq^2-q)=0,$$

$$R\{y(x)\}=-qe^{-qL}D-\ q^2\,e^{-qL}C-\frac{\mathrm{W}}{\mathrm{YI}}\left(L-\frac{1}{q}\right) \quad (10)$$

Taking inverse Rohit transform, we have

$$\mathrm{y(x)}=-\mathrm{D}\,(x-L)U(x-L)-\mathrm{C}U(x-L)+\frac{\mathrm{W}}{\mathrm{YI}}(Lx^2-\frac{x^3}{3!})$$

Now, for $\mathrm{x}<\mathrm{L}, U(x-L)=0$.

Therefore,

$$\mathrm{y(x)}=\frac{\mathrm{W}}{\mathrm{YI}}(Lx^2-\frac{x^3}{3!}) \quad (11)$$

Taking, for example, $\frac{\mathrm{W}}{\mathrm{YI}}=32\ and\ L=10$, the graph of y(x) is shown in the Figure 2.

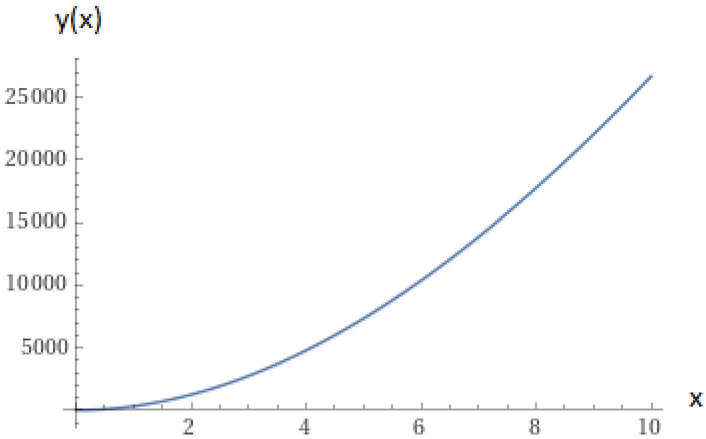

Figure 2. Numerical solution of Equation (8).

4.3 Analysis of Elongated Vertical Column

In this study, consider an elongated vertical column AB (A is at top and B is at bottom) of length 'L' and of uniform cross-section. Let "y" be the lateral deflection of the column section at height "x". We now consider three different cases:

Case-I: When both ends A and B of the column are pinned or hinged

In this case, the bending moment [1], [13] at the section is given by

$$\ddot{y}(x)+k^2y(x)=0, \quad (12)$$

where $\mathrm{k}=\sqrt{\frac{P}{YI}}$.

Taking Rohit transform of Equation (12), we get

$$q^3\int_0^\infty e^{-qx}\,\ddot{\mathrm{y}}(\mathrm{x})\,dx+k^2R\{y(x)\}=\ 0,$$

$$q^3\left[\int_0^L e^{-qx}\,\ddot{\mathrm{y}}(\mathrm{x})\,dx+\int_L^\infty e^{-qx}\,\ddot{\mathrm{y}}(\mathrm{x})\,dx\right]+k^2R\{y(x)\}=0 \quad (13)$$

As 0 < x < L, therefore, $\int_L^\infty e^{-qx}\,\ddot{\mathrm{y}}(\mathrm{x})\,dx=0$.

Thus,

$$q^3 \int_0^L e^{-qx}\, \ddot{y}(\mathrm{x})\, dx + k^2 R\{y(x)\} = 0,$$

$$q^3 \left[e^{-qL} y'(L) - y'(0) + q \int_0^L e^{-qx}\, \mathrm{y}'(x) dx \right] k^2 R\{y(x) = 0,$$

$$q^3 \left[e^{-qL} y'(L) - y'(0) + q\, e^{-qL} y(L) - q\, y(0) + q^2 \int_0^L e^{-qx}\, \mathrm{y}(\mathrm{x})\, dx \right] + k^2 R\{y(x) = 0,$$

$$q^3 e^{-qL} y'(L) - \ q^3 y'(0) + \ q^4\, e^{-qL} y(L) - \ q^4\, y(0) + q^2 R\{y(x)\} + k^2 R\{y(x)\} = 0 \qquad (14)$$

Applying initial conditions: $y(0) = 0$, $\mathrm{y(L)} = 0$, $y'(0) = A, and\ y'(L) = B$. Equation (14) becomes

$$q^3 e^{-qL} B - \ q^3 A + q^2 R\{y(x)\} + k^2 R\{y(x)\} = 0,$$

$$R\{y(x)\}(q^2 + k^2) = -\, q^3 e^{-qL} B + \ q^3 A,$$

$$\mathrm{R}\{y(x)\} = \frac{-\, q^3 e^{-qL} B}{q^2 + k^2} + \frac{q^3 A}{q^2 + k^2} \qquad (15)$$

Taking inverse Rohit transform of Equation (15), we get

$$y(\mathrm{x}) \ = \ -\frac{B}{k} \sin \mathrm{k}\,(\mathrm{x} - \mathrm{L}) \mathrm{U}(\mathrm{x} - \mathrm{L}) + \ \frac{A}{k} \sin\,(\mathrm{k\,x}).$$

Now, for $\mathrm{x} \ < \ \mathrm{L}, U(x - L) = 0.$

Thus,

$$y(x) = \frac{A}{k} sin(kx) \qquad (16)$$

As $y(\mathrm{L}) = 0$, therefore, Equation (16) gives

$$\sin\,(\mathrm{k\,L}) = \ 0,$$

where n is an integer greater than equal to zero.

$$kL = n\,\pi,$$

$$\mathrm{k} = \frac{\mathrm{n}\pi}{L} \qquad (17)$$

The least practical value of n is 1, therefore, considering n = 1, we have

$$k = \frac{\pi}{L},$$

$$\sqrt{\frac{\mathrm{P}}{\mathrm{YI}}} = \ \frac{\pi}{\mathrm{L}},$$

$$\mathrm{P} = \frac{\pi^2 \mathrm{YI}}{\mathrm{L}^2} \qquad (18)$$

The Euler's formula for the critical buckling load of the elongated column with pins at both ends is found in Equation (18).

Case-II: When the bottom end B of the column is fixed and the upper end A is hinged

In this case, the bending moment [4] at the section is given by

$$\ddot{y}(x) + \ k^2 y(x) \ = \ H(L - x) \qquad (19)$$

where $\mathrm{H} = \dfrac{\mathrm{H_0}}{\mathrm{EI}}$. Here, $\mathrm{H_0}$ is horizontal force at the fixed end B.

Taking Rohit transform of Equation (19), we get

$$q^3 \int_0^\infty e^{-qx}\, \ddot{y}(\mathrm{x})\, dx + k^2 R\{y(x)\} = \ H(L\, q^2 - q),$$

$$q^3 \left[\int_0^L e^{-qx}\, \ddot{y}(\mathrm{x})\, dx + \int_L^\infty e^{-qx}\, \ddot{y}(\mathrm{x})\, dx \right] + k^2 R\{y(x)\} = H(L\, q^2 - q).$$

As 0 < x < L, therefore, $\int_L^\infty e^{-qx}\, \ddot{y}(\mathrm{x})\, dx = 0.$

Thus,

$$q^3 \int_0^L e^{-qx}\, \ddot{y}(\mathrm{x})\, dx + k^2 R\{y(x)\} = H(L\, q^2 - q),$$

$$q^3 \left[e^{-qL} y'(L) - y'(0) + q \int_0^L e^{-qx}\, y'(x) dx\right] + k^2 R\{y(x) = H(L\, q^2 - q),$$

$$q^3 \left[e^{-qL} y'(L) - y'(0) + q\, e^{-qL} y(L) - q\, y(0) + q^2 \int_0^L e^{-qx}\, \mathrm{y(x)}\, dx\right] + k^2 R\{y(x) = H(L\, q^2 - q),$$

$$q^3 e^{-qL} y'(L) - q^3 y'(0) + \; q^4 e^{-qL} y(L) - q^4\, y(0) + q^2 R\{y(x)\} + k^2 R\{y(x)\} = H(L\, q^2 - q) \tag{20}$$

Applying initial conditions: $y(0) = 0$, $\mathrm{y(L)} = 0$, $y'(0) = 0, and\ y'(L) = B$. Equation (20) becomes

$$q^3 e^{-qL} B + q^2 R\{y(x)\} + k^2 R\{y(x)\} = H(L\, q^2 - q),$$

$$R\{y(x)\}(q^2 + k^2) = -\, q^3 e^{-qL} B + k^2 d\, q^2,$$

$$R\{y(x)\} = -\frac{q^3 e^{-qL} B}{q^2 + k^2} + \frac{HL\, q^2}{q^2 + k^2} - \frac{Hq}{q^2 + k^2},$$

$$R\{y(x)\} = -\frac{q^3 e^{-qL} B}{q^2 + k^2} + \frac{HL}{k^2}\left[q^2 - \frac{q^4}{q^2 + k^2}\right] - \frac{H}{k^2}\left[q - \frac{q^3}{q^2 + k^2}\right] \tag{21}$$

Taking inverse Rohit transform of Equation (21), we get

$$y(\mathrm{x}) = -\frac{B}{k}\, \sin \mathrm{k}\, (\mathrm{x} - \mathrm{L}) \mathrm{U}(\mathrm{x} - \mathrm{L}) + \; \mathrm{H}\left[\frac{L}{k^2} - \frac{L}{k^2} \cos\, (\mathrm{k\, x})\right] - \mathrm{H}\left[\frac{x}{k^2} - \frac{\sin kx}{k^3}\right] \tag{22}$$

Now for $\mathrm{x} < \mathrm{L}, U(x - L) = 0$.

Thus, from Equation (22), we have

$$y(\mathrm{x}) = \mathrm{H}\left[\frac{L}{k^2} - \frac{L}{k^2} \cos\, (\mathrm{k\, x}) - \frac{x}{k^2} + \frac{\sin kx}{k^3}\right] \tag{23}$$

Applying the condition: $y(\mathrm{L}) = 0$. Equation (23) gives

$$\mathrm{H}\ \left[\frac{L}{k^2} - \frac{L}{k^2} \cos\, (\mathrm{k\, L}) - \frac{L}{k^2} + \frac{\sin kL}{k^3}\right] = 0,$$

$$\left[-\frac{L}{k^2}\, \cos\, (\mathrm{k\, L}) + \frac{\sin kL}{k^3}\right] = 0,$$

$$\frac{L}{k^2} \cos\, (\mathrm{k\, L}) = \frac{\sin kL}{k^3}$$

$$\tan\, (\mathrm{k\, L}) = \; \mathrm{kL} \tag{24}$$

On expanding $\tan kL$ upto 5^{th} power of kL and solving, we get

$kL = 4.5$ radians,

$$\sqrt{\frac{\mathrm{P}}{\mathrm{YI}}} L = 4.5 \text{ radians}$$

$$\mathrm{P} = \frac{20.25\mathrm{YI}}{L^2},$$

$$\mathrm{P} = \frac{2\pi^2 \mathrm{YI}}{L^2} \tag{25}$$

The Equation (25) is Euler's formula for the critical buckling load of the elongated column with a fixed lower end and a fixed upper end.

Case-III: When both the ends A and B of the column are fixed

In this case, the bending moment at the section is given by

$$\ddot{y}(x) + k^2 y(x) = \mathrm{M} \tag{26}$$

where $\mathrm{M} = \dfrac{\mathrm{M_0}}{\mathrm{EI}}$. Here, $\mathrm{M_0}$ is the restraint moment at each end.

Taking Rohit transform of Equation (26), we get

$$q^3 \int_0^\infty e^{-qx}\, \ddot{y}(\mathrm{x})\, dx + k^2 R\{y(x)\} = M\, q^2,$$

$$q^3 \left[\int_0^L e^{-qx}\, \ddot{y}(\mathrm{x})\, dx + \int_L^\infty e^{-qx}\, \ddot{y}(\mathrm{x})\, dx\right] + k^2 R\{y(x)\} = M\, q^2.$$

As 0 < x < L, therefore, $\int_L^\infty e^{-qx}\, \ddot{y}(\mathrm{x})\, dx = 0.$

Thus,

$$q^3 \int_0^L e^{-qx}\, \ddot{y}(\mathrm{x})\, dx + k^2 R\{y(x)\} = M\, q^2,$$

$$q^3 \left[e^{-qL} y'(L) - y'(0) + q \int_0^L e^{-qx}\, \mathrm{y}'(x) dx\right] + k^2 R\{y(x) = M\, q^2,$$

$$q^3 \left[e^{-qL} y'(L) - y'(0) + q\, e^{-qL} y(L) - q\, y(0) + q^2 \int_0^L e^{-qx}\, \mathrm{y}(\mathrm{x})\, dx\right] + k^2 R\{y(x) = M\, q^2,$$

$$q^3 e^{-qL} y'(L) - q^3 y'(0) + q^4\, e^{-qL} y(L) - q^4\, y(0) + q^2 R\{y(x)\} + k^2 R\{y(x)\} = M\, q^2 \quad (27)$$

Applying initial conditions: $y(0) = 0$, y(L) = 0, $y'(0) = 0, and\ y'(L) = B$. Equation (27) becomes

$$q^3 e^{-qL} B + q^2 R\{y(x)\} + k^2 R\{y(x)\} = M\, q^2,$$

$$R\{y(x)\}(q^2 + k^2) = -\, q^3 e^{-qL} B + k^2 d\, q^2,$$

$$R\{y(x)\} = -\frac{q^3 e^{-qL} B}{q^2 + k^2} + \frac{M\, q^2}{q^2 + k^2},$$

$$R\{y(x)\} = -\frac{q^3 e^{-qL} B}{q^2 + k^2} + \frac{M}{k^2}\left[q^2 - \frac{q^4}{q^2 + k^2}\right] \quad (28)$$

Taking inverse Rohit transform of Equation (28), we get

$$\mathrm{y(x)} = -\frac{B}{k} \sin \mathrm{k\,(x - L)U(x - L)} + \frac{M}{k^2}\,[1 - \cos(\mathrm{k\,x})] \quad (29)$$

Now, for x $<$ L, $U(x - L) = 0$.

Thus, from Equation (29), we have

$$\mathrm{y(x)} = \frac{M}{k^2}\,[1 - \cos(\mathrm{k\,x})] \quad (30)$$

Applying the condition: y(L) = 0. Equation (30) gives

$$\frac{M}{k^2} - \frac{M}{k^2} \cos(\mathrm{k\,L}) = 0,$$

$$\cos(\mathrm{k\,L}) = 1,$$

$$k = \frac{2\mathrm{n}\pi}{L} \quad (31)$$

The least practical value of n is 1, therefore, considering n = 1, we have

$$k = \frac{2\pi}{L},$$

$$\sqrt{\frac{\mathrm{P}}{\mathrm{YI}}} = \frac{2\pi}{\mathrm{L}},$$

$$\mathrm{P} = \frac{4\pi^2 \mathrm{YI}}{\mathrm{L}^2} \quad (32)$$

The Equation (32) is the Euler's formula for critical buckling load for the elongated column whose both ends are fixed.

5. Discussion

The integral Rohit transform has effectively handled the analysis of a beam supported at both ends and loaded in the middle, as well as cantilevers and elongated columns with modest buckling axial stress. Figures 1 and 2 make it abundantly evident that the depression grows as the cantilever and beam lengths that are loaded in the middle and supported at both ends rise. An attempt has been made to provide an example of the Rohit transform in order to analyze the elongated column with low axial buckling loads and derive the buckling load Euler's formula. It is found that the critical buckling load for elongated columns subjected to axial loads is inversely related to the square of length of the column in all of the cases that were studied.

6. Conclusion

According to the calculus approach described in the literature [14-21], the results obtained by the integral Rohit transform are accurate. This demonstrates the efficacy and ability of the method to analyze beams supported at both ends and loaded in the middle, as well as cantilevers and elongated columns with minimal axial buckling loads. In contrast to the calculus method, the suggested method presents an alternate approach for the analysis of loaded beams supported at their ends, cantilevers, and elongated columns with low buckling axial loads. The dominance of integral Rohit transform over other methods available in the literature is in terms of simplicity, speed, and accuracy. It involves simple formulation and less computational work compared to other methods available in the literature.

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Conflict of Interest

The authors declare that they have no conflicts of interest.

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