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On $G_\alpha{}^*$-open sets

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Abstract

This study investigates new concepts in grill topological spaces by employing specified sets in which the α-open sets are defined. Many scholars, such as Choquet, who is considered the first to lay the foundation stone for the concept of a grill and formulate its definition, were interested in studying it. After that, many attempts have been made to study the properties associated with this concept and to understand the relationships among these properties. There are several types of grill topological spaces, including discrete and cofinite topologies. Properties of this set and certain relationships are studied, as well as examining a group of functions, like open, closed, and continuous functions, determining their relation with each other and giving examples and properties associated with this set. This shall serve as the beginning of examining numerous topological properties with this set.

Keywords: Grill , α -o sets , $G_\alpha{}^*$-o set , α-c sets , $G_\alpha{}^*$-of , $G_\alpha{}^*$-cf .

1. Introduction

Choquet first proposed the idea of a grill on a topological space in 1947(1-4). A grill is a collection of non-empty subsets of $(\wp, \tau)$. If i. $\mu \in \mathfrak{G}$ and $\mu \subseteq \upsilon$ implies $\upsilon \in \mathfrak{G}$, ii. $\mu, \upsilon \subseteq \wp$ and $\mu \cup \upsilon \in \mathfrak{G}$, then $\mu \in \mathfrak{G}$ or $\upsilon \in \mathfrak{G}$. A grill topological space is denoted by $(\wp, \tau, \mathfrak{G})$ (5-7). Scholars investigated topological concepts and defined a unique topology via a grill. Let $\varphi:\mathcal{P}(\wp) \to \mathcal{P}(\wp)$ be a mapping, and it is referred to as $\varphi(\mu) = \{ x \in \wp: \mu \cap U \in \mathfrak{G}$ for every $U \in \tau_\wp$ for every $\mu \in \mathcal{P}(\wp)$ and $\tau_\wp$ which is all open set containg x. Suppose that $\Theta: \mathcal{P}(\wp) \to \mathcal{P}(\wp)$ is a mapping, and it is referred to as $\Theta(\mu) = \mu \cup \varphi(\mu)$ for every $\mu \in \mathcal{P}(\wp)$ (8-12). The map Θ satisfies the closure axioms of Kuratowski (13-15)

i. $\Theta(\emptyset)=\emptyset$.
ii. when $\mu \subseteq \upsilon$, then $\Theta(\mu) \subseteq \Theta(\upsilon)$.
iii. when $\mu \subseteq \wp$, then $\Theta(\Theta(\mu))= \Theta(\mu)$.

iv. when $\mu, \upsilon \subseteq \wp$, then $\Theta(\mu \cup \upsilon) = \Theta(\mu) \cup \Theta(\upsilon)$. There are several types of grill topological spaces, including discrete and cofinite topologies. We can locate $\tau_{\mathfrak{G}}$ in a grill $\mathfrak{G}$ on a topological space $(\wp, \tau)$ via the base given by the following $B(\tau_{\wp}, \wp) = \{v - \mu : v \in \tau, \mu \notin \mathfrak{G}\}$.There exists a unique topology with the formula $\tau_{\mathfrak{G}} = \{u \subseteq \wp : \Psi(\wp - u) = (\wp - u)\}$, for any $\mu \subseteq \wp. \Psi(\mu) = \mu \cup \varphi(\mu) = \tau_G$ - $\varsigma l(\mu)$ and $\tau \subseteq \tau_G$ (4). Every set belongs to τ_G is $\mathfrak{G}$-open sets, and its complement is $\mathfrak{G}$-closed sets; the family of all $\mathfrak{G}$-closed is denoted by (16-19). For instance, suppose that $(\wp, \tau)$ is a topological space, if $\mathfrak{G} = \mathcal{P}(\wp)\setminus\{\varphi\}$, then $\tau_G = \tau$.

Let $(\wp, \tau)$ be a topological space; suppose that $\mu \subseteq \wp$, if $\mu \subseteq \mathrm{Int}(\varsigma l(\mathrm{Int}(\mu)))$, then μ is α-open set. Suppose $\upsilon \subseteq \wp$ it is α-closed set if $(\wp - \upsilon)$ is an α-open set. The collection of all α-open (respectively, α-closed) sets in $(\wp, \tau)$ will be symbolized by τ_α(respectively, $\alpha c(\wp)$). Many academics have employed these combinations with the aim of producing novel generalizations (20,21). In this study, Int (μ) is employed to represent the interior(μ), and the symbol cl(μ) represents a closure of the set μ (22-25).

2. Materials and Methods On $G_\alpha{}^*$-open sets

Definition 2.1: let $(\varrho, \tau, \mathfrak{G})$ is a G.T.S , $\varsigma \in \tau$. The closure grill alpha of ς denoted by $\varsigma l_{\mathfrak{G}\,\alpha}(\varsigma)$ is defined by;

$\varsigma l_{\mathfrak{G}\,\alpha}(\varsigma) = \cap\{V \subseteq \varrho;\ V \textit{ is a closed} \text{ set in } \tau_{\mathfrak{G}_\alpha} \text{ whenever } \varsigma \subseteq V\}$.

Definition 2.2: let A be a subset of $(\wp, \tau, \mathfrak{G})$, A is a G_α -open set if there exists $\varsigma \in \tau$; $\varsigma \subseteq A \subseteq \mathrm{Int}\,\mathrm{cl}_{G\alpha}(\varsigma)$. The complement of $G_\alpha -$ open set is the G_α *-closed* set, the set of all G_α *-open* set denoted by G_α -$o(\wp)$ and the complement denoted by G_α -$c(\wp)$.

Definition 2.3: A sub set A of a grill topological space $(\varrho, \tau, \mathfrak{G})$ is said to be α-open set via grill if there exists $\varsigma \in \tau$; $\varsigma - A \notin \mathfrak{G}$ and $A - \mathrm{Int}\,\mathrm{cl}_{G\alpha}(\varsigma) \notin \mathfrak{G}$. And denoted by $G_\alpha{}^*$ *-open*. $\varrho - A$ is $G_\alpha{}^*$*-closed* ,and the set of all $G_\alpha{}^*$*-open* presently by $G_\alpha{}^* o(\varrho)$. the set of all $G_\alpha{}^*$–closed presently by $G_\alpha{}^* c(\varrho)$.

Example 2.4: Let (ϱ, τ, G) is a G.T.S , $\varrho = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3\}$, $\tau = \{\varrho, \emptyset, \{\mathcal{L}_1\}, \{\mathcal{L}_1, \mathcal{L}_2\}\}$,$G = \{\varsigma \subseteq \varrho; \mathcal{L}_2 \in u\}$,$\tau_G = \{\varrho, \emptyset, \{\mathcal{L}_2, \mathcal{L}_3\}, \{\mathcal{L}_1\}, \{\mathcal{L}_2\}, \{\mathcal{L}_1, \mathcal{L}_2\}\}$,

$\tau_{G_\alpha} = \{\varrho, \emptyset, \{\mathcal{L}_2, \mathcal{L}_3\}, \{\mathcal{L}_1\}, \{\mathcal{L}_2\}, \{\mathcal{L}_1, \mathcal{L}_2\}\}$

$F_{G_\alpha} = \{\varrho, \emptyset, \{\mathcal{L}_2, \mathcal{L}_3\}, \{\mathcal{L}_1\}, \{\mathcal{L}_3\}, \{\mathcal{L}_1, \mathcal{L}_3\}\}$

$G_\alpha{}^* o(\varrho) = \{\varrho, \emptyset, \{\mathcal{L}_1\}, \{\mathcal{L}_1, \mathcal{L}_2\}, \{\mathcal{L}_1, \mathcal{L}_3\}\}$.

Proposition 2.5.

i. Each open set is also a $G_\alpha{}^*$-open set.

ii. Each closed set is a $G_\alpha{}^*$-closed set.

Proof. (i) Suppose $A \in \tau$, let $\varsigma \in \tau$, *such that* $\varsigma \subseteq \mathrm{Int}\,\mathrm{cl}_{G\alpha}(\varsigma)$, whenever $\varsigma = A \in \tau$ so,$\varsigma - A = \phi \notin \mathfrak{G}$ $\wedge A - \mathrm{Int}\,\mathrm{cl}_{G\alpha} A = \phi \notin \mathfrak{G}$.

(ii) Let **A** is a closed set, thus $A^c \in \tau$, $A^c \in G_\alpha{}^* o(\varrho)$, then $A \in G_\alpha{}^* c(\varrho)$.

Proposition 2.6. whenever A is a G_α *-open* set, then A is a $G_\alpha{}^*$- *open* set.

Proof. Suppose A is a G_α *-open*, then there exists ς is an open set such that $\varsigma \subseteq A \subseteq \mathrm{Int}\,\mathrm{cl}_{G\alpha}(\varsigma)$, thus $\varsigma - A = \emptyset \wedge A - \mathrm{Int}\,\mathrm{cl}_{G\alpha}(\varsigma) = \emptyset$ and $\varsigma - A \notin \mathfrak{G} \wedge A - \mathrm{Int}\,\mathrm{cl}_{G\alpha}(\varsigma) \notin \mathfrak{G}$. then A is a $G_\alpha{}^*$-open set.

Proposition 2.7. In G.T.S $(\varrho, \tau, \mathfrak{G})$, A is an G_α -o set if and only if A is a $G_\alpha{}^*$-o set whenever $\mathfrak{G} = \mathcal{P}(\varrho)\setminus\{\emptyset\}$.

Proof. Suppose A is a G_α -open set, then A is a $G_\alpha{}^*$-open set by (theorem 2.5). Conversely, let A is a $G_\alpha{}^*$-open set so, there exists $\varsigma \in \tau$ such that $\varsigma - A \notin G \wedge A - \natural nt\,cl_{G\alpha}(\varsigma) \notin G$ .Thus, $\varsigma - A = \emptyset \wedge A - \natural nt\,cl_{G\alpha}(\varsigma) = \emptyset$ then $\varsigma \subseteq A$ and $A \subseteq \natural nt\,cl_{G\alpha}(\varsigma)$, *so* $\varsigma \subseteq A \subseteq \natural nt\,cl_{G\alpha}(\varsigma)$. There fore A is a G_α -open.

Note. $G_\alpha{}^*$-o with α -o set are independent.

Example 2.8: Suppose (ϱ, τ, G) is any Grill topology and $\varrho = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3\}$, $\tau = \{\varrho, \emptyset, \{\mathcal{L}_1\}\}$, $G = \{u \subseteq \varrho; \mathcal{L}_1 \in \varsigma\}$, $\tau_G = \{\varrho, \emptyset, \{\mathcal{L}_2, \mathcal{L}_3\}, \{\mathcal{L}_1\}, \{\mathcal{L}_2\}, \{\mathcal{L}_1, \mathcal{L}_2\}\}$, $G_\alpha{}^*\ o(\varrho) = \mathcal{P}(\varrho)$, and $\alpha o(\varrho) = \{\varsigma \sqsubseteq \varrho; \mathcal{L}_1 \in \varsigma\}$, hence it is clear that $\{\mathcal{L}_2\} \in G_\alpha{}^* o(\varrho)$ but $\{\mathcal{L}_2\} \notin \tau_\alpha$.

Proposition 2.9: Suppose A is a $G_\alpha{}^*$-open set and $H \subseteq X$ such that $A \subseteq H \subseteq \natural nt\,cl_{G\alpha}(\varsigma)$ for each $\varsigma \in \tau$, then H is a $G_\alpha{}^*$-open set.

Proof. Whenever A is a $G_\alpha{}^*$-open set, then $\exists\ \varsigma \in \tau$ such that $\varsigma - A \notin \mathfrak{G} \wedge A - \natural nt\,cl_{G\alpha}(\varsigma) \notin G$, and since $A \subseteq H$,so $\varsigma - H \subseteq \varsigma - A \notin G$,then there exist $\varsigma \in \tau$ such that $\varsigma - H \notin G$, and since $H \subseteq \natural nt\,cl_{G\alpha}(\varsigma)$ for each $\varsigma \in \tau$, then $H - \natural nt\,cl_{G\alpha}(\varsigma) = \emptyset \notin G$, there fore H is a $G_\alpha{}^*$-open set.

Proposition 2.10: Suppose that $(\varrho, \tau, \mathfrak{G})$ is a grill topological space and A is a sub set of ϱ. If $\mathfrak{G} = \mathbb{P}(\varrho) \setminus \{\emptyset\}$. A is a $G_\alpha{}^*$-o set if and only if A *is* α-o set .

Proof. Suppose A is a $G_\alpha{}^*$–open *set*, there exists $\varrho \in \tau$; $\varrho - A \notin G$ and $A - \natural nt\,cl_{G\alpha}(\varsigma) \notin G$, $\varsigma - A = \emptyset \notin G$ and $A - \natural nt\,cl_{G\alpha}(\varsigma) = \emptyset$, $\varsigma \subseteq A$ and $A \subseteq \natural nt\,cl_{G\alpha}(\varsigma)$, $\varsigma \subseteq A \subseteq \natural nt\,cl_{G\alpha}(\varsigma)$, and since $cl_{G\alpha}(\varsigma) = cl_G(\varsigma)$,Then $\varsigma \subseteq A \subseteq \natural nt\,cl_G(\varsigma)$. Therefore A is an α -open set. Conversely, let A be an α -open set it is clear that $\varsigma \subseteq A \subseteq \natural nt\,cl_G(\varsigma)$, since $\varsigma \subseteq A$ and $A \subseteq \natural nt\,cl_G(\varsigma)$, then $\varsigma - A = \emptyset$ and $A - \natural nt\,cl_G(\varsigma) = \emptyset \notin \mathfrak{G}$, since $cl_{G\alpha}(\varsigma) = cl_G(\varsigma)$, $\varsigma \subseteq A \subseteq \natural nt\,cl_{G\alpha}(\varsigma)$ So A is a $G_\alpha{}^*$–open *set* .

Lemma 2.11: $(\cup_{\mathfrak{t} \in \mathfrak{f}} (\natural nt\,cl_{G\alpha}(A_t))) \subseteq (\natural nt\,cl_{G\alpha}(\cup_{\mathfrak{t} \in \mathfrak{f}} A_t)$

Proof. $A_t \subseteq \cup_{\mathfrak{t} \in \mathfrak{f}} (A_t)$,

so $\natural nt\,cl_{G\alpha}(A_t) \subseteq \natural nt\,cl_{G\alpha}(\cup_{\mathfrak{t} \in \mathfrak{f}} (A_t))$,

$\cup_{\mathfrak{t} \in \mathfrak{f}} (\natural nt\,cl_{G\alpha}(A_t)) \subseteq (\natural nt\,cl_{G\alpha}(\cup_{\mathfrak{t} \in \mathfrak{f}} A_t)$.

Proposition 2.12: The union of the collectin of $G_\alpha{}^*$-o set also $G_\alpha{}^*$-o.

Proof. Suppose that A_i is a $G_\alpha{}^*$-open set for each i to show $\cup_{i \in \mathfrak{f}} A_i$ is $G_\alpha{}^*$-o, since A_i is $G_\alpha{}^*$-o set, then there exists $\varsigma_i \in \tau$, $(\varsigma_i - A_i) \notin \mathfrak{G}$, and$(A_i - \natural nt\,cl_{G\alpha}(\varsigma_i)) \notin \mathfrak{G}$. Now, since $(\varsigma_i - A_i) \subseteq \cup_{i \in \mathfrak{f}} (\varsigma_i - A_i)$ and $(\varsigma_i - A_i) \notin \mathfrak{G}\ \forall i$, then by condition two of definition of the grill $\cup_{i \in \mathfrak{f}} (\varsigma_i - A_i) \notin \mathfrak{G}$ but $(\cup_{i \in \mathfrak{f}} \varsigma_i - \cup_{i \in \mathfrak{f}} A_i) \subseteq \cup_{i \in \mathfrak{f}} (\varsigma_i - A_i) \notin \mathfrak{G}$. Therefore, $(\cup_{i \in \mathfrak{f}} \varsigma_i - \cup_{i \in \mathfrak{f}} A_i) \notin \mathfrak{G}$ and $\cup_{i \in \mathfrak{f}} \varsigma_i \in \tau$. Now, to proof $(\cup_{i \in \mathfrak{f}} A_i - \natural nt\,cl_{G\alpha}(\cup_{i \in \mathfrak{f}} \varsigma_i)) \notin \mathfrak{G}$, hence $(A_i - \natural nt \cup_{i \in \mathfrak{f}} (\varsigma_i)) \notin \mathfrak{G}$ for each i so, by condition two of the definition of the grill. $\cup_{i \in \mathfrak{f}} (A_i - (\natural nt\,cl_{G\alpha}(\cup_{i \in \mathfrak{f}} \varsigma_i)) \notin \mathfrak{G}$ and since $(\cup_{i \in \mathfrak{f}} A_i - \cup_{i \in \mathfrak{f}} (\natural nt\,cl_{G\alpha}(\varsigma_i)) \subseteq \cup_{i \in \mathfrak{f}} (A_i - \natural nt\,cl_{G\alpha}(\varsigma_i)) \notin \mathfrak{G}$, therefore $(\cup_{i \in \mathfrak{f}} A_i - (\natural nt\,cl_{G\alpha} \cup_{i \in \mathfrak{f}} (\varsigma_i)) \notin \mathfrak{G}$ by (lemma 2.11) $\cup_{i \in \mathfrak{f}} A_i$ is a $G_\alpha{}^*$-open set .

Note: A collection of every $G_\alpha{}^*$-o sets is supra topology.

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3. Some kinds of $G_\alpha{}^*$-o Functions

Definition 3.1: Suppose $\mathfrak{f}:(\varrho, \tau, G) \rightarrow (\vartheta, \tau', G')$ is a function then $\mathfrak{f}$ is :

(1) $G_\alpha{}^*$-open function symbolizes "$G_\alpha{}^*$-o f " if $\mathfrak{f}(s) \in G_\alpha{}^* o(\vartheta)$ since $s \in G_\alpha{}^* o(\varrho)$.

(2) $G_\alpha{}^{**}$- open function symbolizes "$G_\alpha{}^{**}$-o f " if $\mathfrak{f}(s) \in G_\alpha{}^*\ o(\vartheta)$ since $s \in \tau$.

(3) $G_\alpha{}^{***}$- open function symbolizes "$G_\alpha{}^{***}$-o f " if $\mathfrak{f}(s) \in \tau'$ since $s \in G_\alpha{}^*\ o(\varrho)$.

Proposition 3.2: Let $\mathfrak{f}:(\varrho,\tau,G)\rightarrow(\vartheta,\tau',G')$ be a function then

(1) $\mathfrak{f}$ is an o-f since $\mathfrak{f}$ is a $G_\alpha{}^{***}$-*o* f.

(2) $\mathfrak{f}$ is $G_\alpha{}^{*}$-*o* f since $\mathfrak{f}$ is a $G_\alpha{}^{***}$ -*o* f.

(3) $\mathfrak{f}$ is $G_\alpha{}^{**}$-*o* f since $\mathfrak{f}$ is a $G_\alpha{}^{*}$-*o* f.

(4) $\mathfrak{f}$ is $G_\alpha{}^{**}$-*o* f whenever $\mathfrak{f}$ is an o-f.

Proof. (1) Suppose $\varsigma\in\tau$ by (proposition 2.5 (i)), $\varsigma\in G_\alpha{}^{*}(\varrho)$. Since $\mathfrak{f}$ is a $G_\alpha{}^{***}$-*o* f, $\mathfrak{f}(\varsigma)$ is an open set (ϑ,τ'). Hence, $\mathfrak{f}$ is an o-f.

(2) Suppose $u\in G_\alpha{}^{*}o(\varrho)$ since $\mathfrak{f}$ is a $G_\alpha{}^{**}$-*o* f then, $\mathfrak{f}(u)$ is an open set in (ϑ,τ'). By (proposition 2.5 (i)), $\mathfrak{f}(u)\in G_\alpha{}^{*}\ o(\varrho)$. Hence, $\mathfrak{f}$ is a $G_\alpha{}^{*}$-*o* f.

(3) Let $\varsigma\in\tau$ by (proposition 2.5 (i)), $\varsigma\in G_\alpha{}^{*}\ o\ (\varrho)$. Since $\mathfrak{f}$ is a $G_\alpha{}^{*}$-*o* f, then $\mathfrak{f}(\varsigma)\in G_\alpha{}^{*}\ o\ (\vartheta)$.so $\mathfrak{f}$ is a $G_\alpha{}^{**}$-*o* f.

(4) Let $\varsigma\in\tau$ and since $\mathfrak{f}$ is an o- f so that $\mathfrak{f}(\varsigma)\in\tau'$. By (proposition 2.5 (i)), $\varsigma\in G_\alpha{}^{*}(\varrho)$. So $\mathfrak{f}$ is a $G_\alpha{}^{**}$ -o f.

The reverse direction of (proposition 3.2) is not true in genera as the examples.

Example 3.3: Let $\varrho=\{\mathcal{L}_1,\mathcal{L}_2,\mathcal{L}_3\}$, $\tau=\{\varrho,\ \phi,\{\mathcal{L}_2\}\}$, $\mathfrak{G}=\mathrm{P}(\varrho)\backslash\{\emptyset\}$, $\mathfrak{f}:(\varrho,\tau,\ G)\rightarrow(\varrho,\tau,G)$, $\mathfrak{f}(\mathcal{L})=\mathcal{L}$, $\mathcal{L}\in\varrho$. It is clear that $\mathfrak{f}$ is an open function, $G_\alpha{}^{*}o(\varrho)=\{\varsigma\subseteq\varrho\ ;\mathcal{L}_2\in\varsigma\}\cup\{\emptyset\}$, there exist $\{\mathcal{L}_1,\mathcal{L}_2\}\in G_\alpha{}^{*}o(\varrho)$, $\mathfrak{f}(\{\mathcal{L}_1,\mathcal{L}_2\})\notin\tau$.Then $\mathfrak{f}$ is not $G_\alpha{}^{***}$-o f.

Example 3.4: Let $\varrho=\{\mathcal{L}_1,\ \mathcal{L}_2,\ \mathcal{L}_3\}$, $\tau=\{\varrho,\ \phi\ ,\ \{\mathcal{L}_1\},\ \{\mathcal{L}_1,\mathcal{L}_2\}\}$, $\mathfrak{G}=\{u\subseteq\varrho\ ;\mathcal{L}_3\in u\}$, $\tau_G=\mathrm{P}(\varrho)$, $G_\alpha{}^{**}o(\varrho)=\mathrm{P}(\varrho)$, $\mathfrak{f}:(\varrho,\tau,\ G)\rightarrow(\varrho,\tau,\ G)$, $\mathfrak{f}(\mathcal{L}_1)=\{\mathcal{L}_2\}$,$\mathfrak{f}(\mathcal{L}_2)=\{\mathcal{L}_1\}$,$\mathfrak{f}(\mathcal{L}_3)=\{\mathcal{L}_3\}$.We observe that the function is $G_\alpha{}^{*}$-o f and $G_\alpha{}^{**}$-o f but it is not open and not $G_\alpha{}^{***}$-o f since $\mathfrak{f}(\mathcal{L}_1)=\{\mathcal{L}_2\}\notin\tau$.

Example 3.5: Let $\mathfrak{f}:(\varrho,\tau,G)\rightarrow(\varrho,\tau,G)$, $\varrho=\{\mathcal{L}_1,\mathcal{L}_2,\mathcal{L}_3\}$,
$\tau=\{\varrho,\phi,\{\mathcal{L}_1,\ \mathcal{L}_2\}\}$, $G=\mathrm{P}(\varrho)\backslash\{\{\emptyset\}\cup\{\mathcal{L}_1\}\}$, $\mathfrak{f}(\mathcal{L}_2)=\{\mathcal{L}_3\}$, $\mathfrak{f}(\mathcal{L}_1)=\{\mathcal{L}_2\}$,$\mathfrak{f}(\mathcal{L}_3)=\{\mathcal{L}_1\}$,
$\tau_G=\{\varrho,\phi,\{\mathcal{L}_1,\ \mathcal{L}_2\},\{\mathcal{L}_2,\ \mathcal{L}_3\},\{\mathcal{L}_2\}\}$,
$G_\alpha{}^{*}o(\varrho)=\{\varrho,\phi,\{\mathcal{L}_1,\mathcal{L}_2\},\{\mathcal{L}_2,\mathcal{L}_3\},\{\mathcal{L}_2\},\{\mathcal{L}_1\}\}$. $\mathfrak{f}$ is a $G_\alpha{}^{**}$ of but $\mathfrak{f}$ is not $G_\alpha{}^{*}$ o f since there exists $\{\mathcal{L}_2,\mathcal{L}_3\}\in G_\alpha\ o(\varrho)$, but $\mathfrak{f}(\{\mathcal{L}_2,\mathcal{L}_3\})=\{\mathcal{L}_1,\mathcal{L}_3\}\notin G_\alpha\ o(\varrho)$, and its not $G_\alpha{}^{**}$ o f since there exists $\{\mathcal{L}_2\}\in G_\alpha\ o(\varrho)$ but $\mathfrak{f}\{\mathcal{L}_2\}=\{\mathcal{L}_3\}\notin\tau$.

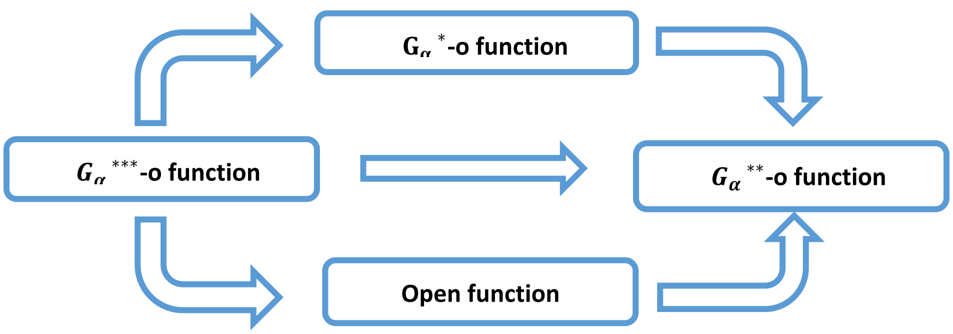

Figure 1. Functions *via* $G_\alpha{}^{*}$-o f

Definition 3.6: Suppose $\mathfrak{f}:(\varrho,\tau,G)\rightarrow(\vartheta,\tau',G')$ is a function then :

(1) $G_\alpha{}^{*}$-*closed* function, denoted by "$G_\alpha{}^{*}$-*c* f " if $\mathfrak{f}(\varsigma)\in G_\alpha{}^{*}\ c(\vartheta)$ since $\varsigma\in G_\alpha{}^{*}\ c(\varrho)$.

(2) $G_\alpha{}^{**}$*closed* function, denoted by "$G_\alpha{}^{**}$-*c* f " if $\mathfrak{f}(u)\in G_\alpha{}^{*}c(\vartheta)$ since ς is a closed set in (ϱ,τ) .

(3) $G_\alpha{}^{***}$-c f, denoted by "$G_\alpha{}^{***}$-*c* f " if $\mathfrak{f}(\varsigma)$ is a closed set in (ϑ,τ') since $\varsigma\in G_\alpha{}^{*}c(\varrho)$.

Proposition 3.7: Suppose that $\mathfrak{f}:(\varrho,\tau,G)\rightarrow(\vartheta,\tau',G')$ is a function

(1) $\mathfrak{f}$ is a c-f since $\mathfrak{f}$ is a $G_\alpha{}^{***}$-*c* f.

(2) ƒ *is* $G_\alpha{}^{*}$-*c* f since ƒ *is* a $G_\alpha{}^{***}$-*c* f.
(3) ƒ *is* $G_\alpha{}^{**}$-*c* f since ƒ *is* a $G_\alpha{}^{*}$-*c* f.
(4) ƒ *is* $G_\alpha{}^{**}$-*c* f since ƒ *is* a c- f.

Proof by the some way of proposition 3.2 the proof has been completed.

The reverse direction of this proposition is not true. See (Example 3.3) (Example 3.4) and (Example 3.5) .

Remark 3.8: If ƒ *is* onto function then:
(1) $G_\alpha{}^{*}$-c f , $G_\alpha{}^{*}$-o f are equivalents.
(2) $G_\alpha{}^{**}$-c f , $G_\alpha{}^{**}$-o f are equivalents.
(3) $G_\alpha{}^{***}$-c f , $G_\alpha{}^{***}$-o f are equivalents.

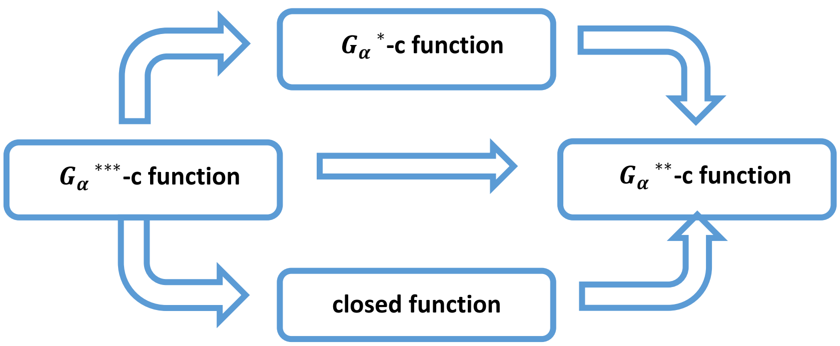

Figure 2. functions *v*ia $G_\alpha{}^{*}$-c function

4.Some kinds of $G_\alpha{}^{*}$- Continuous Functions.

Definition 4.1: Suppose ƒ:(ϱ,τ,G)⟶(ϑ, τ',G′) is a function then ƒ is said to be
1. $G_\alpha{}^{*}$- cont function, denoted by " $G_\alpha{}^{*}$-cont f "*if* $ƒ^{-1}$(ς)∈ $G_\alpha{}^{*}$ o(ϱ) for each ς ∈ τ'.
2. Strongly $G_\alpha{}^{*}$-continuous function, denoted by “s $G_\alpha{}^{*}$-cont f ” *if* $ƒ^{-1}$ (ς)∈τ, for each ς ∈$G_\alpha{}^{*}$ o(ϑ).
3. $G_\alpha{}^{*}$-irresolute function, denoted by "$G_\alpha{}^{*}$-irr f " if $ƒ^{-1}$ (ς)∈ $G_\alpha{}^{*}$*o*(ϱ), for each ς ∈$G_\alpha{}^{*}$*o*(Y).

Theorem 4.2: Suppose that ƒ:(ϱ,τ,G)⟶(ϑ,τ',G′) is a function denoted that:
1.ƒ *is* $G_\alpha{}^{*}$-irr f whenever ƒ is a S $G_\alpha{}^{*}$-cont f
2. ƒ is cont-f whenever ƒ is a s $G_\alpha{}^{*}$-cont f.
3.ƒ *is* $G_\alpha{}^{*}$-cont-f whenever ƒ is *a* cont-f.
4. ƒ *is* a $G_\alpha{}^{*}$-cont-f whenever ƒ is a $G_\alpha{}^{*}$-irr f.

Proof.
1. let ς ∈ $G_\alpha{}^{*}$*o*(ϑ) since ƒ is a s $G_\alpha{}^{*}$-cont f, then $ƒ^{-1}$ (ς)∈τ *by* (proposition 2.5(i)) $ƒ^{-1}$ (ς)∈ $G_\alpha{}^{*}$*o*(ϱ). This implies ƒ is a $G_\alpha{}^{*}$-irr f.
2. Suppose that ς is an open set in (ϑ, τ'). By (proposition 2.5(i)), ς ∈Ꞔα*o*(ϑ) *since* ƒ is s $G_\alpha{}^{*}$-cont f , then $ƒ^{-1}$ (ς) *is an open set in* (ϱ,τ), Implies that ƒ is a cont- f.
3. Let ς ∈τ' since ƒ is a cont- f then $ƒ^{-1}$ (ς) is an open set in (ϱ,τ). By (proposition 2.5(i)) $ƒ^{-1}$ (ς)∈ $G_\alpha{}^{*}$*o*(ϱ) *so* ƒ *is* $G_\alpha{}^{*}$− cont f.
4. Let ς ∈τ by (proposition 2.5 (i)), ς ∈$G_\alpha{}^{*}$*o*(ϱ) since ƒ is $G_\alpha{}^{*}$-irr f. Then, $ƒ^{-1}$ (ς)∈ $G_\alpha{}^{*}$ *o*(ϱ), *so* ƒ *is* $G_\alpha{}^{*}$-cont f.

The inverse of this theorem is not true by the example.

Example 4.3: Suppose ʄ:(ϱ,τ,G)⟶(ϑ,τ, G′) is a function such that ʄ ($\mathcal{L}$)= $\mathcal{L}$ for each $\mathcal{L} \in \varrho$ where $\varrho=\{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3\}$,$\tau=\{\varrho,\emptyset,\{\mathcal{L}_2\}\}$, Ǥ =Ƥ($\varrho$)\{Ø}, Ǥ′={ ς; $\mathcal{L}_2 \in \varsigma$ },

$G_\alpha{}^{*}o(\vartheta)=\{\varsigma; \mathcal{L}_2 \in \varsigma\} \cup \{\emptyset\}$, $G_\alpha{}^{*}o(\varrho)$=Ƥ(ϱ) .

So that, ʄ is $G_\alpha{}^{*}$-cont f and continuous function but it is not $G_\alpha{}^{*}$ o(ϱ)-irr f and it is not $G_\alpha{}^{*}$o(ϱ) s cont f since there exists$\{\mathcal{L}_2, \mathcal{L}_3\}\in$Ǥ′$_\alpha{}^{**}$($\varrho$)but ʄ$^{-1}$ $\{\mathcal{L}_2, \mathcal{L}_3\}=\{\mathcal{L}_2, \mathcal{L}_3\}\notin G_\alpha{}^{*}$o($\varrho$)

and $\{\mathcal{L}_2, \mathcal{L}_3\} \notin \tau$.

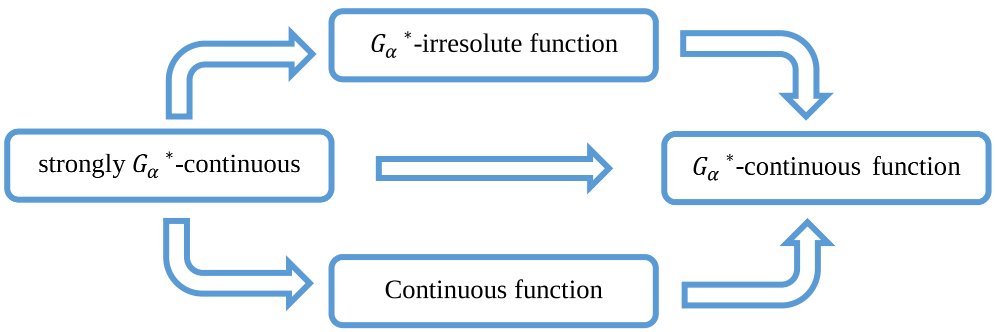

Figure 3. Continuity via $G_\alpha{}^{*}$-open set

5. Conclusions

Through our research, unique characteristics was found that this group has. Also, investigated the continuity of this group, identified the relationships between the groups associated with it, and illustrated these relationships using diagrams. As described earlier. In the future, we can also study the properties of the group we selected in other spaces, such as topological fuzzy space or topological nano space, and we can also study the relationships between the properties of this group. In the future, we can study some properties of coverage and correlation across open groups.

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Conflict of Interest

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