The Construction of Complete (k_n,n)-Arcs in The Projective Plane PG(2,11) by Geometric Method, with the Related Blocking Sets and Projective Codes

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Abstract

In this paper,we construct complete (k_n,n)-arcs in the projective plane PG(2,11), n = 2,3,…,10,11 by geometric method, with the related blocking sets and projective codes.

Keywords: Complete arcs, blocking set, projective cod.

Introduction

A (k,n)-arc is a set of k points of PG(2,q) for some n, but not n + 1 of them, are collinear.

A (k,n)-arc is complete if it is not contained in a (k + 1,n)-arc.

Let PG(2,q) be the projective plane over Galois field GF(q). The points of PG(2,q) are the non-zero vectors of the vector space V(3,q) with the rule that $X(x_1,x_2,x_3)$ and $Y(\lambda x_1,\lambda x_2,\lambda x_3)$ are the same point, where $\lambda \in GF(q)\backslash\{0\}$.

Similarly, $x[x_1,x_2,x_3]$ and $y[(\lambda x_1,\lambda x_2,\lambda x_3]$ are the same line, where $\lambda \in GF(q)\backslash\{0\}$.

The point $X(x_1,x_2,x_3)$ is on the line $Y[y_1,y_2,y_3]$ if and only if $x_1y_1 + x_2y_2 + x_3y_3 = 0$.

In PG(2,q), there are $q^2 + q + 1$ points and $q^2 + q + 1$ lines, every line contains exactly q + 1 points and every point is on exactly q + 1 lines. Many researcher worked on the construction and classification of the(k,n)-arcs in projective planes PG(2,q),Hirschfeld showed the construction and classification of (k,2)-arcs in PG(2,q),q≤9,Brune showed the relation between the (k,n)-arc and the blocking (b, t) set,

1. Definition: [1]

A (k_n,n)-arc K is in PG(2,q) is a set of k_n points such that some lines of the plane meet K in n points but no line meets K in more than n points, where $n \geq 2$.

2. Definition:[2]

A (k,n)-arc is complete if it is not contained in a (k+1,n)-arc. The maximum number of points that (k,2)-arc can have is m(2,q) and this arc is an oval.

3. Theorem:[3]

In PG(2,q), $m(2,q) = \begin{cases} q+1 & \text{for } q \text{ odd} \\ q+2 & \text{for } q \text{ even} \end{cases}$

4. Definition:[1]

A line ℓ in PG(2,q) is an i-secant of a (k,n)-arc K if $|\ell \cap K| = i$.

5. Definition:[1]

A variety V(F) of PG(2,q) is a subset of PG(2,q) such that

$V(F) = \{P(A) \in PG(2,q) \mid F(A) = 0\}$.

6. Definition:[1]

Let Q(2,q) be the set quadrics in PG(2,q), that is the varieties V(F), where:

$$F = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 \qquad \ldots(1)$$

If V(F) is non-singular, then the quadric is a conic, that is, if

$$A = \begin{bmatrix} a_{11} & \frac{a_{12}}{2} & \frac{a_{13}}{2} \\ \frac{a_{12}}{2} & a_{22} & \frac{a_{23}}{2} \\ \frac{a_{13}}{2} & \frac{a_{23}}{2} & a_{33} \end{bmatrix}$$

Is non singular, then the quadric (1) is a conic.

7. Theorem:[3]

In PG(2,q), with q odd, every oval is a conic.

8. Definition:[3]

A point N which is not on a (k,n)-arc has index i if there exactly i(n-secants) of the arc through N, the number of the points N of index i is denoted by N_i.

9. Remark:[3]

The (k,n)-arc is complete if and only if $N_0 = 0$. Thus the arc is complete if and only if every point of PG(2,q) not on the arc lies on some n-secant of the arc.

10. Definition:[4]

An (b,t)-blocking set β in PG(2,q) is a set of b points such that every line of PG(2,q) intersects β in at least t points, and there is a line intersecting β in exactly t points.

If β contains a line, it is called trivial, thus β is a subset of PG(2,q) which meets every line but contains no line completely; that is $t \le |\beta \cap \ell| \le q$ for every line ℓ in PG(2,q). So β is a blocking set if and only if PG(2,q)\β is also blocking set. We may note that a blocking set is merely a (k,n)-arc with $n \le q$ and no 0-secants. A blocking set β is minimal if β \{p} is not blocking set for every $p \in \beta$.

11.The Relation Between the Blocking (b,t)-set and the (k,n)-arc: [4]

The (k,n)-arcs and the (b,t)-blocking sets are each complement to the other in the projective plane PG(2,q), that is, $n + t = q + 1$ and $k + b = q^2 + q + 1$. Thus the complement of the (b,t)-blocking set is the set of points that intersects every line in at most n points which represents the (k,n)-arc. Also finding minimal (b,t)-blocking set is equivalent to find maximal (k,n)-arc in PG(2,q).Blocking sets were studied in details by Di Paola who determined the minimum size of non-trivial blocking set in PG(2,q), $q \le 9$.

12. Definition:[3]

In PG(2,q), let β contains a line ℓ minus a point P plus a set of q points one on each of the q lines through P other than ℓ but not all collinear; then β is minimal (2q,1)-blocking set. Blocking sets of this kind are called rédéi-type studied by [Bruen, A.A. and Thas, J.A. (1977)] and in [Blockhuis, A.A. and Brouwer, E. and S.Z. “onyi, T. (1995)].

13. Definition:[5,6]

Let V(n,q) denote the vector space of all ordered n-tuples over GF(q). A linear code C over GF(q) of length n and dimension k is a k-dimensional subspace of V(n,q). The vectors of C are called codewords. The Hamming distance between two codewords is defined to be the number of coordinate places in which they differ. The minimum distance of a code is the smallest distance between distinct codewords. Such a code is called an $[n,k,d]_q$ code if its minimum hamming distance is d.

There exists a relationship between complete (n,r)-arcs in PG(2,q) and $[n,3,d]_q$ codes, given by the next theorem.

14. Theorem:[5]

There exists a projective $[n,3,d]_q$ code if and only if there exists an (n,n – d)-arc in PG(2,q).

The Projective Plane PG(2,11)

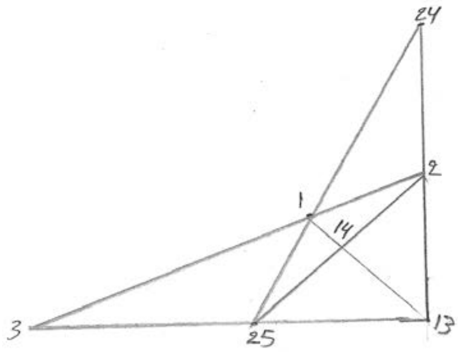
In this paper,we consider the case q = 11 and the elements of GF(11) which are denoted by 0,1,2,3,4,5,6,7,8,9,10

A projective plane π = PG(2,11) over GF(11) consists of 133 points, 133 lines, each line contains 12 points and through each point there are 12 lines.
Let P_i and L_i be the points and lines of PG(2,11), respectively. i=1,2,……….,133 . Let i stands for the point P_i , and for the line L_i whose coordinates are the same coordinates of the point P_i and by using computer program (2-pl- program) [7] the points and the lines of PG(2,11) are given in table (1).

1. The Constructions of (k,n)-Arcs in PG(2,11)

Let ***A***={1,2,13,25}be the set of reference and unit points in π=PG(2,11) , where
1≡(1,0,0) , 2≡(0,1,0) , 13≡(0,0,1) , 25≡(1,1,1)
A is a(4,2)-arc since no three points of ***A*** are collinear , the points of ***A*** are the vertices of a quadrangle whose sides are the lines:

L_1 = [2,13] ={ 2,13,24,35,46,57,68,79,90,101,112,123}
L_2 = [1,13] ={ 1,13,14,15,16,17,18,19,20,21,22,23}
L_3=[1,2]= {1,2,3,4,5,6,7,8,9,10,11,12}
L_4 = [1,25] ={1,24,25,26,27,28,29,30,31,32,33,34}
L_5 = [2,25] = {2,14,25,36,47,58,69,80,91,102,113,124}
L_6 = [13,25] = {3,13,25,37,49,61,73,85,97,109,121,133}



The diagonal points of *A* are the points 3,14,24 where.
$L_1 \cap L_4$=24
$L_3 \cap L_6$=3
$L_2 \cap L_5$=14

Which are the intersections of the pairs of the opposite sides. Then there are 61 points on the sides of the quadrangle,four of them are the points on the arc ***A***, and three of them are the diagonal points of ***A***. So there are 72 points not on the sides of the quadrangle which are the points of index zero for ***A***, these points are:
{38,39,40,41,42,43,44,45,48,50,51,52,53,54,55,56,59,60,62,63,64,65,66,67,70,71,72,74,75,76,77,
78,81,82,83,84,86,87,88,89,92,93,94,95,96,98,99,100,103,104,105,106,107,108,110,111,114,
115, 116,117,118,119,120,122,125,126,127,128,129,130,131,132}
Hence ***A*** is incomplete(4, 2) – arc

2. The Conics in PG (2,11) Through the Reference and Unit Points

The general equation of the conic is
$a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 = 0$ ...(1)
By substituting the points of ***A*** in (1), we get:
$a_{11}=0$, $a_{22}=0$, $a_{33}=0$
$a_{12}+a_{13}+a_{23}=0$
so the equation (1)becomes:
$a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 = 0$ …(2)
If $a_{12}=0$ then the conic is degenerated . therefore $a_{12} \neq 0$
similarly $a_{13} \neq 0$ and $a_{23} \neq 0$
since $a_{12} \neq 0$ we divide equation (2) by a_{12},we get
$x_1x_2 + \alpha x_1x_3 + \beta x_2x_3 = 0$ …(3)

where $\alpha = \frac{a_{13}}{a_{12}}$, $\beta = \frac{a_{23}}{a_{12}}$

since $1+\alpha+\beta = 0 \pmod{11} \rightarrow \beta = -(1+\alpha)$
then (3) can be written as
$x_1x_2 + \alpha x_1x_3 - (1+\alpha)\, x_2x_3 = 0$…(4)
where $\alpha \neq 0$ and $\alpha \neq 10$ for if $\alpha = 0$ or $\alpha = 10$, we get degenerated conic ,
that is $\alpha = 1,2,3,4,5,6,7,8,9$

3. The Equations and the Points of the Conics of PG(2,11) Through the Reference and Unite Points.

For any value of α , there is a unique conic contains 12 points ,four of them are the reference and unit points

1. If $\alpha = 1$ then the equation of the conic C_1 is
$x_1x_2 + x_1x_3 + 9x_2x_3 = 0$
The points of C_1 are{1,2,13,25,40,53,63,77,87,100,104,116}

2. If $\alpha = 2$ then the equation of the conic C_2 is
$x_1x_2 + 2x_1x_3 + 8x_2x_3 = 0$
The points of C_2 are{1,2,13,25,42,50,59,78,84,96,110,131}

3.If $\alpha = 3$ then the equation of the conic C_3 is
$x_1x_2 + 3x_1x_3 + 7x_2x_3 = 0$
and the points of C_3 are{1,2,13,25,41,48,64,76,89,95,115,132}

4.If $\alpha = 4$ then the equation of the conic $C_4$is
$x_1x_2 + 4x_1x_3 + 6x_2x_3 = 0$
The points of C_4 are {1,2,13,25,44,56,65,72,82,108,118,125}

5.If $\alpha = 5$ then the equation of the conic C_5 is
$x_1x_2 + 5x_1x_3 + 5x_2x_3 = 0$
The points of C_5 are {1,2,13,25,43,51,67,71,99,103,119,127}

6. If $\alpha = 6$ then the equation of the conic C_6 is
$x_1x_2 + 6\, x_1x_3 + 4x_2x_3 = 0$
The points of C_6 are {1,2,13,25,45,52,62,88,98,105,114,126}

7.If $\alpha = 7$ then the equation of the conic C_7 is
$x_1x_2 + 7x_1x_3 + 3x_2x_3 = 0$
The points of C_7 are{1,2,13,25,38,55,75,81,94,106,122,129}

8.If $\alpha = 8$ then the equation of C_8 is
$x_1x_2 + 8\, x_1x_3 + 2x_2x_3 = 0$
The points of C_8 are{1,2,13,25,39,60,74,86,92,111,120,128}

9.If $\alpha = 9$ then the equation of C_9 is
$x_1x_2 + 9x_1x_3 + x_2x_3 = 0$
and the points of C_9 are{1,2,13,25,54,66,70,83,93,107,117,130}

Thus we found eleven conics, two of them are degenerated and the remaining nine conics $C_1,C_2,C_3,C_4,C_5,C_6,C_7,C_8$ and C_9 are non –degenerated ,which are complete (k,2)-arcs.

4. The Construction of Complete (k_n,n)-Arcs in PG(2,11) and the Related Blocking Setsand Projective Codes.

The complete (k,n)-arcs in PG(2,11) can be constructed by eliminating the conics given above from PG(2,11) as follows:

a. The Construction of Complete $(k_{11},11)$-Arc .

Let π = PG(2,11)
We take a conic say C_1 , where
C_1 ={1,2,13,25,40,53,63,77,87,100,104,116}
Let
K = π-C_1
={3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,38,39,41,42,43,44,45,46,47,48,49,50,51,52,54,55,56,57,58,59,60,61,62,64,65,66,67,68,69,70,71,72,73,74,75,76,78,79,80,81,82,83,84,85,86,88,89,90,91,92,93,94,95,96,97,98,99,101,102,
103,105,106,107,108,109,110,111,112,113,114,115,117,118,119,120,121,122,123,124,125,126, 127,128,129,130,131,132,133}

This points in table(3.1) represent the elements of the projective plane PG(2,11)=π ,some points of π are deleted to satisfy the definition of the arc which is mentioned in definition 1.1 ,and some of these deleted points are of index zero therefore we must add them to the remained points in π to make the arc K_{11} complete as mentioned in remark 1.9 .

The construction of complete $(k_{11},11)$ –arc must satisfy the following :
1.Any line of π must intersect the arc in at most 11 points .
2. Every point not in the arc is on at least one (11-secant) of the arc.
We eliminate twenty points from K which are :
7,8,10,12,17,24,26,27,28,31,35,46,47,57,68,79,90,101,112,124 to satisfy (1)
since the points {1,116,104,100,87,77,63,53,40,25} are of index zero therefore we add them to K to satisfy (2)
Then
K_{11}={1,3,4,5,6,9,11,14,15,16,18,19,20,21,22,23,25,29,30,32,33,34,36,37,38,39,40,41,42,43,44,45,48,49,50,51,52,53,54,55,56,58,59,60,61,62,63,64,65,66,67,69,70,71,72,73,74,75,76,77,78,80,81,82,83,84,85,86,87,88,89,91,92,93,94,95,96,97,98,99,100,102,103,104,105,106,107,108,109,110,111,113,114,115,116,117,118,119,120,121,122,123,125,126,127,128,129,130,131,132,133}.
K_{11} is a complete (111,11)-arc **.**
By theorem 1.14, there exists a projective [111,3,100] code which is equivalent to the complete (111,11)-arc K_{11}
Let β_1= π – K_{11}
β_1 = {2,7,8,10,12,13,17,24,26,27,28,31,35,46,47,57,68,79,90,101,112,124}
β_1 is (22,1)–blocking set of size (2q)which is of Redei-type ,(figure 1) contains the line L_1,where $L_1\setminus\{123\}$ = {2,13,24,35,46,57,68,79,90,101,112} and one point on each Line through the point 123 other than L_1 which are non-collinear points:
7,8,10,12,17,26,27,28,31,47,124.
Note that each line in π intersect β_1 in at least one point .

b. The Construction of Complete $(k_{10},10)$-Arc

Let π =PG(2,11)
In this section we construct $(k_{10},10)$-arc by eliminating the union of two conics, say C_1 and $C_2$where
C_1= {1,2,13,25,40,53,63,77,87,100,104,116}

C_2= {1,2,13,25,42,50,59,78,84,96,110,131}
Let $K = \pi - C_1 \cup C_2$
={3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,38,
39,41,43,44,45,46,47,48,49,51,52,54,55,56,57,58,60,61,62,64,65,66,67,68,69,70,71,72,73,74,75,76,
79,80,81,82,83,85,86,88,89,90,91,92,93,94,95,97,98,99,101,102,103,105,106,107,108,109,111,112,113,114,115,117,118,119,120,121,122,123,124,125,126,127,128,129,130,132,133}

The construction of a complete $(k_{10},10)$-arc must satisfy the following :

1.Any line of π intersects K in at most 10 points .

2. Every point not in K is on at least one 10 -secant .

we have to eliminate twenty points of K which are
5,7,21,26,35,46,47,54,57,68,79,83,90,99,101,105,112,115,123,126 to satisfy(1).

there are six points of index zero which are 1,2,50,77,87,116 therefore we add them to K to satisfy (2) then
K_{10}={1,2,3,4,6,8,9,10,11,12,14,15,16,17,18,19,20,22,23,24,27,28,29,30,31,32,33,34,36,37,38,
39,41,43,44,45,48,49,50,51,52,55,56,58,60,61,62,64,65,66,67,69,70,71,72,73,74,75,76,77,80,
81,82,85,8687,88,89,91,92,93,94,95,97,98,102,103,106,107,108,109,111,113,114,116,117,118,119,120,121,122, 124,125, 127,128,129,130,132,133}

then K_{10} is a complete (99,10) –arc

By theorem 1.14, there exists a projective [99,3,89] code
which is equivalent to the complete (99,10) –arc K_{10}

Let $\beta_2 = \pi - K_{10}$
={5,7,13,21,25,26,35,40,42,46,47,53,54,57,59,63,68,78,79,83,84,90,96,99,100,101,104,105,110, 112,115,123,126,131}

β_2 is (34,2)-blocking set. Note that each line in π intersects β_2 in at least two points .

c. The Construction of Complete (k9,9)-Arc

In this section, we eliminate the union of three conics, say C_1,C_2, and C_3 where
C_1= {1,2,13,25,40,53,63,77,87,100,104,116}
C_2= {1,2,13,25,42,50,59,78,84,96,110,131}
C_3= {1,2,13,25,41,48,64,76,89,95,115,132}
Let$K= \pi - C_1 \cup C_2 \cup C_3$
={3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,
37,38,39,43,44,45,46,47,49,51,52,54,55,56,57,58,60,61,62,65,66,67,68,69,70,71,72,73,74,75,
79,80,81,82,83,85,86,88,90,91,92,93,94,97,98,99,101,102,103,105,106,107,108,109,111,112,
,113,114,117,118,119,120,121,122,123,124,125,126,127,128,129,130,133}.

The construction of a complete $(k_9,9)$-arc must satisfy the following :

1. Any line in π intersects K in at most 9 point.

2. Every point not on K is on at least one (9-secant).

we eliminate twenty five points of K which are
4,23,24,27,28,33,35,46,52,58,60,61,68,69,74,79,88,90,93,101,106,112,123,126,129, to satisfy (1)

we add the points50,58 which are of index zero to K to satisfy (2), then
K_9={3,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,26,29,30,31,32,34,36,37,38,39,43,44,45
,47,49,50,51,54,55,56,57,58,62,65,66,67,70,71,72,73,75,80,81,82,83,85,86,91,92,94,97,98,99,
102,103, 105,107,108,109,111,113,114,117,118,119,120,121,122,124,125,127,128,130,133}

Then K_9 is a complete (82,9)-arc .

By theorem 1.14, there exists a projective [82,3,73] code which is equivalent to the complete (82,9)-arc K_9.

Let $\beta_3 = \boldsymbol{\pi} - K_9$

={1,2,4,13,23,24,25,27,28,33,35,40,41,42,46,48,52,53,59,60,61,63,64,68,69,74,76,77,78,79,84,87, 88,89,90,93,95,96,100,101,104,106,110,112,115,116,123,126,129,131,132}
Then β_3 is a(51,3)-blocking set ,note that each line intersects β_3 in at least three points .

d. The Construction of Complete (k8,8)-Arc

In this section ,we take the union of four conics, say C_1, C_2, C_3 ,and C_4 where
C_1={1,2,13,25,40,53,63,77,87,100,104,116}
C_2={1,2,13,25,42,50,59,78,84,96,110,131}
C_3={1,2,13,25, 41,48,64,76,89,95,115,132}
C_4={1,2,13,25,44,56,65,72,82,108,118,125}
Let $K=\pi-C_1\cup C_2\cup C_3\cup C_4$
={3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,38,39,43,45,46,47,49,51,52,54,55,57,58,60,61,62,66,67.68,69,70,71,73,74,75,79,80,81,83,,85,86,88,90,91,92,93,94,97,98,99,101,102,103,105,106,107,109,111,112,113,114,117,119,120,121,122,123,124,126,127,128,129,130,133}.
The construction of complete(k_8,8)-arc must satisfy the following:
1.Any line in π intersects K in at most 8 points.
2. Every point not in K is on at least one (8-secant).
we eliminate thirty points from K which are 3,5,6,7,17,23,24,27,28,29,35,36,45,46,57,61,68,70,86,88,90,97,101,105,112,113,114,123,126, 129 to satisfy (1).
K is incomplete arc since there are points of index zero which are 7,44 we add them to K to satisfy(2),then
K_8={4,7,8,9,10,11,12,14,15,16,18,19,20,21,22,26,30,31,32,33,34,37,38,39,43,44,47,49,51,52, 54,55,58,60,62,66,67,69,71,73,74,75,79,80,81,83,85,91,92,93,94,98,99,102,103,106,107,109, 111,117,119120,121,122,124,127,128,130,133}
K_8 is a complete(69,8)-arc .
By theorem 1.14 there exists a projective [69,3,61]code which is equivalent to the complete(69,8)-arc K_8
Let $\beta_4=\pi-K_8$

={1,2,3,5,6,13,17,23,24,25,27,28,29,35,36,40,41,42,45,46,48,50,53,56,57,59,61,63,64,65,68, 70,72,
76,77,78,82,84,86,87,88,89,90,95,96,97,100,101,104,105,108,110,112,113,114,115,116,118,123, 125,126,129,131,132}
Then β_4 is a(64,4)-blocking set .Note that each line in π intersects β_4 in at least four points .

e. The Construction of Complete (k7,7)-Arc

In this section ,we take the union of five conics, say C_1, C_2, C_3, C_4, and $C_5$where
C_1={1,2,13,25,40,53,63,77,87,100,104,116}
C_2={1,2,13,25,42,50,59,78,84,96,110,131}
C_3={1,2,13,25,41,48,64,76,89,95,115,132}
C_4={1,2,13,25,44,56,65,72,82,108,118,125}
C_5={1,2,13,25,43,51,67,71,99,103,119,127}
Let $K=\pi-C_1\cup C_2\cup C_3\cup C_4\cup C_5$

={3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36, 37,38
,39,45,46,47,49,52,54,55,57,58,60,61,62,66,68,69,70,73,74,75,79,80,81,83,85,86,88,90,91,92, 93,94

,97,98,101,102,105,106,107,109,111,112,113,114,117,120,121,122,123,124,126,128,129,130,133}

The construction of complete(k_7,7)-arc must satisfy the following:

1. Any line in π intersects K in at most 7 points.

2. Every point not in K is on at least one(7-secant).

We eliminate thirty-seven points of K which are 3,8,12,14,19,20,21,27,31,34,35,38,39,46,49,55,57,58,60,68,69,74,79,86,90,92,93,98,101,109,111,112,117,120,123,126,128, to satisfy(1)

K is incomplete arc since there are the points 53,56,63,82,87,95 of index zero therefore we add them to K to satisfy (2), then

K_7={4,5,6,7,9,10,11,15,16,17,18,22,23,24,26,28,29,30,32,33,36,37,45,47,52,53,54,56,61,62,63,66,70,73,75,80,81,82,83,85,87,88,91,94,95,97,102,105,106,107,113,114,121,122,124,129,130,133}.

Then K_7 is a complete(58,7)-arc .

By theorem 1.14, there exists a projective [58,3,51]code which is equivalent to the complete (58,7)-arc K_7

Let $\beta_5=\pi-K_7$

={1,2,3,8,12,13,14,19,20,21,25,27,31,34,35,38,39,40,41,42,43,44,46,48,49,50,51,55,57,58,59,60,64,65,67,68,69,71,72,74,76,77,78,79,84,86,89,90,92,93,96,98,99,100,101,103,104,108,109,110, 111,112,115,116,117,118,119,120,123,125,126,127,128,131,132}.

β_5 is a (75,5)-blocking set. Note that each line in π intersects β_5 in at least five points .

f. The Construction of Complete (k6,6)-Arc

In this section, we take the union of six conics say C_1,C_2,C_3,C_4,C_5, and $C_6$where

C_1={1,2,13,25,40,53,63,77,87,100,104,116}

C_2={1,2,13,25,42,50,59,78,84,96,110,131}

C_3={1,2,13,25, 41,48,64,76,89,95,115,132}

C_4={1,2,13,25,44,56,65,72,82,108,118,125}

C_5={1,2,13,25, 43,51,67,71,99,103,119,127}

C_6={1,2,13,25,45,52,62,88,98,105,114,126}

Let $K=\pi-C_1\cup C_2\cup C_3\cup C_4\cup C_5\cup C_6$

={3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37 38,39,46,47,49,54,55,57,58,60,61,66,68,69,70,73,74,75,79,80,81,83,85,86,90,91,92,93,94,97,101,102,106,107,109,111,112,113,117,120,121,122,123,124,128,129,130,133}.

The construction of complete (k_6,6)-arc must satisfy the following:

1. Any line in π intersects K in at most 6 points.

2. Every point not in K is on at least one (6-secant).

We eliminate thirty seven points from K which are 3,8,10,11,14,17,18,20,24,26,28,29,37,39,46,47,57,60,61,66,68,70,73,79,83,90,91,101,111,112,113,117,120,122,123,129,133 to satisfy(1).

since the points 73,132 of index zero therefore we add them to K to satisfy(2) ,then

K_6={4,5,6,7,9,12,15,16,19,21,22,23,27,30,31,32,33,34,35,36,38,49,54,55,58,69,73,74,75,80,81,85,86,92,93,94,97,102,106,107,109,121,124,128,130,132}.

then K_6 is a complete(46,6)-arc .

By theorem 1.14, there exists a projective[46,3,40]code which is equivalent to the complete(46,6)-arc K_6
Let $\beta_6=\pi-K_6$

={1,2,3,8,10,11,13,14,17,18,20,24,25,26,28,29,37,39,40,41,42,43,44,45,46,47,48,50,51,52,53,56
57,59,60,61,62,63,64,65,66,67,68,70,71,72,76,77,78,79,82,83,84,87,88,89,90,91,95,96,98,99,100,
101,103,104,105,108,110,111,112,113,114,115,116,117,118,119,120,122,123,125,126,127,129,131, 133}
then β_6 is a(87,6)-blocking set. Note that each line in π intersects β_6 in at least six points.

g. The Construction of Complete(k5,5)-Arc

In this section, we take the union of seven conics, say C_1,C_2,C_3,C_4,C_5,C_6 and C_7
C_1={1,2,13,25,40,53,63,77,87,100,104,116}
C_2={1,2,13,25,42,50,59,78,84,96,110,131}
C_3={1,2,13,25,41,48,64,76,89,95,115,132}
C_4={1,2,13,25,44,56,65,72,82,108,118,125}
C_5={1,2,13,25,43,51,67,71,99,103,119,127}
C_6={1,2,13,25,45,52,62,88,98,105,114,126}
C_7={1,2,13,25,38,55,75,81,94,106,122,129}
Let $K=\pi-C_1\cup C_2\cup C_3\cup C_4\cup C_5\cup C_6\cup C_7$

={3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,
39,46,47,49,54,57,58,60,61,66,68,69,70,73,74,79,80,83,85,86,90,91,92,93,97,101,102,107,109,111,112,113,117,120,121,123,124,128,130,133}
The construction of complete (k_5,5)-must satisfy the following:
1. Any line in π intersects K in at most 5 points.
2. Every point not in K is on at least one (5-secant).
We eliminate forty points from K which are
3,5,10,11,12,14,15,20,22,23,28,29,31,32,33,36,39,46,49,58,60,66,68,70,73,74,86,90,92,111,112,
113,117,120,121,123,124,128,130,133 to satisfy(1)
K is incomplete arc since the points43,45,65are of index zero, therefore we add themt to K to satisfy(2), then
K_5={4,6,7,8,9,16,17,18,19,21,24,26,27,30,34,35,37,43,45,47,54,57,61,65,69,79,80,83,85,91,93,97, 101,102,107,109}
Then K_5 is a complete(36,5)-arc .
By theorem 1.14, there exists a projective[36,3,31]code which is equivalent to the complete(36,5)-arc K_5
Let $\beta_7=\pi-K_5$
={1,2,3,5,10,11,12,13,14,15,20,22,23,25,28,29,31,32,33,36,38,39,40,41,42,44,46,48,49,50,51,
52,53,55,56,58,59,60,62,63,64,66,67,68,70,71,72,73,74,75,76,77,78,81,82,84,86,87,88,89,90,
92,94,95,96,98,99,100,103,104,105,106,108,110,111,112,113,114,115,116,117,118,119,120,121,122,123, 124,125,126,127,128,129,130,131,132,133}
Then β_7 is a (97,7)-blocking set , which is trivial since β_7 contains some lines completely. Note that each line in π intersects β_7 in at least seven points .

h. The Construction of Complete (k4,4)-Arc

In this section, we take the union of eight conics say C_1, C_2, C_3, C_4, C_5, C_6, C_7, and C_8 where

C_1={1,2,13,25,40,53,63,77,87,100,104,116}
C_2={1,2,13,25,42,50,59,78,84,96,110,131}
C_3={1,2,13,25,41,48,64,76,89,95,115,132}
C_4={1,2,13,25,44,56,65,72,82,108,118,125}
C_5={1,2,13,25,43,51,67,71,99,103,119,127}
C_6={1,2,13,25,45,52,62,88,98,105,114,126}
C_7={1,2,13,25,38,55,75,81,94,106,122,129}
C_8={1,2,13,25,39,60,74,86,92,111,120,128}

Let $K=\pi-C_1\cup C_2\cup C_3\cup C_4\cup C_5\cup C_6\cup C_7\cup C_8$
={3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,46,47,49,54,57,58,61,66,68,69,70,73,79,80,83,85,90,91,93,97,101,102,107,109,112,113,117,121,123,124,130,133}.

The construction of complete(k_4,4)-arc must satisfy the following:

1. Any line in π intersects K in at least 4 points.

2. Every point not in K is on at least one 4-secant.

We eliminate thirty nine points from K which are
3,5,9,10,11,12,14,15,17,20,22,23,24,27,28,29,32,33,35,36,37,46,47,49,57,66,83,85,91,102,107,112,113,117,121,123,124,130,133 to satisfy(1).

K is incomplete arc since the point 82 is of index zero therefore we add it to K to satisfy(2), then

K_4={4,6,7,8,16,18,19,21,26,30,31,34,54,58,61,68,69,70,73,79,80,82,90,93,97,101,109}.

Then K_4 is a complete (27,4)-arc .

By theorem 1.14, there exists a projective [27,3,23]code which is equivalent to the complete(27,4)-arc K_4

Let $\beta_8=\pi-K_4$
={1,2,3,5,9,10,11,12,13,14,15,17,20,22,23,24,25,27,28,29,32,33,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,55,56,57,59,60,62,63,64,65,66,67,71,72,74,75,76,77,78,81,83,84,85,86,87,88,89,91,92,94,95,96,98,99,100,102,103,104,105,106,107,108,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133}.

Then β_8 is a (106,8)-blocking set which is trivial since β_8 contains some lines completely. Note that each line in π intersects β_8 in at least eight points .

i. The Construction of a Complete (k3,3)-Arc

In this section, we take the union of nine conics
$C_1,C_2,C_3,C_4,C_5,C_6,C_7,C_8$ and C_9,where

C_1={1,2,13,25,40,53,63,77,87,100,104,116}
C_2={1,2,13,25,42,50,59,78,84,96 ,110,131 }
C_3={1,2,13,25,41,48,64,76,89,95,115,132 }
C_4={1,2,13,25,44,56,65,72,82,108,118,125}
C_5={1,2,13,25,43,51,67,71,99,103,119,127}
C_6={1,2,13,25,45,52,62,88,98,105,114,126 }
C_7={1,2,13,25,38,55,75,81,94,106,122,129 }
C_8={1,2,13,25,39,60,74,86,92,111,120,128 }
C_9={1,2,13,25,54,66,70,83,93,107,117,130 }

Let $K=\pi -C_1\cup C_2\cup C_3\cup C_4\cup C_5\cup C_6\cup C_7\cup C_8\cup C_9$
K={3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,46,47,49,57,58,61,68,69,73,79,80,85, 90,91,97,101,102,109,112,113,121,123,124,133}

The construction of a complete (k_3,3)-arc must satisfy the following:

1. Any line in π intersects the arc in at most three points.
2. Every point not in the arc is on at least one (3-secant).
We eliminate forty two points from K which are
3,5,8,9,10,11,12,14,15,16,17,18,20,22,23,24,27,28,29,30,32,33,35,36,46,47,49,57,58,61,68,73,79,85,90,91,102,112,113, 121,123,133 to satisfy(1).
K is incomplete arc since there are three points of index zero which are 15, 74, 130 therefore we add them to K to satisfy (2).
Then K_3={4,6,7,15,19,21,26,31,34,37,69,74,80,97,101,109,124,130}
K_3 is a complete (18,3)-arc .
By theorem 1.14, there exists a projective [18,3,15]code which is equivalent to the complete (18,3)-arc K_3
Let $\beta_9=\pi-K_3$
={1,2,3,5,8,9,10,11,12,13,14,16,17,18,20,22,23,24,25,27,28,29,30,32,33,35,36,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,70,71,72,73,75,76,77,78,79,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,98,99,100,102,103,104,105,106,107,108,110,111,112,113,114,115,116,117,118,119,120,121,122,123,125,126,127,128,129,131,132,133}.

β_9 is (115,9)-blocking set which is trivial since β_9 contains some line completely, each line in π intersects β_9 in at least nine points .

j. The Construction of Complete (k2,2)-Arc

In this section, we construct a complete (k_2,2)-arc by using the arc K_3 which is constructed above .
The construction of complete (k_2,2)-arc must satisfies the following:
1.Any line in π intersects the arc in at most 2 points.
2.Every point not in the arc is on at least one (2-secant).
We eliminate seventeen points from K_3 which are
4,6,7,15,19,21,26,31,34,37,69,74,80,97,101,124,130 to satisfy (1)
Let $K_2=K_3\setminus$ {4,6,7,15,19,21,26,31,34,37,69,74,80,97,101,124,130}.
K_2={109}
K is incomplete arc since there are eleven points of index zero which are
133,132,122,120,110,95,90,75,35,20,15 , therefore we add them to K to satisfy (2),then K_2={15,20,35,75,90,95,109,110,120,122,132,133}
K_2 is a complete (12,2)-arc .
By theorem 1.14 there exists a projective [12,3,10]code which is equivalent to the complete (12,2)-arc ,K_2.
Let $\beta_{10}=\pi-K_2$
={1,2,3,4,5,6,7,8,9,10,11,12,13,14,16,17,18,19,21,22,23,24,25,26,27,28,29,30,31,32,33,34,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,76,77,78,79,80,81,82,83,84,85,86,87,88,89,91,92,93,94,96,97,98,99,100,101,102,103,104,105,106,107,108,111,112,113,114,115,116,117,118,119,121,123,124,125,126,127,128,129,130,131},thenβ_{10} is (121,10)-blocking set which is trivial since β_{10} contains some lines completely, we notice that each line in π intersects β_{10} in at least ten points.

Conclusion

1.We can construct complete(k_n,n)-arcs by eliminating some conics, wheren=11,10,…,3.
2.We can construct a complete(k_2,2)-arc by eliminating some points from a complete(k_3,3)-arc

Table (1) : Points and Lines of PG(2,11)

i	P_i	L_i											
1	(1,0,0)	2	13	24	35	46	57	68	79	90	101	112	123
2	(0,1,0)	1	13	14	15	16	17	18	19	20	21	22	23
3	(1,1,0)	12	13	34	44	54	64	74	84	94	104	114	124
4	(2,1,0)	7	13	29	45	50	66	71	87	92	108	113	129
5	(3,1,0)	9	13	31	38	56	63	70	88	95	102	120	127
6	(4,1,0)	10	13	32	40	48	67	75	83	91	110	118	126
7	(5,1,0)	4	13	26	39	52	65	78	80	93	106	119	132
8	(6,1,0)	11	13	33	42	51	60	69	89	98	107	116	125
9	(7,1,0)	5	13	27	41	55	58	72	86	100	103	117	131
10	(8,1,0)	6	13	28	43	47	62	77	81	96	111	115	130
11	(9,1,0)	8	13	30	36	53	59	76	82	99	105	122	128
12	(10,1,0)	3	13	25	37	49	61	73	85	97	109	121	133
13	(0,0,1)	1	2	3	4	5	6	7	8	9	10	11	12
14	(1,0,1)	2	23	34	45	56	67	78	89	100	111	122	133
15	(2,0,1)	2	18	29	40	51	62	73	84	95	106	117	128
16	(3,0,1)	2	20	31	42	53	64	75	86	97	108	119	130
17	(4,0,1)	2	21	32	43	54	65	76	87	98	109	120	131
18	(5,0,1)	2	15	26	37	48	59	70	81	92	103	114	125
19	(6,0,1)	2	22	33	44	55	66	77	88	99	110	121	132
20	(7,0,1)	2	16	27	38	49	60	71	82	93	104	115	126
21	(8,0,1)	2	17	28	39	50	61	72	83	94	105	116	127
22	(9,0,1)	2	19	30	41	52	63	74	85	96	107	118	129
23	(10,0,1)	2	14	25	36	47	58	69	80	91	102	113	124
24	(0,1,1)	1	123	124	125	126	127	128	129	130	131	132	133
25	(1,1,1)	12	23	33	43	53	63	73	83	93	103	113	123
26	(2,1,1)	7	18	34	39	55	60	76	81	97	102	118	123
27	(3,1,1)	9	20	27	45	52	59	77	84	91	109	116	123
28	(4,1,1)	10	21	29	37	56	64	72	80	99	107	115	123
29	5,1,1)	4	15	28	41	54	67	69	82	95	108	121	123
30	(6,1,1)	11	22	31	40	49	58	78	87	96	105	114	123
31	(7,1,1)	5	16	30	44	47	61	75	89	92	106	120	123
32	(8,1,1)	6	17	32	36	51	66	70	85	100	104	119	123
33	(9,1,1)	8	19	25	42	48	65	71	88	94	111	117	123
34	(10,1,1)	3	14	26	38	50	62	74	86	98	110	122	123
35	(0,2,1)	1	68	69	70	71	72	73	74	75	76	77	78
36	(1,2,1)	11	23	32	41	50	59	68	88	97	106	115	124
37	(2,2,1)	12	18	28	38	48	58	68	89	99	109	119	129
38	(3,2,1)	5	20	34	37	51	65	68	82	96	110	113	127
39	(4,2,1)	7	21	26	42	47	63	68	84	100	105	121	126
40	(5,2,1)	6	15	30	45	49	64	68	83	98	102	117	132
41	(6,2,1)	9	22	29	36	54	61	68	86	93	111	118	125
42	(7,2,1)	8	16	33	39	56	62	68	85	91	108	114	131
43	(8,2,1)	10	17	25	44	52	60	68	87	95	103	122	130
44	(9,2,1)	3	19	31	43	55	67	68	80	92	104	116	128
45	(10,2,1)	4	14	27	40	53	66	68	81	94	107	120	133
46	(0,3,1)	1	90	91	92	93	94	95	96	97	98	99	100
47	(1,3,1)	10	23	31	39	47	66	74	82	90	109	117	125

i	P_i	L_i											
48	(2,3,1)	6	18	33	37	52	67	71	86	90	105	120	124
49	(3,3,1)	12	20	30	40	50	60	70	80	90	111	121	131
50	(4,3,1)	4	21	34	36	49	62	75	88	90	103	116	129
51	(5,3,1)	8	15	32	38	55	61	78	84	90	107	113	130
52	(6,3,1)	7	22	27	43	48	64	69	85	90	106	122	127
53	(7,3,1)	11	16	25	45	54	63	72	81	90	110	119	128
54	(8,3,1)	3	17	29	41	53	65	77	89	90	102	114	126
55	(9,3,1)	9	19	26	44	51	58	76	83	90	108	115	133
56	(10,3,1)	5	14	28	42	56	59	73	87	90	104	118	132
57	(0,4,1)	1	101	102	103	104	105	106	107	108	109	110	111
58	(1,4,1)	9	23	30	37	55	62	69	87	94	101	119	126
59	(2,4,1)	11	18	27	36	56	65	74	83	92	101	121	130
60	(3,4,1)	8	20	26	43	49	66	72	89	95	101	118	124
61	(4,4,1)	12	21	31	41	51	61	71	81	91	101	122	132
62	(5,4,1)	10	15	34	42	50	58	77	85	93	101	120	128
63	(6,4,1)	5	22	25	39	53	67	70	84	98	101	115	129
64	(7,4,1)	3	16	28	40	52	64	76	88	100	101	113	125
65	(8,4,1)	7	17	33	38	54	59	75	80	96	101	117	133
66	(9,4,1)	4	19	32	45	47	60	73	86	99	101	114	127
67	(10,4,1)	6	14	29	44	48	63	78	82	97	101	116	131
68	(0,5,1)	1	35	36	37	38	39	40	41	42	43	44	45
69	(1,5,1)	8	23	29	35	52	58	75	81	98	104	121	127
70	(2,5,1)	5	18	32	35	49	63	77	80	94	108	122	125
71	(3,5,1)	4	20	33	35	48	61	74	87	100	102	115	128
72	(4,5,1)	9	21	28	35	53	60	78	85	92	110	117	124
73	(5,5,1)	12	15	25	35	56	66	76	86	96	106	116	126
74	(6,5,1)	3	22	34	35	47	59	71	83	95	107	119	131
75	(7,5,1)	6	16	31	35	50	65	69	84	99	103	118	133
76	(8,5,1)	11	17	26	35	55	64	73	82	91	111	120	129
77	(9,5,1)	10	19	27	35	54	62	70	89	97	105	113	132
78	(10,5,1)	7	14	30	35	51	67	72	88	93	109	114	130
79	(0,6,1)	1	112	113	114	115	116	117	118	119	120	121	122
80	(1,6,1)	7	23	28	44	49	65	70	86	91	107	112	128
81	(2,6,1)	10	18	26	45	53	61	69	88	96	104	112	131
82	(3,6,1)	11	20	29	38	47	67	76	85	94	103	112	132
83	(4,6,1)	6	21	25	40	55	59	74	89	93	108	112	127
84	(5,6,1)	3	15	27	39	51	63	75	87	99	111	112	124
85	(6,6,1)	12	22	32	42	52	62	72	82	92	102	112	133
86	(7,6,1)	9	16	34	41	48	66	73	80	98	105	112	130
87	(8,6,1)	4	17	30	43	56	58	71	84	97	110	112	125
88	(9,6,1)	5	19	33	36	50	64	78	81	95	109	112	126
89	(10,6,1)	8	14	31	37	54	60	77	83	100	106	112	129
90	(0,7,1)	1	46	47	48	49	50	51	52	53	54	55	56
91	(1,7,1)	6	23	27	42	46	61	76	80	95	110	114	129
92	(2,7,1)	4	18	31	44	46	59	72	85	98	111	113	126
93	(3,7,1)	7	20	25	41	46	62	78	83	99	104	120	125
94	(4,7,1)	3	21	33	45	46	58	70	82	94	106	118	130
95	(5,7,1)	5	15	29	43	46	60	74	88	91	105	119	133

i	P_i	L_i											
96	(6,7,1)	10	22	30	38	46	65	73	81	100	108	116	124
97	(7,7,1)	12	16	26	36	46	67	77	87	97	107	117	127
98	(8,7,1)	8	17	34	40	46	63	69	86	92	109	115	132
99	(9,7,1)	11	19	28	37	46	66	75	84	93	102	122	131
100	(10,7,1)	9	14	32	39	46	64	71	89	96	103	121	128
101	(0,8,1)	1	57	58	59	60	61	62	63	64	65	66	67
102	(1,8,1)	5	23	26	40	54	57	71	85	99	102	116	130
103	(2,8,1)	9	18	25	43	50	57	75	82	100	107	114	132
104	(3,8,1)	3	20	32	44	56	57	69	81	93	105	117	129
105	(4,8,1)	11	21	30	39	48	57	77	86	95	104	113	133
106	(5,8,1)	7	15	31	36	52	57	73	89	94	110	115	131
107	(6,8,1)	8	22	28	45	51	57	74	80	97	103	120	126
108	(7,8,1)	4	16	29	42	55	57	70	83	96	109	122	124
109	(8,8,1)	12	17	27	37	47	57	78	88	98	108	118	128
110	(9,8,1)	6	19	34	38	53	57	72	87	91	106	121	125
111	(10,8,1)	10	14	33	41	49	57	76	84	92	111	119	127
112	(0,9,1)	1	79	80	81	82	83	84	85	86	87	88	89
113	(1,9,1)	4	23	25	38	51	64	77	79	92	105	118	131
114	(2,9,1)	3	18	30	42	54	66	78	79	91	103	115	127
115	(3,9,1)	10	20	28	36	55	63	71	79	98	106	114	133
116	(4,9,1)	8	21	27	44	50	67	73	79	96	102	119	125
117	(5,9,1)	9	15	33	40	47	65	72	79	97	104	122	129
118	(6,9,1)	6	22	26	41	56	60	75	79	94	109	113	128
119	(7,9,1)	7	16	32	37	53	58	74	79	95	111	116	132
120	(8,9,1)	5	17	31	45	48	62	76	79	93	107	121	124
121	(9,9,1)	12	19	29	39	49	59	69	79	100	110	120	130
122	(10,9,1)	11	14	34	43	52	61	70	79	99	108	117	126
123	(0,10,1)	1	24	25	26	27	28	29	30	31	32	33	34
124	(1,10,1)	3	23	24	36	48	60	72	84	96	108	120	132
125	(2,10,1)	8	18	24	41	47	64	70	87	93	110	116	133
126	(3,10,1)	6	20	24	39	54	58	73	88	92	107	122	126
127	(4,10,1)	5	21	24	38	52	66	69	83	97	111	114	128
128	(5,10,1)	11	15	24	44	53	62	71	80	100	109	118	127
129	(6,10,1)	4	22	24	37	50	63	76	89	91	104	117	130
130	(7,10,1)	10	16	24	43	51	59	78	86	94	102	121	129
131	(8,10,1)	9	17	24	42	49	67	74	81	99	106	113	131
132	(9,10,1)	7	19	24	40	56	61	77	82	98	103	119	124
133	(10,10,1)	12	14	24	45	55	65	75	85	95	105	115	125

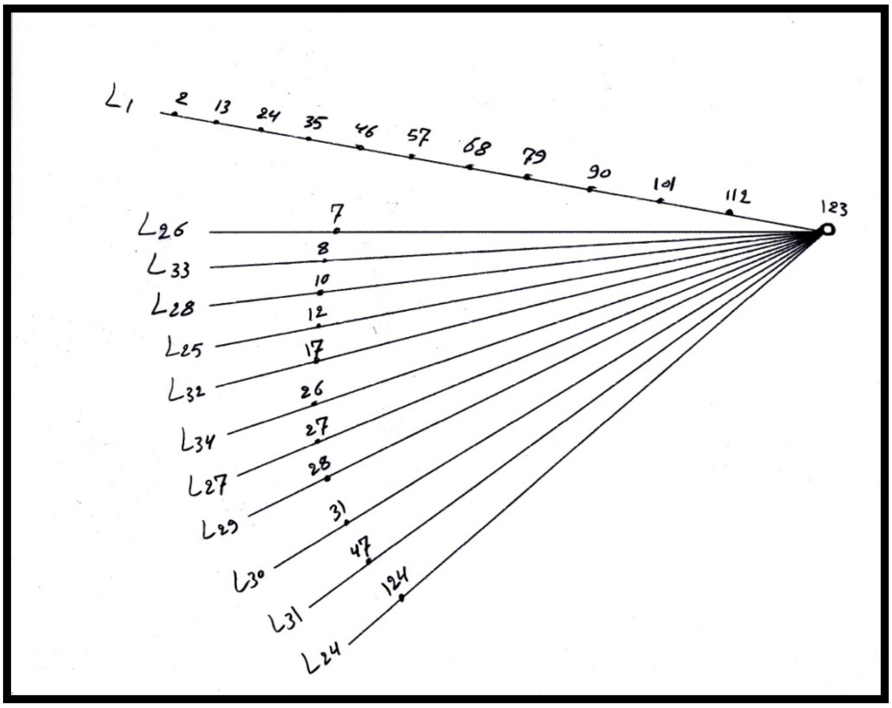


Figure (1)

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بناء الاقواس (k_n,n) في المستوي إلاسقاطي PG(2,11) بطريقة هندسية مع المجموعات القالبية والشفرات الاسقاطية المرتبطة بها

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أستلم البحث في : 17 حزيران 2013 ، قبل للنشرفي : 10 تشرين الاول 2013

الخلاصة

القوس (k,n)هو مجموعة من k من النقاط في PG(2,q),أذ توجد n ولا توجد n + 1 منها على استقامة واحدة، فيكون القوس – (k,n) كاملاً اذا لم يكن محتوى في قوس-(k + 1,n).
في هذا البحث نقوم ببناء اقواس – (k_n,n) كاملة ،إذ n = 2,3,…,10,11 في المستوي الاسقاطي PG(2,11) بطريقة هندسية، مع المجموعات القالبية والشفرات الاسقاطية المرتبطة بها.

كلمات مفتاحية : أقواس كاملة ، مجموعة قالبية، شفرة أسقاطية