



Simulate and Analyze the Effect of the Magnetic System on the Electron Oscillatory Motion and the Formation of the Free Electron Laser Beam

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Abstract

The free electron laser (FEL) represents a significant advance over conventional laser systems by allowing a wide range of operating wavelengths without changing the active laser medium or undergoing extensive redesign. Central to this ability is the Wiggler component, which influences many of the properties of the emitting laser through its wavelength (λ_u). This study investigates the influence of (λ_u) on the FEL's performance metrics, including magnetic field strength, photon wavelength, and laser power, by utilizing MATLAB R2023a for simulation. The study employs a set of equations to explore how variations in (λ_u) affect these parameters. The undulator's role, a critical component of the FEL, is analyzed for its ability to convert high-speed electron kinetic energy into coherent light by manipulating electron trajectories through Lorentz force interactions. Results indicate a direct proportionality between (λ_u) and both the generated magnetic field and output photon wavelengths, suggesting that (λ_u) is a pivotal factor in optimizing FEL performance. The findings highlight potential enhancements in laser power and stability that could benefit a range of applications, from medical to military. This paper not only reaffirms the critical role of (λ_u) in the operational efficiency of FEL systems but also guides future design strategies for advanced laser technologies.

Keywords: Free Electron Laser, Wiggler, Laser power, Lorentz force, Undulator.

1. Introduction

The invention of the free electron laser system fifty years ago this decade caused a tremendous revolution in human life and its general applications, including medical and military applications. (1,2). Laser technologies have been developed that outperform traditional laser systems with unique advantages. This technology is called free electron laser (FEL) (3), as it is characterized by its ability to generate a wide spectrum of laser wavelengths without the need to change the material of the laser active medium or redesign the entire system (4). In a free electron laser system, the active medium consists of particles of negatively charged electrons traveling at very high speeds approaching the speed of light after being supplied with energy from a voltage source (5). The active medium will not be a solid, liquid metal, or semiconductor as in conventional laser systems (6,7). These negatively charged electrons line



up with each other while traveling as a stream in the form of a beam, and this beam or stream of electrons passes through two rows of magnets with certain geometric dimensions called an undulator (8,9). where in the undulator the magnets are arranged periodically, that is, once a south pole is followed by a north pole, and so it continues. That is, it is done, in an alternating and opposite way, by generating a regular periodic magnetic field (10 -12). This alternating magnetic field generated by the undulator obstructs and inhibits the movement of the electron beam passing through it, forcing it to move in an oscillatory manner and bending the path of its movement, where a force called the Lorentz force will arise (13). The wavelength of Wiggler (λ_u) is one of the most important factors that directly affect the magnetic field, which in turn affects the electronic beam in particular, the wavelength of the laser output, and the power of the laser as a whole in general (1,12) By conducting a literature survey and reviewing previous research that devoted its attention and calculations to studying the free electron laser system and working to improve its performance The literature survey and review revealed that the effect of the wavelength of Wiggler (λ_u) on the free-electron laser system had not been studied extensively from various aspects, whether its effect on the magnetic, the wavelength of the laser output, or the total energy of the free-electron laser system. Despite its great importance in influencing the generation of coherent photons and the oscillatory movement of the electron (14). The focus of this work was to analyze the effect of the wavelength of Wiggler (λ_u) on the path of the electron and the generation of photons inside the laser.

2. Materials and Methods

To obtain a wide and variable range of wavelengths in the electromagnetic spectrum, many parameters can be studied in the free-electron laser system. This wide spectrum can be used in many scientific, medical, and military applications. Without the need to replace the system as a whole or change the type of active medium (15,16), but only by making some modifications in the geometric design of the free-electron laser system (17), because the free-electron laser is characterized by the ability to generate a wide range of wavelengths, extending from radio wavelengths to X-rays (1-3). In this paper, certain geometric changes were made in the undulator magnets so that from these changes, we obtain variable values for the wavelength of Wiggler (λ_u). Making these changes leads to a change in the laser output in terms of wavelength and laser output power. Making these changes in the free electron laser is one of the important advantages that add important characteristics to the system, including wide stability, short wavelengths, high brightness, pulsed laser, short pulses, and high energy (18). The working principle of the free electron laser is based on the idea of the presence of an active medium, which is a current beam of electrons that travel at high speeds approaching the speed of light and then passes through a magnetic field (9). These electrons are accelerated through an electron accelerator, and these electrons pass through the magnetic field generated by the undulator (2). The main components of the free electron laser system are the following: The electron gun, which fires negative electron particles and is equipped with a high voltage source, is a system that works to provide the electrons with the necessary kinetic energy; an electron accelerator is a system that increases the speed of electrons to very high speeds. The undulator is a system consisting of magnets arranged in two rows that generate a regular magnetic field through which the electron beam passes. Its working principle is to convert part of the kinetic energy of electrons traveling at high speeds into coherent electromagnetic radiation. A resonator is a system of mirrors that amplifies the laser beam (6, 19-21).

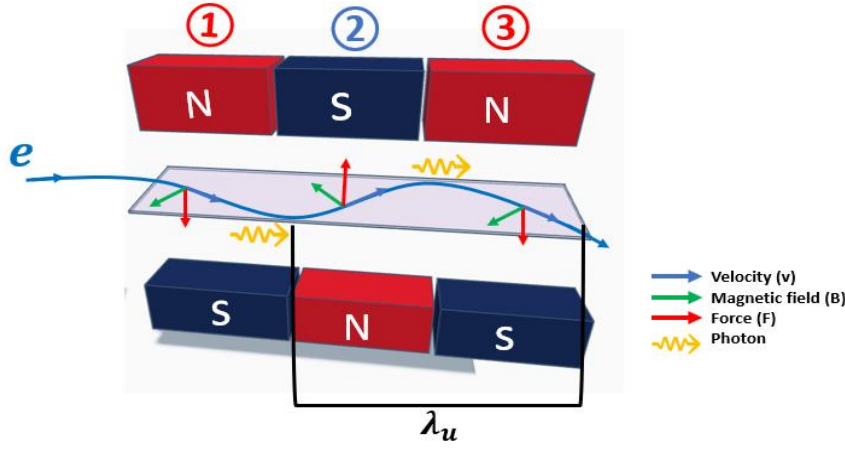


Figure 1. Sketch producing a pair of photons in an undulator in difference (λ_u) (1).

After using equations found in the literature and scientific references and performing some calculations, the following was calculated: The effect of the wavelength of Wiggler (λ_u) on the magnetic field (B), and the relationship between them was found and drawn graphically. The oscillatory parameter (k) and Lorentz factor (γ) were also calculated for energy (500 MeV), which is equivalent to ($8 * 10^{-10}$) in joules. The wavelength of the photon (λ) was also calculated at the same energy mentioned, and the relationship between the wavelength of Wiggler (λ_u) and the wavelength of the resulting photon (λ) was drawn graphically; the initial energy (P_0), photon energy (E_p), number of photons (N_{ph}), and gain were also calculated. gain-length (L_G), laser power (P_z), for the mentioned energy (500 MeV).

$$B = 4.22 \exp\left(-\frac{gu}{\lambda_u} \left(5.08 + 1.54 \frac{gu}{\lambda_u}\right)\right) \quad (1)$$

Where: (B) is the magnetic field, (g_u) is undulator gap, (λ_u) wavelength of the wiggler (22).

$$K = 93.3 \lambda_u \beta_u \quad (2)$$

$$\gamma = \frac{E_e}{m_e c^2} \quad (3)$$

$$\lambda = \lambda_u \left(\frac{1}{2\gamma^2} + \frac{k^2}{4\gamma^2} \right) \quad (4)$$

Where: (k) wiggler parameter, (γ) Lorentz-factor, (E_e) energy of electrons particles, (λ) wavelength of laser (22,23)

$$P_0 = \frac{1}{9} \frac{\chi^2 c E_e}{\lambda} \quad (5)$$

Where: (P_0) initial power, (χ) scaling parameter takes the value (10^{-2} or 10^{-3}) (24,25).

$$E_p = h \frac{c}{\lambda} \quad (6)$$

Where: (E_{ph}) photon energy (22,25).

$$N_{PH} = \frac{\chi E_e}{E_{ph}} \quad (7)$$

Where: (N_{PH}) number of photons (23,26).

$$L_G = \frac{\lambda_u}{4\pi\sqrt{3}\chi} \quad (8)$$

Where: (L_G) gain-length (21,27).

$$P_z = N_{PH} P_0 \exp(z/L_G) \quad (9)$$

Where: (P_z) The laser power, (z) length of undulator (1).

3. Results and Discussion

In this paper, the executive program (FEL) was created and designed using (MATLAB R2023a (9.14.0.2206163)) as shown in **Figure 2**. The mathematical interface of the executive program shown in **Figure 2**. consists of several parameters and equations to simulate the requirements of the calculations that were completed for this paper in terms of the wavelength of Wiggler (λ_u). During the study of this paper, some factors affecting the wavelength formation of the photons generated in the free electron laser system were studied, such as the magnetic field (B), as well as Several other important parameters were also calculated, which are (Lorentz factor (γ), photons wavelength (λ), initial power (P_0), photon energy (E_p), number of photons (N_{ph}), gain-length (L_G), The laser power (P_z), out power from the mirror (P_m)). The undulator is considered an important and main part of the free electron laser system, as it consists of a group of small magnets, the number of which is always even facing each other in two rows, working to generate a regular magnetic field. These pieces of small magnets are arranged among themselves periodically, once at the north pole and then at the next pole. Southern and so on the most important benefit envisaged for the undulator system is the generation of regular magnetic field lines measured in the Tesla unit It works to slow down the very high speeds of the electron beam particles coming from the accelerator after they have been supplied with high energy through a high current voltage source whose range ranges from one megaelectron volt to five hundred megaelectron volts.

The screenshot shows a MATLAB simulation program interface for a Free Electron Laser (FEL). The interface is organized into a grid of input fields and a central result button. The input fields are labeled as follows:

- undulator gap (g_u)
- magnetic field (B)
- wiggler parameter (k)
- Lorentz-factor (γ)
- wavelength of wiggler (λ_u)
- energy of electrons (E_e)
- initial power (P_0)
- number of photons (N_{ph})
- photon energy (E_{ph})
- gain-length (L_G)
- scaling parameter (χ)
- The laser power (P_z)
- out power (P_m)
- R_1
- R_2
- ($G - \psi$)
- laser length (L_z)
- Length of undulator (z)

A large yellow button labeled "result" is positioned at the bottom right of the input grid. At the bottom of the interface, there is a red text box that reads: "Simulate and analyze the effect of the magnetic system on the electron oscillatory motion and the formation of the free electron laser beam".

Figure 2. Shows the simulation program to analyze the effect of wavelength of Wiggler (λ_u) on the output laser.

When the electrons reach very high speeds close to the speed of light and then enter the undulator, the regular magnetic field will obstruct the path of these particles and transform their path from a linear path to a curved path (29). This resulting curved path consists of peaks and troughs, and the distance between every two identical points along the path is called the wavelength of Wiggler (λ_u), and it is repeated along the undulator. Where two photons are generated, one at the top of the wave and the other at the bottom, as shown in **Figure 1**. In this paper, we will study the effect of changing the wavelength of Wiggler (λ_u), which is the distance between any two identical points along the path of electrons inside the undulator. Where calculations and simulations have proven that the magnetic field (B) is directly proportional to the wavelength of Wiggler (λ_u) and there is a positive relationship between the laser wavelengths (λ) to the wavelength of Wiggler (λ_u) where the effect of the wavelength of Wiggler (λ_u) on the magnetic field was calculated through equation (1) when the undulator gap $g_u = 0.03$ and energy of electron particles $E_{BM} = 8 * 10^{-10}$ joule and Lorentz-factor $\gamma = 9771.46$ The calculation results from the simulation are recorded in **Table 1**. To calculate the effect of changing the wavelength of Wiggler (λ_u) on the magnetic field (B) and the laser wavelengths produced in the free electron laser (λ), this was done using equations (1-4) Where the effect of changing the wavelength of Wiggler (λ_u) was studied at the energy of the electron particles ($8 * 10^{-10}$) joules, which is equivalent in the unit of electron volts (500 MeV). The value of the Wiggler parameter (k) was also calculated through equation (2). Where it was considered that the mass of the electron is $m_e = 9.10938356 * 10^{-31}$ (kg) and the typical speed of light is $C = 299,792,458$ (m/s). The values resulting from the calculations and simulations have been recorded in **Table 1**.

Table 1. Magnetic field (B) and Wiggler parameter(k) and Wavelength values (λ) resulting from changing the wavelength of Wiggler (λ_u) at 500 MeV.

$g_u = 0.03$ & $E_{BM} = 8 * 10^{-10} J$ & $\gamma = 9771.46$				
	Wavelength of undulator (λ_u) in (m)	Magnetic field (B) in (T)	Wiggler parameter (k) in Tm	Laser wavelength (λ) in nm
1	0.01	9.71277e-13	9.06201e-13	0.0523662
2	0.012	8.50527e-10	9.5225e-10	0.0628394
3	0.014	6.70848e-08	8.76262e-08	0.0733127
4	0.016	1.37202e-06	2.04815e-06	0.0837859
5	0.018	1.2316e-05	2.06835e-05	0.0942592
6	0.02	6.47378e-05	0.000120801	0.104732
7	0.022	0.000236143	0.000484707	0.115206
8	0.024	0.00066454	0.00148804	0.125679
9	0.026	0.00154613	0.0037506	0.136153
10	0.028	0.00311702	0.0081429	0.14663
11	0.03	0.00562708	0.0157502	0.157118

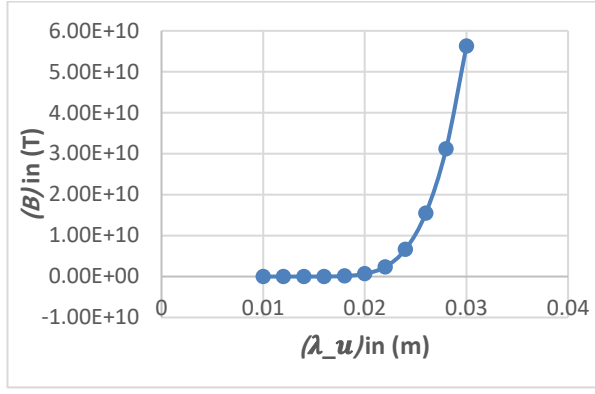


Figure 3. Plot of Change in a magnetic field (B) resulting from changing the wavelength of Wiggler (λ_u) at 500 MeV.

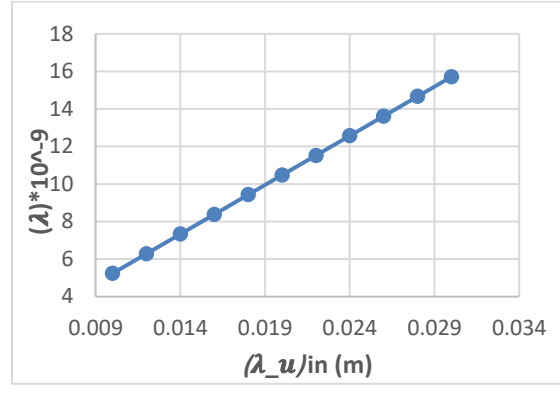


Figure 4. Plot of Change in laser wavelength values (λ) resulting from changing the wavelength of Wiggler (λ_u) at 500 MeV.

Figure 3 shows that there is a direct relationship between the wavelength of Wiggler (λ_u) and the magnetic field (B), which are linked together within equation (1), where the wavelength of Wiggler represents the distance between any two identical points along the curved path of the electron beam. Which is exposed to the magnetic field by the undulator system, where the magnetic field begins at (9.71277e-13) in Tesla when the wavelength of Wiggler (λ_u) is equal to (0.01 m) and ends at (0.00562708 T) if the wavelength of Wiggler (λ_u) is equal to (0.03 m), Through the behavior of the curve, the physical meaning of the curve can be explained by saying that there is a direct interaction between both the negative particle field and the undulator magnetic field. While **Figure 4** shows that there is a linear relationship between the wavelength of Wiggler (λ_u) and laser wavelength values (λ), which are linked together within equation (4), where the simulation showed that any increase in the wavelength of Wiggler will be offset by an increase in laser wavelength. values (λ) and the opposite are true, meaning that whenever the wavelength of Wiggler decreases, there will be a decrease in laser wavelength values (λ). Therefore, the wavelength of Wiggler is considered an important factor in obtaining a wide spectrum of wavelengths for the laser output, as can be seen from the diagram that laser wavelength values (λ) will be (0.0523662 nm) when the wavelength of Wiggler equals (0.01 m) while laser wavelength values (λ) will be equal to (0.157118 nm) when the wavelength of Wiggler equals (0.03 m), Through the behavior of the curve, the physical meaning of the curve can be explained by saying that every increase in the wavelength of Wiggler (λ_u) It means an increase in the amount of obstruction of electron particles at 500 MeV Therefore, there will be an increase in laser wavelength values (λ). In order to calculate the effect of changing the wavelength of Wiggler (λ_u) on the initial power (P_0) in the free electron laser, this was done using equation (5) At the energy studied (500 MeV), where the values and constants have been substituted, the (χ) scaling parameter takes a value of (0.001). To calculate the laser power (P_z) is done through equation (9) where photon energy (E_{ph}) was calculated through equation (6), and the number of photons (N_{ph}) was also calculated through equation (7). Likewise, gain length (LG) was calculated through equation (8). length of undulator (L_D) was considered (2m). The values resulting from the calculations and simulations have been recorded in **Table 2**.

Table 2. Values of initial power (P_0) and photon energy (E_p) values of (N_{ph}) number of photons resulting from changing the wavelength of Wiggler (λ_u) of energy 500 MeV.

$\chi = 0.001$ & $E_{BM} = 8 * 10^{-10} J$ & $L_D = 2m$						
Laser wavelength (λ) in nm	Initial power (P_0) in (w)	Photon energy (E_{ph}) In FJ	NO. photon (N_{ph})	gain-length (L_G)	The laser power (P_z) In J	
1	0.0523662	508.882	3.79337	210.894	0.459674	8.32257e+06
2	0.0628394	424.069	3.16115	253.072	0.551609	4.0302e+06
3	0.0733127	363.487	2.70955	295.252	0.643543	2.40093e+06
4	0.0837859	318.051	2.37086	337.43	0.735478	1.62804e+06
5	0.0942592	282.712	2.10743	379.609	0.827413	1.2035e+06
6	0.104732	254.442	1.89669	421.787	0.919348	945083
7	0.115206	231.309	1.72426	463.967	1.01128	775497
8	0.125679	212.034	1.58057	506.147	1.10322	657664
9	0.136153	195.723	1.45898	548.328	1.19515	572063
10	0.14663	181.738	1.35473	590.524	1.28709	507610
11	0.157118	169.606	1.2643	632.761	1.37902	457657

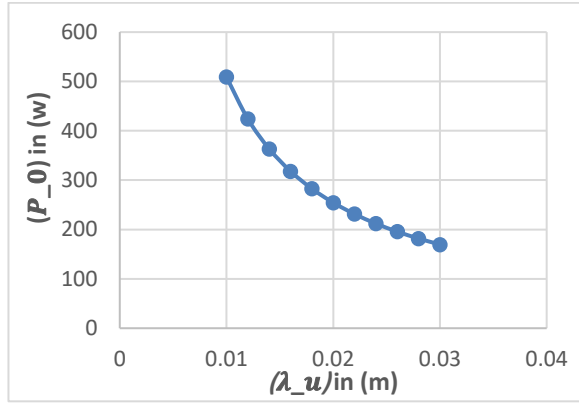


Figure 5-a. Plot of initial power (P_0) resulting from changing the wavelength of Wiggler (λ_u) at 500 MeV.

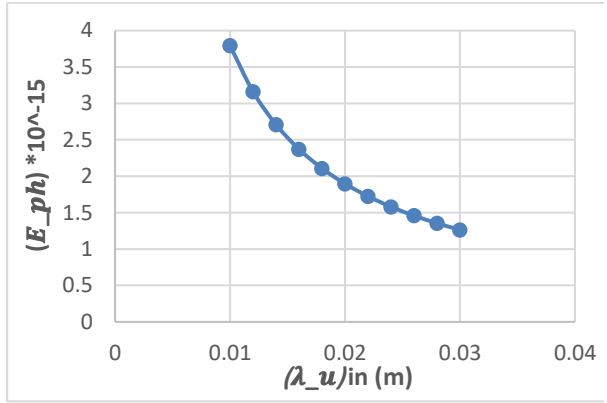


Figure 5-b. Plot of Photon Energy (E_{ph}) resulting from changing the wavelength of Wiggler (λ_u) at 500 MeV.

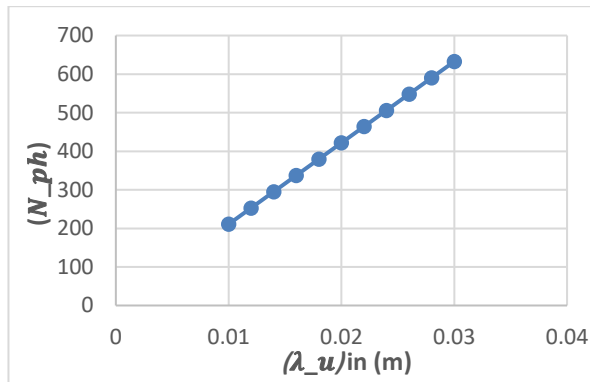


Figure 5-c. Plot of values of a number of photons (N_{ph}) resulting from changing the wavelength of Wiggler (λ_u) at 500 MeV.

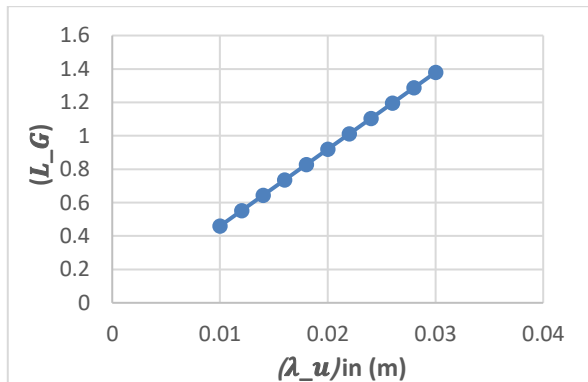


Figure 5-d. Plot of values of gain-length (L_G) resulting from changing the wavelength of Wiggler (λ_u) at 500 MeV.

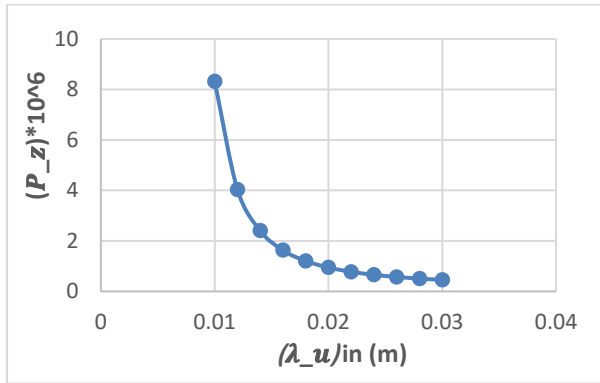


Figure 5-e. Plot of the laser power (P_z) resulting from changing the wavelength of Wiggler (λ_u) at 500 MeV.

Figures (5-a) and (5-b) show through simulation calculations that there is an inverse relationship between the wavelength of Wiggler (λ_u) with both initial power (P_0) and Photon Energy (E_{ph}) at the energy studied. Whereas, the initial power is the integrated shot noise incoherently radiated into the dominant growing mode by the electrons passing through the first gain length, Likewise, Photon Energy (E_{ph}) started at (210.894 femtojoules) and decreased to (1.2643 femtojoules) when the wavelength of Wiggler (λ_u) was increased. This proves that the wavelength of Wiggler (λ_u) plays an important role in both important factors. **Figures (5-c) and (5-d)** show through simulation calculations that there is a direct relationship between the wavelength of Wiggler (λ_u) with both the number of photons (N_{ph}) and the gain-length (LG), and that any change in the amount of Gain-length will affect laser power (P_z), because gain-length (LG) is one of the requirements for calculating equation (9). As a result, this means that the wavelength of Wiggler (λ_u) affects them greatly. **Figure (5-e)** Simulation calculations found that there is a negative relationship between the wavelength of Wiggler (λ_u) with the laser power (P_z). The resulting values were recorded through equation (9) based on calculating all the requirements of the aforementioned equation at the same energy studied. As for changing the wavelength of Wiggler (λ_u), it depends mainly on the geometry and size of the magnet pieces that make up the undulator system, as whenever the poles are changed along the path of the electron, the direction of the top or bottom of the path changes, and all of this is subject to the influence of the Lorentz force.

4. Conclusion

Through the simulations that were conducted using scientific equations related to the subject of the free electron laser mentioned in the references and sources, and through the simulation program that was designed, the calculations that were conducted showed that there is a large and direct effect of the wavelength of Wiggler (λ_u), on each of the laser output wavelengths. The laser's power also plays a significant role. As the electron beam flows into the undulator system, many options can be used to obtain a change in the behavior of the laser system as a whole. This paper states that the magnetic field (B), Laser Wavelength (λ), initial power (P_0), Photon Energy (E_{ph}), number of photons (N_{ph}), the gain length (LG), and the laser power (P_z) are affected when there is any change in the wavelength of Wiggler (λ_u), and that there is a positive proportionality between the wavelength of Wiggler (λ_u) with the magnetic field (B), Laser Wavelength (λ), number of photons (N_{ph}) and the gain-length (LG), while there is negative proportionality with initial power (P_0), Photon Energy (E_{ph}), and the laser power (P_z) at the energy studied in this paper of 500 MeV.

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Conflict of Interest

The authors declare that they have no conflicts of interest.

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