ON Weak D_{s^*g}-Sets And Associative Separation Axioms

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Abstract

In this paper, we introduce new classes of sets called D_{s^*g}-sets , $D_{\alpha-s^*g}$-sets , D_{pre-s^*g}-sets , D_{b-s^*g}-sets and $D_{\beta-s^*g}$-sets . Also, we study some of their properties and relations among them . Moreover, we use these sets to define and study some associative separation axioms .

Keywords: s*g- D_i **-spaces ,** α **-s*g-** D_i **-spaces, pre-s*g-** D_i **-space , b-s*g-** D_i **-spaces,** β **-s*g-** D_i **-spaces for** $i = 0,1,2$ **.**

Introduction

Tong, J . [1] , Calads,M. [2], Calads,M. and et.al. [3] , Jafari, S. [4] and Keskin,A. and Noiri, T. [5] introduced the notion of D-sets, D_s-sets , D_α-sets , D_{pre}-sets and D_b-sets respectively by using open sets, semi-open sets, α-open sets, pre-open sets and b-open sets respectively and used the notion to define some associative separation axioms . Khan,M. and et.al.[6] introduced and investigated s*g-closed sets by using the concept of semi-open sets . In this paper we introduce and investigate new notions called D_{s^*g}-sets , $D_{\alpha-s^*g}$-sets , D_{pre-s^*g}-sets , D_{b-s^*g}-sets and $D_{\beta-s^*g}$-sets . Moreover, we use these notions to define some associative separation axioms . Recall that a subset A of a topological space (X,τ) is called semi-open [7] (resp. α-open [8], pre-open [9] , b-open[10] and β-open [11]) set if $A \subseteq cl(int(A))$ (resp . $A \subseteq int(cl(int(A)))$, $A \subseteq int(cl(A))$, $A \subseteq int(cl(A)) \bigcup cl(int(A))$ and $A \subseteq cl(int(cl(A)))$) . Also, a subset A of a topological space (X,τ) is called s*g-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X [6] . The complement of an s*g-closed set is defined to be s*g-open . The family of all s*g-open subsets of (X,τ) is denoted by $S^*GO(X,\tau)$ [6], this family from a topology on X which is finer than τ [6] .

The s*g-closure of A, denoted by $cl_{s^*g}(A)$ is the intersection of all s*g-closed subsets of X which contains A and the s*g-interior of A, denoted by $int_{s^*g}(A)$ is the union of all s*g-open sets in X which are contained in A [6] . A function $f:(X,\tau) \to (Y,\sigma)$ is called s*g-continuous [12] (resp. s*g-irresolute [12]) if the inverse image of every open (resp. s*g-open) subset of Y is an s*g-open set in X. Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) represent non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

Preliminaries

First we recall the following definitions .

Definition(1.1)[1]: A subset A of a topological space (X,τ) is called a D-set if there are two open sets U and V in X such that $U \neq X$ and $A = U \setminus V$.

Definition(1.2)[1]: A topological space (X,τ) is called a D_0-space if for any two distinct points x and y of X, there exists a D-set of X containing one of the points but not the other .

Definition(1.3)[1]: A topological space (X,τ) is called a D_1-space if for any two distinct points x and y of X, there exists a D-set of X containing x but not y and a D-set of X containing y but not x .

Definition(1.4)[1]: A topological space (X,τ) is called a D_2-space if for any two distinct points x and y of X, there are two D-sets U and V of X such that $x \in U$, $y \in V$ and $U \bigcap V = \phi$.

Definition(1.5)[13]: A topological space (X,τ) is called a door space if each subset of X is either open or closed .

Theorem(1.6): Let (X,τ) be a topological space and $A, B \subseteq X$. Then:-

i) $int(A) \subseteq int_{s^*g}(A) \subseteq A$ and $A \subseteq cl_{s^*g}(A) \subseteq cl(A)$.

ii) $int_{s^*g}(A)$ is an s*g-open set in X and $cl_{s^*g}(A)$ is an s*g-closed set in X .

iii) If $A \subseteq B$, then $int_{s^*g}(A) \subseteq int_{s^*g}(B)$ and $cl_{s^*g}(A) \subseteq cl_{s^*g}(B)$.

iv) A is s*g-open iff $int_{s^*g}(A) = A$ and A is s*g-closed iff $cl_{s^*g}(A) = A$.

v) $int_{s^*g}(A \cap B) = int_{s^*g}(A) \cap int_{s^*g}(B)$ and $cl_{s^*g}(A \cup B) = cl_{s^*g}(A) \cup cl_{s^*g}(B)$.

vi) $int_{s^*g}(int_{s^*g}(A)) = int_{s^*g}(A)$ and $cl_{s^*g}(cl_{s^*g}(A)) = cl_{s^*g}(A)$.

vii) $X - cl_{s^*g}(A) = int_{s^*g}(X - A)$ and $X - int_{s^*g}(A) = cl_{s^*g}(X - A)$.

viii) $x \in int_{s^*g}(A)$ iff there is an s*g-open set U in X s.t $x \in U \subseteq A$.

ix) $x \in cl_{s^*g}(A)$ iff for every s*g-open set U containing x , $U \cap A \neq \phi$.

x) $\bigcup_{\alpha \in \wedge} cl_{s^*g}(U_\alpha) \subseteq cl_{s^*g}(\bigcup_{\alpha \in \wedge} U_\alpha)$ and $\bigcup_{\alpha \in \wedge} int_{s^*g}(U_\alpha) \subseteq int_{s^*g}(\bigcup_{\alpha \in \wedge} U_\alpha)$.

Proof: It is a obvious .

In this paper we introduce and investigate new notions called α-s*g-open sets , pre -s*g-open sets , b-s*g-open sets and β-s*g-open sets which are weaker than s*g-open . Moreover, we use these notions to define some associative separation axioms .

2. Weak Forms Of s*g-Open Sets

In this section we introduce the following notions.

Definitions(2.1): A subset A of a topological space (X,τ) is said to be :

i) An α-s*g-open set if $A \subseteq int_{s^*g}(cl(int_{s^*g}(A)))$.

ii) A pre-s*g-open set if $A \subseteq int_{s^*g}(cl(A))$.

iii) A b-s*g-open set if $A \subseteq int_{s^*g}(cl(A)) \cup cl(int_{s^*g}(A))$.

iv) A β-s*g-open set if $A \subseteq cl(int_{s^*g}(cl(A)))$.

Lemma(2.2): Let (X,τ) be a topological space , then the following properties hold:

i) Every α-open (resp. pre-open, b-open, β-open) set is α-s*g-open (resp. pre-s*g-open, b-s*g-
open, β-s*g-open) set .

ii) Every s*g-open set is α-s*g-open .

iii) Every α-s*g-open set is pre-s*g-open .

iv) Every pre-s*g-open set is b-s*g-open .

v) Every b-s*g-open set is β-s*g-open .

Proof: It is obvious .

Since every open set is s*g-open, then we have the following diagram for some types of open sets and s*g-open set .

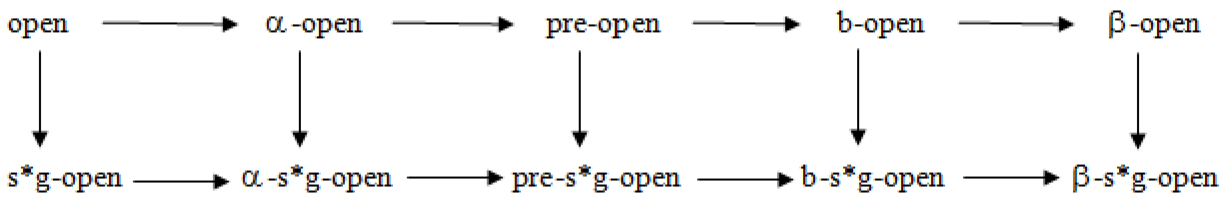


Figure No. (1): Relations between some types of open sets and s*g-open sets

The converses need not be true in general as shown by the following examples .

Example(2.3): Let $X=\{a,b,c\}$ with the indiscrete topology $\tau = I = \{X,\phi\}$. Then $\{a,b\}$ is an s*g-open (resp. α-s*g-open) set, but it is not open (resp. not α-open) set .

Example(2.4): Let $X=\{a,b,c,d\}$ & $\tau=\{X,\phi,\{a\},\{a,c\},\{a,b,d\}\}$. Then $\{a,c,d\}$ is an α-s*g-open set , but it is not s*g-open .

Example(2.5): Let $X=\Re$ with the usual topology τ. Let $A=Q$ be the set of all rational numbers. Then A is an pre-s*g-open set (since A is pre-open set) which is not α-s*g-open .

Example(2.6): Let $X=\Re$ with the usual topology τ. Let $A=(0,1]$. Then A is an b-s*g-open set (since A is b-open set) which is not pre-s*g-open .

Example(2.7): Let $X=\Re$ with the usual topology τ. Let $A=Q\cap[0,1]$. Then A is a β-s*g open set (since A is β-open set) which is not b-s*g-open .

Theorem(2.8): If A is a pre-s*g-open subset of a topological space (X,τ) such that $U\subseteq A\subseteq cl(U)$ for a subset U of X , then U is an pre-s*g-open set .

Proof: Since $A\subseteq cl(U)\Rightarrow cl(A)\subseteq cl(cl(U))=cl(U)\Rightarrow int_{s^*g}(cl(A))\subseteq int_{s^*g}(cl(U))$. Since $A\subseteq int_{s^*g}(cl(A))$ and $U\subseteq A\Rightarrow U\subseteq int_{s^*g}(cl(U))$. Thus U is an pre-s*g-open set .

Theorem(2.9): A subset A of a topological space (X,τ) is semi-open if and only if A is β-s*g-open and $int_{s^*g}(cl(A))\subseteq cl(int(A))$.

Proof: Let A be semi-open, then $A\subseteq cl(int(A))\subseteq cl(int_{s^*g}(cl(A)))$ and hence A is β-s*g-open. Also, since $A\subseteq cl(int(A))\Rightarrow cl(A)\subseteq cl(int(A))\Rightarrow int_{s^*g}(cl(A))\subseteq cl(int(A))$.
Conversely, let A be β-s*g-open and $int_{s^*g}(cl(A))\subseteq cl(int(A))$. Then $A\subseteq cl(int_{s^*g}(cl(A)))\subseteq cl(cl(int(A)))=cl(int(A))$ and hence A is semi-open .

Lemma(2.10)[13]: Let (X,τ) be a topological space . If U is an open set in X, then $U\cap cl(A)\subseteq cl(U\cap A)$ for any subset A of X .

Propositions(2.11): Let (X,τ) be a topological space , then:
i) The intersection of a pre-s*g-open set and an open set is pre-s*g-open .
ii) The intersection of a β-s*g-open set and an open set is β-s*g-open .

iii) The intersection of a b-s*g-open set and an open set is b-s*g-open .
iv) The intersection of an α-s*g-open set and an open set is α-s*g-open .

Proof: We prove only the first case since the other cases are similarly shown .
i) Let A be a pre-s*g-open set and U be an open set in X . Since every open set is s*g-open , then $A \subseteq int_{s*g}(cl(A))$ and $U = int_{s*g}(U)$. By Lemma (2.10), we have
$U \bigcap A \subseteq int_{s*g}(U) \bigcap int_{s*g}(cl(A)) = int_{s*g}(U \bigcap cl(A)) \subseteq int_{s*g}(cl(U \bigcap A))$.
Therefore $A \bigcap U$ is pre-s*g-open .

Remark(2.12): We note that the intersection of two pre-s*g-open (resp. b-s*g-open, β-s*g-open, α-s*g-open) sets need not be pre-s*g-open (resp. b-s*g-open, β-s*g-open, α-s*g-open) as can be seen from the following examples:

Example(2.13): Let $X = \Re$ with the usual topology τ. Let $A = Q$ and $B = Q^c \bigcup \{1\}$, then A and B are pre-s*g-open , but $A \bigcap B = \{1\}$ which is not β-s*g-open since
$cl(int_{s*g}(cl(\{1\})))$
$= cl(int_{s*g}(\{1\})) = cl(\{\phi\}) = \phi$.

Example(2.14): Let $X = \{a, b, c\}$ & $\tau = \{X, \phi, \{b, c\}\}$. Then $\{a, b\}$ and $\{a, c\}$ are α-s*g-open sets, but $\{a, b\} \bigcap \{a, c\} = \{a\}$ is not α-s*g-open .

Theorem(2.15): If $\{A_\alpha : \alpha \in \wedge\}$ is a collection of b-s*g-open (resp. pre-s*g-open, β-s*g-open, α
-s*g-open) sets of a topological space (X, τ), then $\bigcup_{\alpha \in \wedge} A_\alpha$ is b-s*g-open (resp. pre-s*g-open, β-s*g-open, α-s*g-open).

Proof: We prove only the first case since the other cases are similarly shown .
Since $A_\alpha \subseteq int_{s*g}(cl(A_\alpha)) \bigcup cl(int_{s*g}(A_\alpha))$ for every $\alpha \in \wedge$, we have:

$$\bigcup_{\alpha \in \wedge} A_\alpha \subseteq \bigcup_{\alpha \in \wedge} [int_{s*g}(cl(A_\alpha)) \bigcup cl(int_{s*g}(A_\alpha))] = [\bigcup_{\alpha \in \wedge} int_{s*g}(cl(A_\alpha))] \bigcup [\bigcup_{\alpha \in \wedge} cl(int_{s*g}(A_\alpha))]$$
$$\subseteq [int_{s*g}(\bigcup_{\alpha \in \wedge} cl(A_\alpha))] \bigcup [cl(\bigcup_{\alpha \in \wedge} int_{s*g}(A_\alpha))] \subseteq [int_{s*g}(cl(\bigcup_{\alpha \in \wedge} A_\alpha))] \bigcup [cl(int_{s*g}(\bigcup_{\alpha \in \wedge} A_\alpha))]$$

Therefore $\bigcup_{\alpha \in \wedge} A_\alpha$ is b-s*g-open .

Proposition(2.16): Let (X, τ) be a topological space and $A \subseteq X$. If A is a b-s*g-open set such that $int_{s*g}(A) = \phi$, then A is pre-s*g-open .

Proof: Since A is b-s*g-open, then $A \subseteq int_{s*g}(cl(A)) \bigcup cl(int_{s*g}(A))$. Since $int_{s*g}(A) = \phi$, then
$cl(int_{s*g}(A)) = \phi$, therefore $A \subseteq int_{s*g}(cl(A))$. Thus A is a pre-s*g-open set .

Propositions(2.17): If (X, τ) is a door space , then:
i) Every pre-s*g-open set is s*g-open .
ii) Every β-s*g-open set is b-s*g-open .

Proof: i) Let A be an pre-s*g-open set. If A is open, then A is s*g-open. Otherwise, A is closed and hence $A \subseteq int_{s^*g}(cl(A)) = int_{s^*g}(A)$. Therefore, $A = int_{s^*g}(A)$ and thus A is an s*g-open set.

ii) Let A be an β-s*g-open set . If A is open , then A is b-s*g-open . Otherwise , A is closed and hence $A \subseteq cl(int_{s^*g}(cl(A))) = cl(int_{s^*g}(A)) \subseteq int_{s^*g}(cl(A)) \bigcup cl(int_{s^*g}(A))$.
Therefore A is an b-s*g-open set .

Definitions(2.18): A subset A of a topological space (X,τ) is called:
i) An s*g-t-set if $int(A) = int_{s^*g}(cl(A))$.
ii) An s*g-B-set if $A = U \bigcap V$, where $U \in \tau$ and V is an s*g-t-set .

Proposition(2.19): Let A and B be subsets of a topological space (X,τ). If A and B are s*g-t-sets, then $A \bigcap B$ is an s*g-t-set .

Proof: Let A and B be s*g-t-sets . Then we have:
$int_{s^*g}(cl(A \bigcap B)) \subseteq int_{s^*g}(cl(A) \bigcap cl(B)) = int_{s^*g}(cl(A)) \bigcap int_{s^*g}(cl(B)) = int(A) \bigcap int(B)$
$= int(A \bigcap B)$. Since $int(A \bigcap B) \subseteq int_{s^*g}(cl(A \bigcap B))$, then $int(A \bigcap B) = int_{s^*g}(cl(A \bigcap B))$ and hence $A \bigcap B$ is an s*g-t-set .

From the following example one can deduce that a pre-s*g-open set and a s*g-B-set are independent .

Example(2.20): Let X = R with the usual topology τ. Then $R \setminus Q$ is pre-s*g-open, but it is not an s*g-B-set (since $R \setminus Q = R \bigcap R \setminus Q$, where $R \in \tau$,but $R \setminus Q$ is not an s*g-t-set) and (0,1] is an s*g-B-set (since $(0,1] = R \bigcap (0,1]$, where $R \in \tau$ and (0,1] is an s*g-t-set) which is not pre-s*g-open (since $(0,1] \not\subset int_{s^*g}(cl((0,1])) = (0,1)$) .

Proposition(2.21): Let (X,τ) be a topological space and $A \subseteq X$. Then the following are equivalent:
i) A is open .
ii) A is pre-s*g-open and an s*g-B-set .

Proof: (i) $\Rightarrow$ (ii) **.** Let A be open. Then $A = int_{s^*g}(A) \subseteq int_{s^*g}(cl(A))$ and A is pre-s*g-open . Also , $A = A \bigcap X$, where $A \in \tau$ and X is an s*g-t-set and hence A is an s*g-B-set .
(ii) $\Rightarrow$ (i) . Since A is an s*g-B-set, we have $A = U \bigcap V$, where $U \in \tau$ and V is an s*g-t-set .
By the hypothesis, A is also pre-s*g-open and we have:
$A \subseteq int_{s^*g}(cl(A)) = int_{s^*g}(cl(U \bigcap V)) \subseteq int_{s^*g}(cl(U) \bigcap cl(V)) = int_{s^*g}(cl(U)) \bigcap int_{s^*g}(cl(V))$
$= int_{s^*g}(cl(U)) \bigcap int(V)$

Hence

$A = U \bigcap V = (U \bigcap V) \bigcap U \subseteq (int_{s^*g}(cl(U)) \bigcap int(V)) \bigcap U = (int_{s^*g}(cl(U)) \bigcap U) \bigcap int(V)$
$= U \bigcap int(V) = int(U) \bigcap int(V) = int(U \bigcap V) = int(A)$. Therefore $A = int(A)$ and A is open .

Definitions(2.22): A subset A of a topological space (X,τ) is called :
i) An s*g- t_α -set if $int(A)=int_{s*g}(cl(int_{s*g}(A)))$.
ii) An s*g- B_α -set if $A=U\bigcap V$, where $U\in\tau$ and V is an s*g- t_α -set .

Proposition(2.23): Let A and B be subsets of a topological space (X,τ) . If A and B are s*g- t_α -sets, then $A\bigcap B$ is an s*g- t_α -set .

Proof: Let A and B be s*g- t_α -sets . Then we have:
$int_{s*g}(cl(int_{s*g}(A\bigcap B))=int_{s*g}(cl(int_{s*g}(A)\bigcap int_{s*g}(B)))\subseteq int_{s*g}(cl(int_{s*g}(A)\bigcap cl(int_{s*g}(B)))$
$=int_{s*g}(cl(int_{s*g}(A)))\bigcap int_{s*g}(cl(int_{s*g}(B)))=int(A)\bigcap int(B)=int(A\bigcap B)$.
Since $int(A\bigcap B)\subseteq int_{s*g}(cl(int_{s*g}(A\bigcap B)))$, then $int(A\bigcap B)=int_{s*g}(cl(int_{s*g}(A\bigcap B))$ and hence
$A\bigcap B$ is an s*g- t_α -set .

From the following example one can deduce that an α-s*g-open set and an s*g- B_α -set are independent .

Example(2.24): Let X = R with the usual topology τ. Then (0,1] is an s*g- B_α -set which is not α-s*g-open . Also, in Example (2.3), $A=\{a,b\}$ is an α-s*g-open set, but is not an s*g- B_α -set.

Proposition(2.25): Let (X,τ) be a topological space and $A\subseteq X$. Then the following are equivalent:
i) A is open .
ii) A is α-s*g-open and an s*g- B_α -set .

Proof: (i) $\Rightarrow$ (ii) **.** Let A be open. Then $A=int_{s*g}(A)\subseteq cl(int_{s*g}(A))$ and
$A\subseteq int_{s*g}(cl(int_{s*g}(A))$
Therefore A is α-s*g-open. Also, $A=A\bigcap X$, where $A\in\tau$ and X is an s*g- t_α -set and hence A is an s*g- B_α -set .
(ii) $\Rightarrow$ (i) . Since A is an s*g- B_α -set, we have $A=U\bigcap V$, where $U\in\tau$ and V is an s*g- t_α -set .
By the hypothesis, A is also α-s*g-open, and we have:
$A\subseteq int_{s*g}(cl(int_{s*g}(A)))=int_{s*g}(cl(int_{s*g}(U\bigcap V)))=int_{s*g}(cl(int_{s*g}(U)\bigcap int_{s*g}(V)))$
$\subseteq int_{s*g}(cl(int_{s*g}(U)\bigcap cl(int_{s*g}(V)))=int_{s*g}(cl(int_{s*g}(U)\bigcap int_{s*g}(cl(int_{s*g}(V)))$
$\subseteq int_{s*g}(cl(U))\bigcap int(V)$

Hence

$A=U\bigcap V=(U\bigcap V)\bigcap U\subseteq(int_{s*g}(cl(U))\bigcap int(V))\bigcap U=(int_{s*g}(cl(U))\bigcap U)\bigcap int(V)$
$=U\bigcap int(V)=int(U)\bigcap int(V)=int(U\bigcap V)=int(A)$. Therefore $A=int(A)$ and A is open .

Definition(2.26): A subset A of a topological space (X,τ) is called an s*g-set if $A = U \cap V$, where $U \in \tau$ and $int(V) = int_{s^*g}(V)$.

From the following example one can deduce that an s*g-open set and an s*g-set are independent .

Example(2.27): Let X = R with the usual topology τ. Then $A = (0,1) \cap Q$ is an s*g-set which is not s*g-open . Also, in Example (2.3) , $A = \{a, b\}$ is an s*g-open set, but is not an s*g-set .

Proposition(2.28): Let (X,τ) be a topological space and $A \subseteq X$. Then the following are equivalent:
i) A is open .
ii) A is s*g-open and an s*g-set .

Proof: (i) $\Rightarrow$ (ii) **.** This is obvious .
(ii) $\Rightarrow$ (i) . Since A is an s*g-set, we have $A = U \cap V$, where $U \in \tau$ and $int(V) = int_{s^*g}(V)$.
By the hypothesis, A is also s*g-open and we have:
$A = int_{s^*g}(A) = int_{s^*g}(U \cap V) = int_{s^*g}(U) \cap int_{s^*g}(V) = U \cap int(V) = int(U) \cap int(V)$
$= int(U \cap V) = int(A)$. Therefore A is open .

Definitions(2.29): A topological space (X,τ) is said to satisfy:
i) The s*g-condition if every s*g-open set is s*g-t-set .
ii) The s*g-B_α-condition if every α-s*g-open set is s*g-B_α-set .
iii) The s*g-B-condition if every pre-s*g-open set is s*g-B-set .

Definition(2.30): A topological space (X,τ) is called an s*g-T_0-space [14] (resp. α-s*g-T_0-space, pre-s*g-T_0-space, b-s*g-T_0-space, β-s*g-T_0-space) if for any two distinct points x and y of X, there exists an s*g-open (resp. α-s*g-open, pre-s*g-open, b-s*g-open, β-s*g-open) set of X containing one of the points but not the other .

Definition(2.31): A topological space (X,τ) is called an s*g-T_1-space [14] (resp. α-s*g-T_1-space, pre-s*g-T_1-space, b-s*g-T_1-space, β-s*g-T_1-space) if for any two distinct points x and y of X, there exists an s*g-open (resp. α-s*g-open, pre-s*g-open, b-s*g-open, β-s*g-open) set of X containing x but not y and an s*g-open (resp. α-s*g-open, pre-s*g-open, b-s*g-open, β-s*g-open) set of X containing y but not x .

Definition(2.32): A topological space (X,τ) is called an s*g-T_2-space [14] (resp. α-s*g-T_2-space, pre-s*g-T_2-space, b-s*g-T_2-space, β-s*g-T_2-space) if for any two distinct points x and y of X, there are two s*g-open (resp. α-s*g-open, pre-s*g-open, b-s*g-open, β-s*g-open) sets U and V of X such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

3. Weak D_{s^*g}-Sets And Associative Separation Axioms

In this section we introduce and investigate new notions called D_{s^*g}-sets , $D_{\alpha-s^*g}$-sets , D_{pre-s^*g}-sets , D_{b-s^*g}-sets and $D_{\beta-s^*g}$-sets and we use these notions to define and study some associative separation axioms .

Definition(3.1): A subset A of a topological space (X,τ) is called an D_{s^*g}-set (resp. $D_{\alpha-s^*g}$-set,
D_{pre-s^*g}-set, D_{b-s^*g}-set, $D_{\beta-s^*g}$-set) if there are two s*g-open (resp. α-s*g-open, pre-s*g-open, b-s*g-open, β-s*g-open) sets U and V in X such that $U \neq X$ and $A = U \setminus V$.

Remark(3.2): In definition (3.1), if $U \neq X$ and $V = \phi$, then every proper s*g-open (resp. α-s*g-open, pre-s*g-open, b-s*g-open, β-s*g-open) subset U of X is an D_{s^*g}-set (resp. $D_{\alpha-s^*g}$-set ,
D_{pre-s^*g}-set, D_{b-s^*g}-set, $D_{\beta-s^*g}$-set) .

Proposition(3.3): In any topological space (X,τ) .
i) Any D-set is D_{s^*g}-set .
ii) Any D_{s^*g}-set is $D_{\alpha-s^*g}$-set .
iii) Any $D_{\alpha-s^*g}$-set is D_{pre-s^*g}-set .
iv) Any D_{pre-s^*g}-set is D_{b-s^*g}-set .
v) Any D_{b-s^*g}-set is $D_{\beta-s^*g}$-set .

Proof: Follows from Lemma (2.2) .

Proposition(3.4): In any door space (X,τ) .
i) Any D_{pre-s^*g}-set is D_{s^*g}-set .
ii) Any $D_{\beta-s^*g}$-set is D_{b-s^*g}-set .

Proof: Follows from Proposition (2.17) .

Proposition(3.5): In any topological space satisfies s*g-condition any D_{s^*g}-set is D-set .

Proof: Suppose that A is an D_{s^*g}-set , then there are two s*g-open sets U and V in X such that $U \neq X$ and $A = U \setminus V$. Hence $U = int_{s^*g}(U) \subseteq int_{s^*g}(cl(U))$ and
$V = int_{s^*g}(V) \subseteq int_{s^*g}(cl(V))$. Since X is satisfy the s*g-condition, then U and V are s*g-t-sets . Therefore $U \subseteq int(U)$ and $V \subseteq int(V)$. Hence U and V are open-sets . Thus A is D-set .

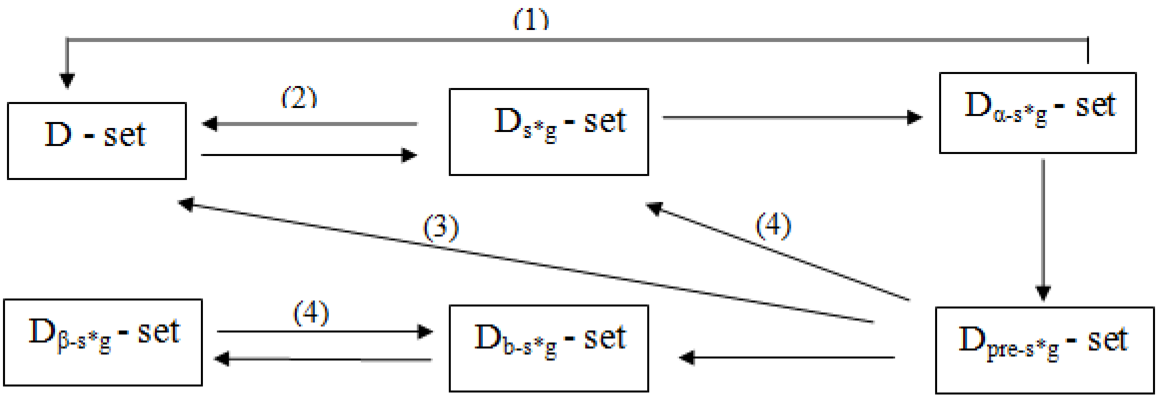
Proposition(3.6): In any topological space satisfies s*g-B_α-condition any $D_{\alpha-s^*g}$-set is D-set
.

Proof: Follows from Proposition (2.25) .

Proposition(3.7): In any topological space satisfies s*g-B-condition any $D_{pre-s*g}$-set is D-set .

Proof: Follows from Proposition (2.21) .

From above propositions we can get the following diagram .



(1) s*g-B_α-Condition (2) s*g-Condition (3) s*g-B-Condition (4) door space

Figure No. (2): Relations among the weak D_{s*g} - sets

Definition(3.8): A function $f:(X,\tau)\to(Y,\sigma)$ is said to be α-s*g-continuous (resp. pre-s*g-continuous, b-s*g-continuous, β-s*g-continuous) if $f^{-1}(V)$ is α-s*g-open (resp. pre-s*g-open, b-s*g-open, β-s*g-open) set in X for each open set V in Y .

Definition(3.9): A function $f:(X,\tau)\to(Y,\sigma)$ is said to be α-s*g-irresolute (resp. pre-s*g-
irresolute ,b-s*g-irresolute, β-s*g-irresolute) if $f^{-1}(V)$ is α-s*g-open (resp. pre-s*g-open, b-s*g-open, β-s*g-open) set in X for each α-s*g-open (resp. pre-s*g-open, b-s*g-open, β-s*g-open) set V in Y .

Theorem(3.10): If $f:(X,\tau)\to(Y,\sigma)$ is an α-s*g-continuous (resp. s*g-continuous, pre-s*g-continuous, b-s*g-continuous, β-s*g-continuous) surjective function and S is a D-set in Y, then the inverse image of S is an $D_{\alpha-s*g}$-set (resp. D_{s*g}-set, $D_{pre-s*g}$-set, D_{b-s*g}-set, $D_{\beta-s*g}$-set) in X .

Proof: Let S be a D-set in Y , then there are two open sets U_1 and U_2 in Y such that $S=U_1\setminus U_2$ and $U_1\neq Y$. Since f is α-s*g-continuous ,then $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are α-s*g-open sets in X. Since $U_1\neq Y$ and f is surjective , then $f^{-1}(U_1)\neq X$. Hence $f^{-1}(S)=f^{-1}(U_1)\setminus f^{-1}(U_2)$ is a $D_{\alpha-s*g}$-set in X . By the same way we can prove that other cases .

Theorem(3.11): If $f:(X,\tau)\to(Y,\sigma)$ is an α-s*g-irresolute (resp. s*g-irresolute , pre-s*g-irresolute , b-s*g-irresolute , β-s*g-irresolute) surjective function and S is an $D_{\alpha-s*g}$-set (resp.

D_{s*g}-set , $D_{pre-s*g}$-set , D_{b-s*g}-set , $D_{\beta-s*g}$-set) in Y, then the inverse image of S is an $D_{\alpha-s*g}$-set (resp. D_{s*g}-set , $D_{pre-s*g}$-set , D_{b-s*g}-set , $D_{\beta-s*g}$-set) in X .

Proof: Let S be an $D_{\alpha-s*g}$-set in Y , then there are two α-s*g-open sets U_1 and U_2 in Y such that $S = U_1 \setminus U_2$ and $U_1 \neq Y$. Since f is α-s*g- irresolute , then $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are α-s*g-open sets in X . Since $U_1 \neq Y$ and f is surjective , then $f^{-1}(U_1) \neq X$. Hence $f^{-1}(S) = f^{-1}(U_1) \setminus f^{-1}(U_2)$ is an $D_{\alpha-s*g}$-set in X. By the same way we can prove that other cases.

Definitions(3.12): A topological space (X, τ) is called:

(i) An s*g-D_0-space (resp. α-s*g-D_0-space, pre-s*g-D_0-space, b-s*g-D_0-space, β-s*g-D_0-space) if for any two distinct points x and y of X, there exists an D_{s*g}-set (resp. $D_{\alpha-s*g}$-set, $D_{pre-s*g}$-set , D_{b-s*g}-set, $D_{\beta-s*g}$-set) of X containing one of the points but not the other .

(ii) An s*g-D_1-space (resp. α-s*g-D_1-space, pre-s*g-D_1-space, b-s*g-D_1-space, β-s*g-D_1-space) if for any two distinct points x and y of X, there exists an D_{s*g}-set (resp. $D_{\alpha-s*g}$-set, $D_{pre-s*g}$-set , D_{b-s*g}-set, $D_{\beta-s*g}$-set) of X containing x but not y and an D_{s*g}-set (resp. $D_{\alpha-s*g}$-set, $D_{pre-s*g}$-set , D_{b-s*g}-set, $D_{\beta-s*g}$-set) of X containing y but not x .

(iii) An s*g-D_2-space (resp. α-s*g-D_2- space, pre-s*g-D_2-space, b-s*g-D_2-space, β-s*g-D_2-space) if for any two distinct points x and y of X, there are two D_{s*g}-sets (resp. $D_{\alpha-s*g}$-sets, $D_{pre-s*g}$-sets, D_{b-s*g}-sets, $D_{\beta-s*g}$-sets) U and V of X such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

Theorem(3.13): (i) Every s*g-T_i-space (resp. α-s*g-T_i-space , pre-s*g-T_i-space , b-s*g-T_i-space, β-s*g-T_i-space) is s*g-T_{i-1}-space (resp. α-s*g-T_{i-1}-space, pre-s*g-T_{i-1}-space, b-s*g-T_{i-1}-space, β-s*g -T_{i-1}-space) , $i = 1,2$.

(ii) Every s*g-T_i-space (resp. α-s*g-T_i-space , pre-s*g-T_i-space , b-s*g-T_i-space , β-s*g-T_i-space) is s*g-D_i-space (resp. α-s*g-D_i-space, pre-s*g-D_i-space, b-s*g-D_i-space, β-s*g-D_i-space) , $i = 0,1,2$.

(iii) Every s*g-D_i-space (resp. α-s*g-D_i-space , pre-s*g-D_i-space , b-s*g-D_i-space , β-s*g-D_i-space) is s*g-D_{i-1}-space (resp. α-s*g-D_{i-1}-space, pre-s*g-D_{i-1}-space, b-s*g-D_{i-1}-space, β-s*g -D_{i-1}-space) , $i = 1,2$.

Proof: (i) It is obvious . (ii) Follows from Remark (3.2). (iii) It is obvious .

Remark(3.14): The converse of theorem (3.13), no. (i) may not be true . Consider the following examples:

Example(3.15): Let X be any infinte set and let $\tau = \{ U \subseteq X : U^c \text{ is finite} \} \cup \{\phi\}$. Then (X, τ) is an s*g-T_1-space (resp. α-s*g-T_1-space , pre-s*g-T_1-space , b-s*g-T_1-space , β-s*g-T_1-space) , but is not an s*g-T_2-space (resp. α-s*g-T_2-space , pre-s*g-T_2-space , b-s*g-T_2-space , β-s*g-T_2-space) .

Example(3.16): Let $X = \{a, b\}$ and $\tau = \{X, \phi, \{a\}\}$. Then (X, τ) is s*g-T_0-space (resp. α-s*g-T_0-space, pre-s*g-T_0-space, b-s*g-T_0-space, β-s*g-T_0-space), but is not s*g-T_1-space (resp. α-s*g-T_1-space, pre-s*g-T_1-space, b-s*g-T_1-space, β-s*g-T_1-space) .

Remark(3.17): The converse of theorem (3.13), no.(ii) may not be true . Consider the following examples:

Example(3.18): Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. Then s*g-open sets in X = open sets in X . Hence (X, τ) is s*g-D_i-space , but is not s*g-T_i-space , $i = 1,2$.

Example(3.19): Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$. Then α-s*g-open sets in X = pre-s*g-open sets in X = b-s*g-open sets in X = β-s*g-open sets in X $= \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. Hence (X, τ) is α-s*g-D_i-space (resp. pre-s*g-D_i-space , b-s*g-D_i-space , β-s*g-D_i-space), but is not α-s*g-T_i-space (resp. pre-s*g-T_i-space , b-s*g-T_i-space , β-s*g-T_i-space) , $i = 1,2$.

Remark(3.20): The converse of theorem (3.13), no. (iii) may not be true . In example (3.16) , (X, τ) is s*g-D_0-space (resp. α-s*g-D_0- space, pre-s*g-D_0-space, b-s*g-D_0-space, β-s*g-D_0-space) , but is not s*g-D_1-space (resp. α-s*g-D_1-space, pre-s*g-D_1-space, b-s*g-D_1-space, β-s*g-D_1-space) .

Theorem(3.21): A topological space (X, τ) is an α-s*g-D_0-space (resp. s*g-D_0-space, pre-s*g-D_0-space, b-s*g-D_0-space, β-s*g-D_0-space) if and only if it is an α-s*g-T_0-space (resp. s*g-T_0- space, pre-s*g-T_0-space, b-s*g-T_0-space, β-s*g-T_0-space) .

Proof: Sufficiency. Follows from Theorem (3.13), no. (ii) .
Necessity. Let $x, y \in X$ such that $x \neq y$. Since (X, τ) is α-s*g-D_0-space, then there exists an $D_{\alpha - s^*g}$-set U such that $x \in U$, $y \notin U$. Let $U = P_1 \setminus P_2$, where $P_1 \neq X$ and P_1, P_2 are α-s*g-open sets in X . By $y \notin U$ we have two cases: (i) $y \notin P_1$ (ii) $y \in P_1$ and $y \in P_2$.
In case (i) $y \notin P_1$ and $x \in U = P_1 \setminus P_2 \Rightarrow x \in P_1$ and $y \notin P_1$.
In case (ii) $y \in P_1$ and $y \in P_2$ and $x \in P_1 \setminus P_2 \Rightarrow x \notin P_2 \Rightarrow y \in P_2$ and $x \notin P_2$.
Thus in both cases, we obtain that (X, τ) is an α-s*g-T_0-space .
By the same way we can prove that other cases .

Theorem(3.22): A topological space (X, τ) is an α-s*g-D_1-space (resp. s*g-D_1-space, pre-s*g-D_1-space, b-s*g-D_1-space, β-s*g-D_1-space) if and only if it is an α-s*g-D_2-space (resp. s*g-D_2- space, pre-s*g-D_2-space, b-s*g-D_2-space, β-s*g-D_2-space) .

Proof: Sufficiency . Follows from Theorem (3.13) , no. (iii) .

Necessity . Let $x, y \in X$ such that $x \neq y$. Since (X, τ) is an α-s*g-D_1-space, then there exists $D_{\alpha-s*g}$-sets U and V in X such $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$. Let $U = P_1 \setminus P_2$ and $V = P_3 \setminus P_4$,where P_1, P_2, P_3, P_4 are α-s*g-open sets in X and $P_1 \neq X$ and $P_3 \neq X$. By $x \notin V$ we have two cases : (i) $x \notin P_3$ (ii) $x \in P_3$ and $x \in P_4$.

In case (i): $x \notin P_3$. By $y \notin U$ we have two subcases: (a) $y \in P_1$ and $y \in P_2$ (b) $y \notin P_1$.

Subcase (a): $y \in P_1$ and $y \in P_2$. We have $x \in P_1 \setminus P_2$, $y \in P_2$ and $(P_1 \setminus P_2) \bigcap P_2 = \phi$. Observe that $P_2 \neq X$ since $U \neq \phi$, thus by Remark (3.2) P_2 is an $D_{\alpha-s*g}$-set .

Subcase (b): $y \notin P_1$. Since $x \in P_1 \setminus P_2$ and $x \notin P_3$, then $x \in P_1 \setminus (P_2 \bigcup P_3)$ and since $y \in P_3 \setminus P_4$ and $y \notin P_1$, then $y \in P_3 \setminus (P_4 \bigcup P_1)$. Observe also from theorem (2.15) that $(P_2 \bigcup P_3)$ and $(P_4 \bigcup P_1)$ are α-s*g-open sets . Hence $x \in P_1 \setminus (P_2 \bigcup P_3)$, $y \in P_3 \setminus (P_4 \bigcup P_1)$ and $(P_1 \setminus (P_2 \bigcup P_3)) \bigcap (P_3 \setminus (P_4 \bigcup P_1)) = \phi$.

In case (ii): $x \in P_3$ and $x \in P_4$. We have $y \in P_3 \setminus P_4$, $x \in P_4$ and $(P_3 \setminus P_4) \bigcap P_4 = \phi$. Observe that $P_4 \neq X$ since $V \neq \phi$, thus by Remark (3.2) P_4 is an $D_{\alpha-s*g}$-set .

Hence (X, τ) is an α-s*g-D_2-space . By the same way we can prove that other cases .

Corollary(3.23): If (X, τ) is an α-s*g-D_1-space (resp. s*g-D_1-space, pre-s*g-D_1-space, b-s*g-D_1-space, β-s*g-D_1-space) ,then it is an α-s*g-T_0-space (resp. s*g-T_0-space, pre-s*g-T_0-space , b-s*g-T_0-space, β-s*g-T_0-space) .

Proof: Follows from Theorem (3.13) , no. (iii) and Theorem (3.21) .

Remark(3.24): The converse of Corollary (3.23) may not be true . In example (3.16), (X, τ) is α-s*g-T_0-space (resp. s*g-T_0-space, pre-s*g-T_0-space, b-s*g-T_0-space, β-s*g-T_0-space), but is not an α-s*g-D_1-space (resp. s*g-D_1-space, pre-s*g-D_1-space, b-s*g-D_1-space, β-s*g-D_1-space) .

Propositions(3.25):

(i) Every s*g-D_i-space is α-s*g-D_i-space , $i = 0,1,2$.

(ii) Every α-s*g-D_i-space is pre-s*g-D_i-space , $i = 0,1,2$.

(iii) Every pre-s*g-D_i-space is b-s*g-D_i-space , $i = 0,1,2$.

(iv) Every b-s*g-D_i-space is β-s*g-D_i-space , $i = 0,1,2$.

Remark(3.26): The converse of proposition (3.25), no.(i) may not be true . In example (3.19), (X, τ) is α-s*g-D_i-space , but is not s*g-D_i-space , $i = 0,1,2$.

Remark(3.27): The converse of proposition (3.25), no. (iii) may not be true . Consider the following example:

Example(3.28): Let $X=\{a,b,c\}$ & $\tau=\{X,\phi,\{a\},\{b\},\{a,b\}\}$. Then pre-s*g-open sets in X = open sets in X and b-s*g-open sets in X=$\{X,\phi,\{a\},\{b\},\{a,b\}\{a,c\},\{b,c\}\}$. Hence (X,τ) is b-s*g-D_i-space , but is not pre-s*g-D_i-space , $i=1,2$.
From above we can get the following diagram .

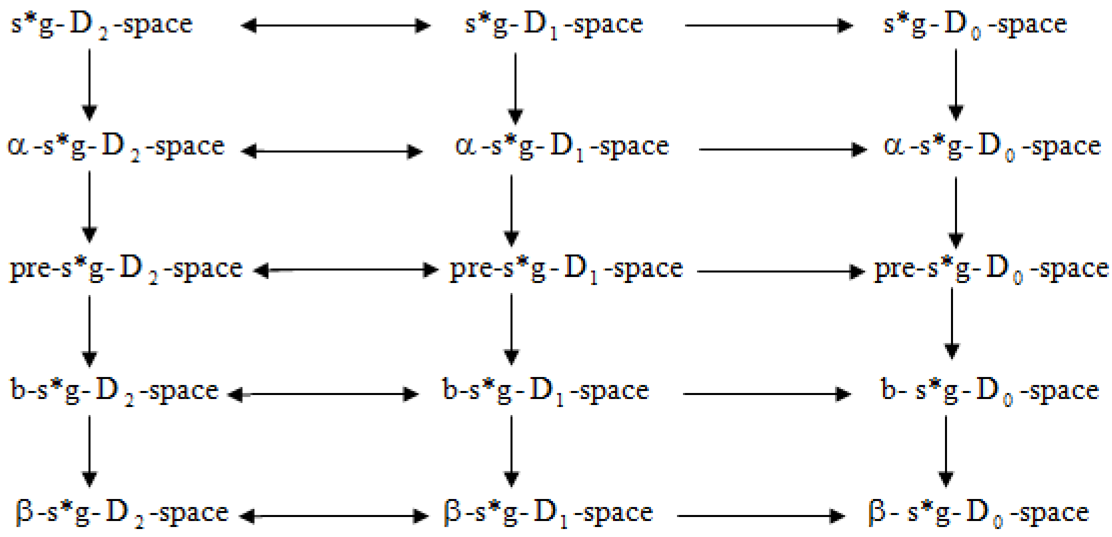


Figure No. (3): Relations among the types of separation axioms

Definition(3.29): A subset A of a topological space (X,τ) is called an α-s*g-neighborhood (resp.
s*g-neighborhood, pre-s*g-neighborhood, b-s*g-neighborhood, β-s*g-neighborhood) of a point x in X if there exists an α-s*g-open (resp. s*g-open, pre-s*g-open, b-s*g-open, β-s*g-open) set U in X such that $x\in U\subseteq A$.

Definition(3.30): Let (X,τ) be a topological space . A point $x\in X$ which has X as the only α-s*g-neighborhood (resp. s*g-neighborhood , pre-s*g-neighborhood , b-s*g-neighborhood , β-s*g-neighborhood) is called an α-s*g-neat (resp. s*g-neat, pre-s*g-neat, b-s*g-neat, β-s*g-neat) point .

Theorem(3.31): For an α-s*g-T_0-space (resp. s*g-T_0-space , pre-s*g-T_0-space , b-s*g-T_0-space , β-s*g-T_0-space) (X,τ) the following are equivalent .
(i) (X,τ) is an α-s*g-D_1-space (resp. s*g-D_1-space, pre-s*g-D_1-space, b-s*g-D_1-space, β-s*g-
D_1-space) .
(ii) (X,τ) has no α-s*g-neat (resp. s*g-neat, pre-s*g-neat, b-s*g-neat, β-s*g-neat) point .

Proof: (i) $\Rightarrow$ (ii) **.** Since (X,τ) is an α-s*g-D_1-space , then each point x of X is contained in a $D_{\alpha-s*g}$-set $G=U\setminus V$, where U and V are α-s*g-open sets and thus in U . By definition $U\neq X$. This implies that x is not an α-s*g-neat point .
(ii) $\Rightarrow$ (i) . If (X,τ) is an α-s*g-T_0-space , then for each distinct points $x,y\in X$, at least one of them, say x has an α-s*g-neighborhood U containing x , but not y . Thus U is different

from X and therefore by Remark (3.2), U is an $D_{\alpha-s*g}$-set . Since X has no α-s*g-neat point, then y is not an α-s*g-neat point . Thus there exists an α-s*g-neighborhood V of y such that $V \neq X$. Therefore , $y \in V \setminus U$, $x \notin V \setminus U$ and $V \setminus U$ is an $D_{\alpha-s*g}$-set . Hence (X, τ) is an α-s*g-D_1-space.

Theorem(3.32): Let $f:(X,\tau)\to(Y,\sigma)$ be an α-s*g-continuous (resp. s*g-continuous , pre-s*g-continuous, b-s*g-continuous, β-s*g-continuous) bijective function . If (Y,σ) is a D_i-space , then (X,τ) is an α-s*g-D_i-space (resp. s*g-D_i-space, pre-s*g-D_i-space, b-s*g-D_i-space, β-s*g-D_i-space), $i = 0,1,2$.

Proof: Suppose that (Y,σ) is a D_2-space . Let $x, y \in X$ such that $x \neq y$. Since f is injective and Y is a D_2-space , then there exists disjoint D-sets G_1 and G_2 of Y such that $f(x) \in G_1$ and $f(y) \in G_2$. By Theorem (3.10), $f^{-1}(G_1)$ and $f^{-1}(G_2)$ are $D_{\alpha-s*g}$-sets in X such that $x \in f^{-1}(G_1)$, $y \in f^{-1}(G_2)$ and $f^{-1}(G_1) \cap f^{-1}(G_2) = \phi$. Hence (X,τ) is an α-s*g-D_2-space .

Theorem(3.33): Let $f:(X,\tau)\to(Y,\sigma)$ be an α-s*g-irresolute (resp. s*g-irresolute , pre-s*g-irresolute, b-s*g-irresolute, β-s*g-irresolute) bijective function . If (Y,σ) is an α-s*g-D_i-space (resp. s*g-D_i-space, pre-s*g-D_i-space, b-s*g-D_i-space, β-s*g-D_i-space) , then (X,τ) is an α-s*g-D_i-space (resp. s*g-D_i-space , pre-s*g-D_i-space , b-s*g-D_i-space , β-s*g-D_i-space) , $i = 0,1,2$.

Proof: Suppose that (Y,σ) is an α-s*g-D_2-space . Let $x, y \in X$ such that $x \neq y$. Since f is injective and Y is an α-s*g-D_2-space , then there exists disjoint $D_{\alpha-s*g}$-sets G_1 and G_2 of Y such that $f(x) \in G_1$ and $f(y) \in G_2$. By Theorem (3.11) , $f^{-1}(G_1)$ and $f^{-1}(G_2)$ are $D_{\alpha-s*g}$-sets in X such that $x \in f^{-1}(G_1)$, $y \in f^{-1}(G_2)$ and $f^{-1}(G_1) \cap f^{-1}(G_2) = \phi$. Hence (X,τ) is an α-s*g-D_2-space .

References

1. Tong, J. (1982) A separation axioms between T_0 and T_1 , Ann. Soc. Sci. Bruxelles 96 II,85- 90 .
2. Calads,M. (1997) A Separation axioms between semi-T_0 and semi-T_1, Mem. Fac. Sci. Kochi Univ. Ser. A Math. 181,37-42.
3. Calads,M. ; Georgiou, D.N. and Jafari, S. (2003) Characterization of low separation axioms via α-open sets and α-closure operator , Bol. Soc. Paran. Mat.(3s), 21(1/2), 1-14 .
4. Jafari, S. On a weak separation axioms ,Far East J. Math. Sci. (to appear) .
5. Keskin,A. and Noiri, T. (2009) On bD-sets and associated separation axioms , Bulletin of the Iranian Math. Soc. 35,1,179-198.
6. Khan,M. ; Noiri, T . and Hussain, M .(2008) On s*g-closed sets and s*-normal spaces , 48 , 31-41 .
7. Levine,N. (1963) Semi-open sets and semi-continuity in topological spaces , Amer. Math. Monthly 70, 36-41 .
8. Njastad,O. (1965) On some classes of nearly open sets , Pacific J. Math. 15, 961-970 .
9. Mashhour, A.S. ; Abd El-Monsef,M.E. and El-Deeb,S.N. (1982) On precontinuous and weak precontinuous functions , Proc. Math. Phys. Soc. Egypt 51, 47-53 .
10. Andrijevic, D. (1996) On b-open sets, Mat. Vesnik , 48 (1-2) , 59-64 .
11. Abd El-Monsef,M.E. ; El-Deeb , S.N. and Mahmoud,R.A. (1983) β-open sets and β-continuous Mappings , Bull. Fac. Sci. Assuit Univ. 12, 77-90 .
12. Veerakumar,M.K.R.S. (2001) $\hat{g}$-closed sets and $G\hat{L}C$-functions , Indian J.Math., 43,2 , 231-247.
13. Kelly,J.L.(1955) General Topology , VAN NOSTRAND , New York .
14. S.I. and Afrah, M . (2010) S*-separation axioms , Iraqi Journal of Science, University of Baghdad, 51, 1 ,145-153 .

حول المجموعات - D_{s*g} الضعيفة وبديهيات الفصل المشتقة منها

صبيحة إبراهيم محمود
قسم الرياضيات - كلية العلوم- الجامعة المستنصرية

أستلم البحث في :3 ايلول 2013 ، قبل البحث في: 19 شباط 2014

الخلاصة

في هذا البحث قمنا بتقديم اصناف جديدة من المجموعات اسميناها بالمجموعات - D_{s*g} , المجموعات - $D_{\alpha-s*g}$, المجموعات $D_{pre-s*g}$, المجموعات - D_{b-s*g} - والمجموعات $D_{\beta-s*g}$ -. كذلك درسنا بعض خواص هذه المجموعات والعلاقات بينهم . فضلا عن ذلك استخدمنا هذه المجموعات في تعريف ودراسه بعض بديهيات الفصل المشتقة منها .

الكلمات المفتاحية : الفضاءات-- D_i s*g , الفضاءات- α -s*g- D_i , الفضاءات--pre-s*g D_i , الفضاءات--b D_i , s*g- الفضاءات- β -s*g- D_i ($i = 0,1,2$).