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Theoretical Investigation Including Averages Fusion Cross-Section, Reactivity for the D-D and D-^{3}He Reactions

Asmaa B. Abd Al-Hameed[1*] and Raad H. Majeed[2]
[1,2] Department of Physics, College of Education for Pure Science (Ibn Al-Haitham), University of Baghdad, Baghdad, Iraq.
*Correspondence Author.

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Abstract

Nuclear fusion technology for power production is considered a renewable and clean energy source, and it is used in many applications such as scientific research, systems for electrical energy production, and fast charge particles such as protons, alpha particles, electrons, beta particles, positrons, etc. In addition, there exists another application such as materials irradiation and artificial elements production. There exist more than 51 fusion reactions in nature. The most important thermonuclear fusion reactions are those used in one of the common hydrogen isotopes such as deuterium, the D-D and D-^{3}He reactions. A laboratory fusion cross-section data fitting equations are calculated theoretically by using the least square fitting technique. Calculations include results for average cross-sections <σ> and the related fusion reaction reactivates <σv> for the above two reactions. Both reactions D-D and D-^{3}He occur in the energy and or temperature range in between energetic ions coming from the measured Maxwellian distribution and atoms at rest.

Keywords: Fusion reaction, Fusion reactivity, Deuterium fuels, Laser pulses, Ion maxwellain distribution, Power introduction.

1. Introduction

Nuclear fusion reaction technology is considered a sustainable energy source because it is available in nature, like the hydrogen isotopes deuterium and tritium, and because of their cheap price (1–3). A nuclear fusion reaction occurs by combining or fusing two light nuclei to form one heavy nucleus and light-charged particles. The sun is a clear, common, and famous source of energy that occurs by the lightest atom (hydrogen or proton) fusion reaction, and it is named the H-H cycle or p-p cycle as explained in the following nuclear reactions.

$$ {}_{1}^{1}H + {}_{1}^{1}H \longrightarrow {}_{1}^{2}H + e^{+} + \upsilon_{e} \quad (1)$$

$$^{1}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{2}He + \gamma \quad (2)$$

$$^{1}_{1}H + ^{3}_{2}He \longrightarrow ^{4}_{2}He + e^{+} + \upsilon_e \quad (3)$$

$$^{3}_{2}He + ^{3}_{2}He \longrightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H \quad (4)$$

Since the range of the nuclear force is $5*10^{-13}$cm, for the fusion process to occur, we need 0.3 MeV to approach the light nuclei to each other against the Coulomb repulsion force (4-6).

It is observed that it produces a high energy by nuclear fusion reaction; one can use it in many applications, such as producing electrical energy and fast-charged particles such as protons and electrons, etc. In addition, materials irradiation and artificial element production (7-9).

The energy or temperature needed to mention the fusion process in a very small range region (penge region) for a very small time interval can be achieved by the recent two following main methods:

- Magnetic confinement fusion:

The plasma comprises charged particles, and by applying an extraordinarily designed magnetic field, it is feasible to keep the plasma locale thermally protected from the environmental elements (10-12).

- Internal confinement fusion:

A little pellet of fuel is caused to collapse so the internal center arrives at such a temperature that it goes through a small nuclear blast, utilizing the radiation of a few extremely strong lasers (4,13). The fusion reaction cross-section is known as the number of reactions per target nucleus per unit of time when the target is struck by a unit flux of projectile particles, and it is entered in many accounts like fusion reaction reactivity, fusion reaction rate, etc. (8,14,15). Experimentally, both D-D and D-^{3}He reactions are stimulated by using a high-power laser pulse with energy (100-180) j and a duration time of about 150 fs a coulomb explosion occurs, which leads to the allocation of low D ions in a highly ionized state, which is considered important for D-D and D-^{3}He reactions (16,17). When a laser hits the clusters, the particles absorb most of its energy, and the positive charge explodes, releasing energy to stimulate the fusion reaction. The resulting energy was measured by the detecting protons at 3.02 MeV and 2.45 MeV neutrons by using Faraday cups, as shown in **Figure 1** (18-20).

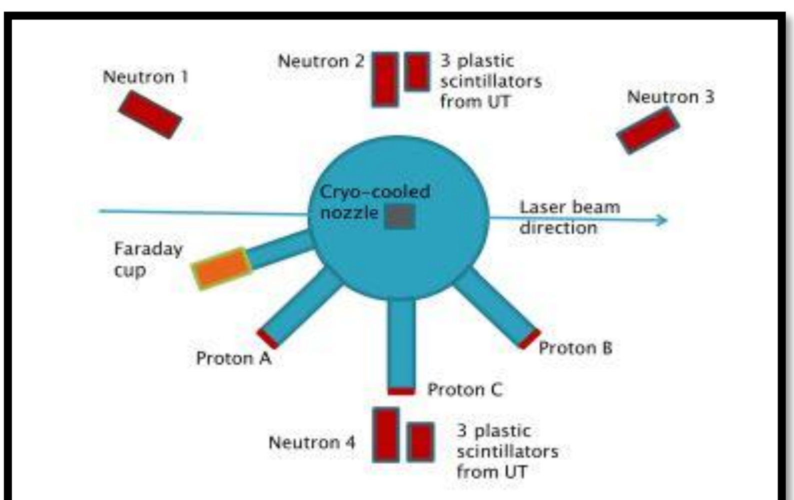


Figure 1. Faraday cups are used to measure the energy of deuterium ions (20).

2. Materials and Methods

The existence of petawatt laser pulses can transfer laser pulses (PW) to small volumes, and this allows the use of the possibilities for studying the reactions of D-D and D-^{3}He through the interaction between high-energy laser pulses and molecular groups in a medium of intense ionization and reduced of electrons, a Coulomb explosion occurs (21-23).Studying the two reactions D-D and D-^{3}He, observing their products under the same conditions, and knowing the properties of these products contribute to the establishment and development of nuclear reactors and knowledge of some important properties of plasma, such as temperature (24,25).The calculations of fusion reaction cross-section and fusion reaction reactivity depend upon suitable empirical formulas.

The fusion reaction cross-section for the two reactions D-D and D-^{3}He is given below (20,26,27).

$$\sigma(E) = \left(\frac{S(E)}{E}\right) exp(\frac{E_G}{E})^{-0.5} \tag{5}$$

Where E: particles kinetic energy in the center of mass in the range (0:200) keV for D-D reaction and (0:350) keV for D-^{3}He.

E_G: Gamma energy in keV.

S (E): astrophysical S factor, given by the following formulas (28).

$$S(E) = a_1 + E(a_2 + E(a_3 + E(a_4 + Ea_5))) \quad \text{for D-D reaction} \tag{6}$$

$$S(E) = \frac{a_1+E(a_2+a_3E)}{1+E(b_1+E(b_2+Eb_3))} \quad \text{for D-}^3\text{He reaction} \tag{7}$$

Table 1 shows the values for the several constant parameters (a_1, a_2, a_3, a_4, a_5, b_1, b_2, b_3) (28).

The average fusion reaction cross-section for the two reactions is given by the following equation [20].

$$< \sigma >_T = \int_0^\infty 2\sqrt{\frac{E}{\pi}} \left(\frac{1}{KT}\right)^{3/2} e^{\frac{-E}{KT}} \sigma(E)dE \tag{8}$$

Where: K: Boltzmann constant = 1.38*10^{-23} joule /mole k°. T: temperature in K°.

One more significant amount is the reactivity, characterized as the prospect of response per unit time per unit density of target cores. The related fusion reactivity for the two reactions is given by the following equations (27-30).

$$R =< \sigma v >_T = \int_0^\infty \sqrt{\frac{2}{\pi}} \left(\frac{m_r}{KT}\right)^{\frac{3}{2}} v^2 e^{\frac{m_r v^2}{2KT}} \sigma(v)\, dv \tag{9}$$

Where m_r: The reduces the mass of the system and is given by

$$M_r = \frac{m_1 * m_2}{m_1 + m_2} \tag{10}$$

m_1: mass of particle 1 in MeV.

m_2: mass of particle 2 in MeV.

K: Boltzmann constant = 1.38*10^{-23} joule /mole k°. T: temperature in K°. v: incident particle velocity in m/s. σ (υ): nuclear reaction cross-section in a unit of barn.

3. Results

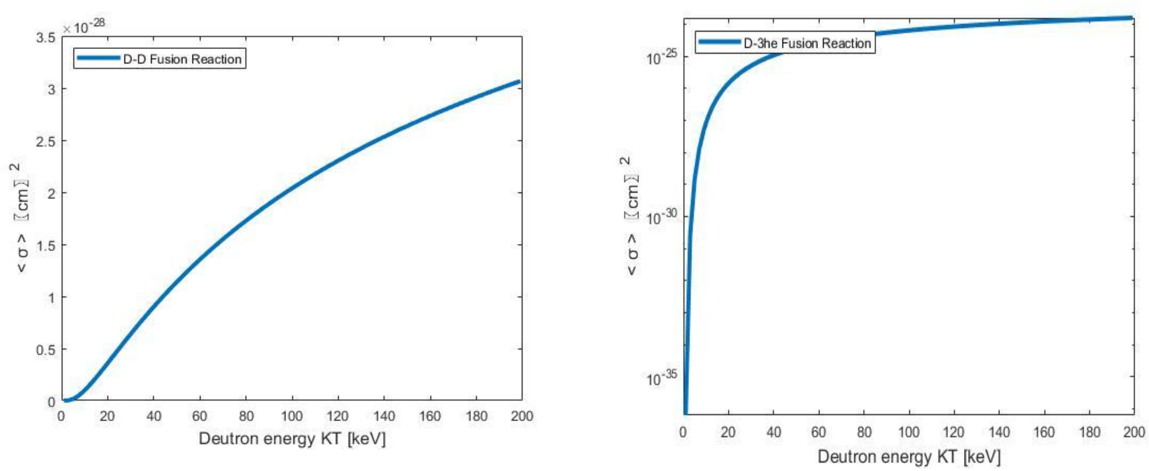
The present calculation includes the average fusion reaction cross-section and their related fusion reactivity, which plays a basic role in determining other fusion parameters such as fusion reaction rate, fusion power density, and fusion product yields (protons and neutrons). Calculations and results for the average fusion reaction cross-section are needed to achieve a previous calculation about the following parameters:

- The fusion reaction cross section is given in equation (5).
- Astrophysical factors, which are given in equations (6,7). The parameters $a_1,a_2,a_3,a_4,a_5,b_1,b_2,b_3$ are chosen from **Table 1** (28).

Table 1. Astrophysical parameter for the fusion cross-section (28).

Coefficient	^{3}He(d,p)^{4}He	D(d,n)^{3}He
$E_G(\sqrt{keV})$	68.7508	31.3970
a_1	$5.7501*10^6$	$5.3701*10^4$
a_2	$2.5226*10^3$	$3.3027*10^2$
a_3	$4.5566*10^1$	$-1.2706*10^{-1}$
a_4	0	$2.9327*10^{-5}$
a_5	0	$-2.5151*10^{-9}$
b_1	$-3.1995*10^{-3}$	0
b_2	$-8.5530*10^{-6}$	0
b_3	$5.9014*10^{-8}$	0

The results for the average fusion reaction cross-section and their physical profiles are explained in the following **Figure 2**.



(a) (b)

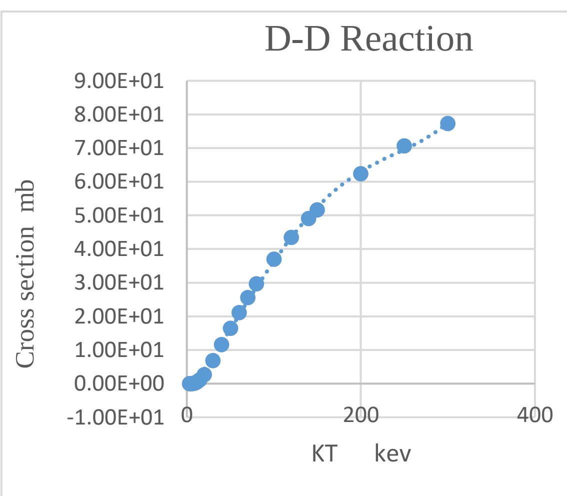
Figure 2. a. Physical profile for the D_D average fusion reaction cross section. **b**. physical profile for the D_^{3}He average fusion reaction cross section.

It is necessary to introduce another result about the fusion cross-section by choosing other tabulated data sets for the cross-section given below (28).

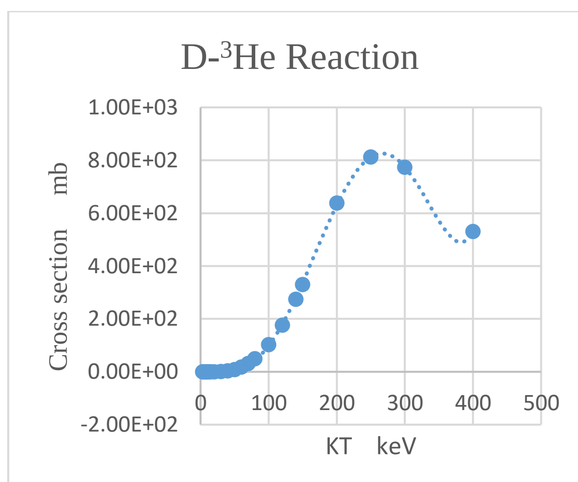
Table 2. Cross-section for D-D and D-^{3}He fusion reactions (28).

E(keV)	^{3}He(d,p)α (mb)	D(d,n)^{3}He (mb)
3	$1.119*10^{-11}$	$2.445*10^{-4}$
4	$1.718*10^{-9}$	$2.093*10^{-3}$
5	$5.199*10^{-8}$	$8.834*10^{-3}$
6	$6.336*10^{-7}$	$2.517*10^{-2}$
7	$4.373*10^{-6}$	$5.616*10^{-2}$
8	$2.058*10^{-5}$	$1.064*10^{-1}$
9	$7.374*10^{-5}$	$1.794*10^{-1}$
10	$2.160*10^{-4}$	$2.779*10^{-1}$
12	$1.206*10^{-3}$	$5.563*10^{-1}$
15	$7.944*10^{-3}$	$1.178*10^{0}$
20	$6.568*10^{-2}$	$2.691*10^{0}$
30	$7.704*10^{-1}$	$6.857*10^{0}$
40	$3.266*10^{0}$	$1.165*10^{1}$
50	$8.688*10^{0}$	$1.649*10^{1}$
60	$1.788*10^{1}$	$2.115*10^{1}$
70	$3.144*10^{1}$	$2.554*10^{1}$
80	$4.978*10^{1}$	$2.694*10^{1}$
100	$1.021*10^{2}$	$3.701*10^{1}$
120	$1.768*10^{2}$	$4.343*10^{1}$
140	$2.740*10^{2}$	$4.904*10^{1}$
150	$3.303*10^{2}$	$5.160*10^{1}$
200	$6.378*10^{2}$	$6.239*10^{1}$
250	$8.122*10^{2}$	$7.071*10^{1}$
300	$7.731*10^{2}$	$7.732*10^{1}$
400	$5.304*10^{2}$	$8.702*10^{1}$

Figure 3 provides a complete description of the fusion reaction cross-section based on fitting the above-tabulated data.



(a)



(b)

Figure 3. a. The fitting physical profile for the D_D fusion reaction cross section. **b.** physical profile for the D_^{3}He fusion reaction cross-section.

So, it is confined to calculating or deducing the fitting formulas by using the mathematical technique of the method of the least square fit, which is given as follows:

$\sigma = 2E-08(KT)^4 - 1E-05(kT)^3 + 0.002(KT)^2 + 0.3027\,KT - 2.3891$ for D-D (11)

$\sigma = 3E-09(KT)^5 - 3E-06(KT)^4 + 0.0007(KT)^3 - 0.052(KT)^2 + 1.4546KT - 8.4732$ (12) for D-^{3}He

Where KT: the incident deuteron energy in a unit of keV. σ*:* fusion reaction cross-section in unit mb the corresponding related fusion reactivity for the two reactions D-D and D-^{3}He is calculated by using equation (9) in which it needs previously calculated data sets for the cross-section and the incident deuteron velocity. The physical behavior profiles of the fusion reactivity for the two fusion reactions are described pictorially in the following **Figure 4**.

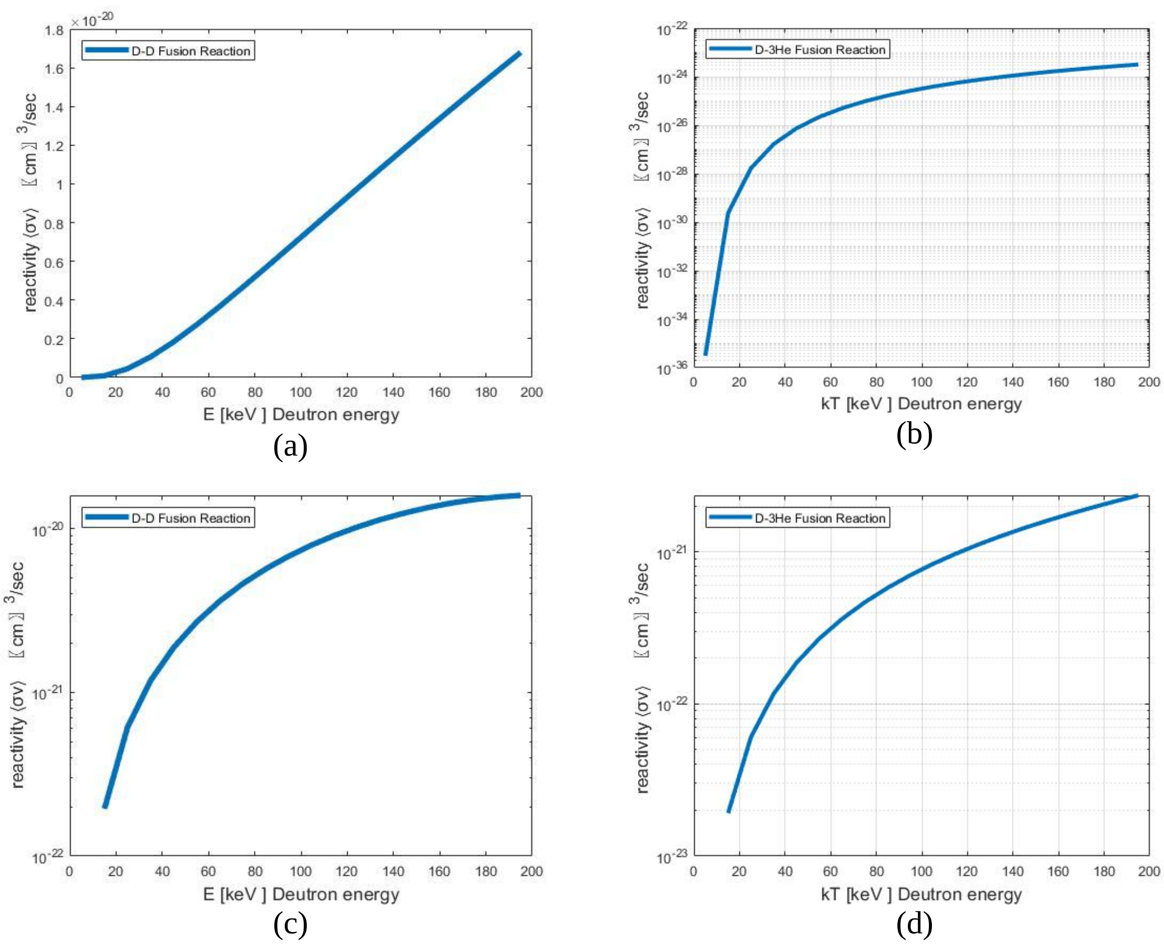


Figure 4. a. Fusion reaction reactivity for D-D reaction according to cross-section data given in equation (5).

b. fusion reaction reactivity for D-^{3}He reaction according to cross-section data given in equation (5).

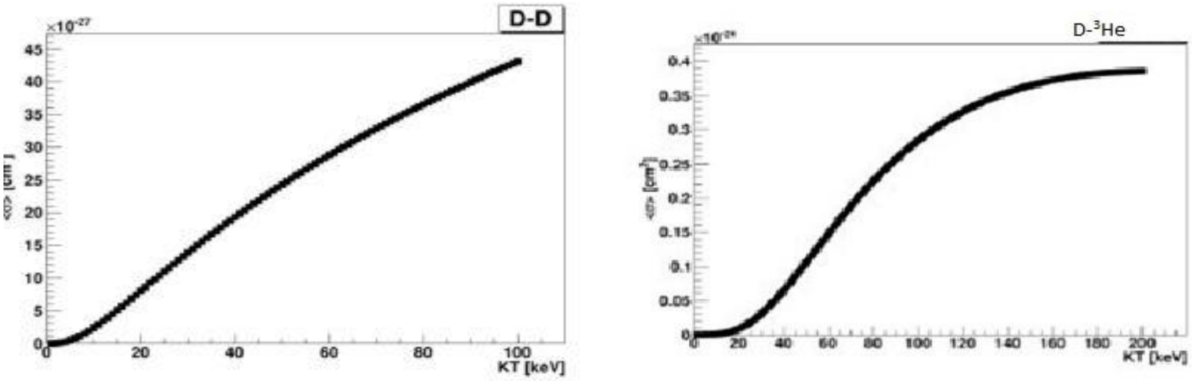
c. fusion reaction reactivity for D-D reaction according to the fitting equation for cross-section.

d. fusion reaction reactivity for D-^{3}He reaction according to the fitting equation for cross-section.

4. Discussion

Fusion reaction Cross-section data are available in several forms, i.e.; as experimental data presented in figures and tables or as semi-experimental formulas and theoretical equations. Our calculations are achieved by using semi-experimental formulas for the cross-section given by equation (5) for both reactions D-D and D-^{3}He and in terms of the average cross-section and the corresponding fusion reactivity are calculated.

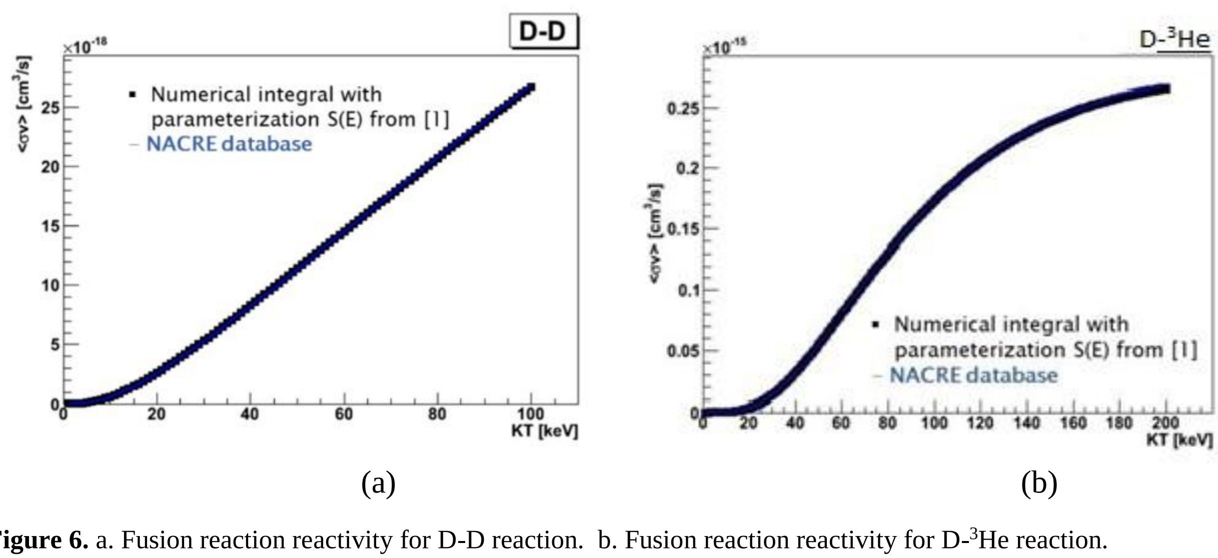
It is necessary to check our results by making a suitable comparison between the present results and the internationally published results. It appears as good or a wild compatible between the calculated average fusion reaction cross section described in **Figure 2** and the corresponding published results given below (20).



(a) (b)

Figure 5. a. Average fusion reaction cross-section for D-D. **b.** Average fusion reaction cross-section for D-^{3}He.where KT is the incident deuteron energy.

Results included the fusion reaction reactivity for the two reactions D-D and D-^{3}He, which are explained in **Figure 4** and are compared with published experimental results given below (20).



(a) (b)

Figure 6. a. Fusion reaction reactivity for D-D reaction. b. Fusion reaction reactivity for D-^{3}He reaction.

Figures 5 and **6** are internationally published results that were used to compare with the results obtained in this research. Based on the above figures, we notice that the difference between the two reactions through the cross-section and the energy range for the products of the two reactions as that the energy resulting from the reaction D-^{3}He reaches 18.35 MeV for cross-section 0.9 barn, so it is considered one of the best fusion reactions, and the resulting energy from the reaction D-D

is by the protons at 3.02 MeV and neutrons at 2.45 MeV. Both reactions are considered stable nuclear reactions that are being developed for wide use in the field of energy production.
There exists a wild agreement between the two profiles' physical behaviours and one can deduce that by using a wild suitable formula or data set that leads to an agreement between the two results.

5. Conclusion

It is known that this presents a naturally small shift between the determined results and the experimental published results, and this case can be interpreted as the existence of laboratory conditions or factors that cannot be introduced or neglected in designing any computer code or programs, and these cases present areal facts in which there exists a mathematical assumption in any numerical programs. Finally, a conclusion leads to the reflection of the existence of good accuracy in results, and these lead to one can be used in determining other fusion parameters such as neutrons, protons yield, and fusion power density.

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Conflict of Interests

The authors declare that they have no conflicts of interest.

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