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Right Truncated Shankar Distribution and its Properties

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Abstract

Since the importance of truncated distributions has increased in many scientific fields in recent years and they provide valuable insights when dealing with censored or truncated data, this paper presents the Right Truncated Shanker Distribution as a new statistical distribution developed for modelling right truncated data. The distribution is defined by specifying its probability density function and cumulative distribution function under the truncation condition, the survival function and the hazard function. In addition, some properties of the right truncated Shanker distribution are derived, such as the moments around the origin, the variance, the coefficients of skewness and kurtosis, the moment generating function, and the mean time to failure. Our statistical properties show that the new distribution has the utility and flexibility to effectively model truncated data scenarios.

Keywords: Truncated distributions, Shanker distribution, probability density function, survival function, moment generating function.

1. Introduction

Over the last few decades, there has been a growing interest in lifetime modeling within distribution theory, leading to the introduction of new models by statisticians. Several of these models have gained popularity and are widely used in fields such as biology, engineering, and agriculture[1]. Many new statistical distributions have presented that are more flexible in representing data-life. Lindley distribution developed by Lindley(2), and the weighted Lindley distribution introduced by Ghitany and Atieh (3). Nadarajah- Haghighi distribution (4). The Modified-Lomax distribution (5) and Komal distribution (6).

The concept of Truncated Distributions (TD) provides a more accurate representation of phenomena, while still maintaining a level of generality. Therefore, truncated distributions are employed when events are restricted to values that are either higher or lower than a certain threshold, or fall within a specific rang (7)

Truncated a distribution within a specific duration is an ideal solution to the problem of the unavailability of information during that period for some reason Singh et al (8) explained that a TD arises In practical statistics when the ability to document or have knowledge of events is restricted to values that fall either above or below a given threshold or within a specified range**.**analyzing TD, its, parameters estimation and applications considered and studied by many authors . Najarzadegan and Alamatsaz,(9) derived truncated Weibull-G distribution. Abid and Abdulrazak (10) Presented [0,1] truncated Frechet-Weibull distribution. Akbarinasab and Arabpour (11) discussed the truncated log-logistic family of distributions. Altawil (12) proposed the [0,1] truncated Lomax – Lomax distribution. Gul et al (13) investigated Weibull-Truncated Exponential distribution. Khaleel et al (14) presented [0,1] Truncated Inverse Weibull Rayleigh distribution. Abbas (15) propose the truncated Weibull exponential distribution. Hussein (16) defined a truncated Lindley-generated family of distributions. Qasim (17) presented the Left Truncated Gumbel-Exponential distribution. (18) investigated the truncated inverse generalized Rayleigh distribution.

This paper aims to precent a right truncated for Shanker distribution and discuss the statistical properties. This article is organized as follows: Section 2 discusses the Right Truncated Shanker Distribution, Section 3 finds the reliability and the hazard functions, Section 4 derives some of the statistical properties, and Section 5 presents the Mean time to failure, finally, Section 6. provides the conclusion.

2. Right Truncated Shanker Distribution

Shanker (19), presented a new lifetime distribution called the "Shanker distribution", by mixture "gamma(2,β) distribution "with "exponential distribution" the pdf and the cdf of shanker distribution are respectively:

$$f_{SH}(x,\vartheta) = \frac{\vartheta^2}{\vartheta^2+1}(\vartheta + x)e^{-\vartheta x};\ x > 0, \vartheta > 0 \tag{1}$$

$$F_{SH}(x,\vartheta) = 1 - \frac{(\vartheta^2+1)+\vartheta x}{\vartheta^2+1}e^{-\vartheta x}\ ;\ x > 0, \vartheta > 0 \tag{2}$$

While the Survival function and hazard rate are defined as**:**

$$S_{SH}(x,\vartheta) = \frac{(\vartheta^2+1)+\vartheta x}{\vartheta^2+1}e^{-\vartheta x} \tag{3}$$

$$h_{sh} = \frac{\vartheta^2(\vartheta^2+1)}{(\vartheta^2+1)+\vartheta x} \tag{4}$$

Let the random variable X belong to the interval $[0\,,1], -\infty < 0 \le x \le 1 < \infty$. Then, the conditional on $0 \le x \le 1$ has a truncated distribution (20) .the pdf and the cdf for $0 \le x \le 1$ where x is followed Shanker distribution are given by

$$w_{RTSH}(x,\vartheta) = \frac{f_{SH}(x,\vartheta)}{F_{sh}(1,\vartheta)} \tag{5}$$

$$W_{RTSH}(x,\vartheta) = \frac{F_{SH}(x,\vartheta)}{F_{sh}(1,\vartheta)} \tag{6}$$

but

$$F_{sh}(1,\vartheta) = \frac{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]}{\vartheta^2+1}e^{-\vartheta}\quad ;\ ,\vartheta > 0 \tag{7}$$

Substituting(1) and (7)in (5) gives the pdf of right truncated Shanker distribution (RTSHD).

$$w_{RTSH}(x,\vartheta) = \frac{\frac{\vartheta^2(\vartheta+x)e^{-\vartheta x}}{\vartheta^2+1}}{\frac{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]}{\vartheta^2+1}e^{-\vartheta}}$$

$$w_{RTSH}(x,\vartheta) = \frac{\vartheta^2(\vartheta+x)e^{-\vartheta x}}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \tag{8}$$

And substitute (2) and (7) in (6) gives the c.d.f of RTSHD as follows:

$$W_{RTSH}(x,\vartheta)=\frac{(\vartheta^2+1)+[\vartheta^2+\vartheta x+1]e^{-\vartheta x}}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \qquad (9)$$

3. Reliability Function and Hazard function

According to the previous equations the reliability function of (RTSHD) can be derived as follows (21):

$$R_{RTSH}(x,\vartheta)=1-W_{RTSH}(x,\vartheta)$$

$$R_{RTSH}(x,\vartheta)=1-\frac{(\vartheta^2+1)+[\vartheta^2+\vartheta x+1]e^{-\vartheta x}}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \qquad (10)$$

By simplifying (10) we obtain;

$$R_{RTSH}(x,\vartheta)=\frac{-[(\vartheta^2+\vartheta+1]e^{-\vartheta}-[\vartheta^2+\vartheta x+1]e^{-\vartheta x}}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \qquad (11)$$

And the hazard rate function is

$$h_{RTSH}(x,\vartheta)=\frac{w_{RTSH}(x,\vartheta)}{S_{RTSH}(x,\vartheta)}$$

$$=\frac{\vartheta^2(\vartheta+x)e^{-\vartheta x}}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\times\frac{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}{-[(\vartheta^2+\vartheta+1]e^{-\vartheta}-[\vartheta^2+\vartheta x+1]e^{-\vartheta x}}$$

Then

$$h_{RTSH}(x,\vartheta)=\frac{\vartheta^2(\vartheta+x)e^{-\vartheta x}}{-[(\vartheta^2+\vartheta+1]e^{-\vartheta}-[\vartheta^2+\vartheta x+1]e^{-\vartheta x}} \qquad (12)$$

4. Statistical Properties of Right Truncated Shanker Distribution

This section presents the derivation and calculation of certain statistical properties of the (RTSHD)

4.1. Moments about origin

Moments about the origin can be derived as bellow (22,23):

$$M'_k(x)=\int_0^1 x^k\, w_{RTSH}(x,\vartheta)dx$$

$$=\int_0^1 x^k\frac{\vartheta^2(\vartheta+x)e^{-\vartheta x}}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}dx \qquad (12)$$

$$=\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\int_0^1 x^k\,(\vartheta+x)e^{-\vartheta x}dx \qquad (13)$$

Depending on Taylor's expansion $e^{-\vartheta x}=\sum_{j=0}^{\infty}\frac{(-\vartheta x)^j}{j!}$

Then Equation(13) can be represented by:

$$M'_k(x)=\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\sum_{j=0}^{\infty}\frac{(-\vartheta)^j}{j!}\int_0^1 x^{k+j}\,(\vartheta+x)dx$$

$$=\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\sum_{j=0}^{\infty}\frac{(-\vartheta)^j}{j!}\int_0^1\left(\vartheta x^{k+j}+x^{k+j+1}\right)dx$$

then

$$M'_k(x)=\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\sum_{j=0}^{\infty}\frac{(-\vartheta)^j}{j!}\left(\frac{\vartheta}{k+j+1}+\frac{1}{k+j+2}\right) \qquad (14)$$

Substitute k by 1,2,3,4 respectively we get:

$$M'_1(x)=E(X)=\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\sum_{j=0}^{\infty}\frac{(-\vartheta)^j}{j!}\left(\frac{\vartheta}{j+2}+\frac{1}{j+3}\right) \qquad (15)$$

$$M'_2(x)=E(X^2)=\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\sum_{j=0}^{\infty}\frac{(-\vartheta)^j}{j!}\left(\frac{\vartheta}{j+3}+\frac{1}{j+4}\right) \qquad (16)$$

$$M_3'(x) = E(X^3) = \frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+4} + \frac{1}{j+5}\right) \quad (17)$$

$$M_4'(x) = E(X^4) = \frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+4} + \frac{1}{j+5}\right) \quad (18)$$

4.2. Variance, Skewness and Kurtosis

To discuss more properties of the truncated Shanker distribution the variance, skewness and kurtosis will be derived respectively :

$var(x) = M_2'(x) - \left(M_1'(x)\right)^2$, then from Equation (16) and (15) we get

$$var_{RTSH}(x) = \frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+3} + \frac{1}{j+4}\right) - \left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+2} + \frac{1}{j+3}\right)\right)^2 \quad (19)$$

The formula of the skewness is

$$sk = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}} = \frac{E(x^3)-3\mu E(x^2)+2\mu^3}{(\sigma^2)^{\frac{3}{2}}}$$

So from Equation (17),(15),(16) and (19) we find that:

$$sk_{RTSH} = \frac{\left\{\begin{array}{c} \frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+4} + \frac{1}{j+5}\right) - \\ 3\left[\left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+2} + \frac{1}{j+3}\right)\right) \times \left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+3} + \frac{1}{j+4}\right)\right)\right] + \\ 2\left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+2} + \frac{1}{j+3}\right)\right)^3 \end{array}\right\}}{\left[\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+3} + \frac{1}{j+4}\right) - \left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+2} + \frac{1}{j+3}\right)\right)^2\right]^{\frac{3}{2}}} \quad (20)$$

The kurtosis of Right Truncated Shanker Distribution can be derived by the same way

$$kr = \frac{\mu_3}{(\mu_2)^2} - 3 = \frac{E(x^4)-4\mu E(x^3)+6\mu^2 E(x^2)-3\mu^4}{(\sigma^2)^2} - 3$$

So

$$kr_{RTSH} = \frac{\left\{\begin{array}{c} \frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+4} + \frac{1}{j+5}\right) - 4\left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+2} + \frac{1}{j+3}\right)\right) \times \\ \left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+4} + \frac{1}{j+5}\right)\right) + 6\left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+2} + \frac{1}{j+3}\right)\right)^2 \times \\ \left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+3} + \frac{1}{j+4}\right)\right) - 3\left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+2} + \frac{1}{j+3}\right)\right)^4 \end{array}\right\}}{\left[\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+3} + \frac{1}{j+4}\right) - \left(\frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}} \sum_{j=0}^{\infty} \frac{(-\vartheta)^j}{j!} \left(\frac{\vartheta}{j+2} + \frac{1}{j+3}\right)\right)^2\right]^2} - 3 \quad (21)$$

4.3. Moment Generating Function *mgf*

In this sub section The *mgf* of the Right truncated Shanker Distribution will be derived as follow:

$$\mathcal{M}_x(t) = E(e^{tx}) = \int_0^1 e^{tx}\ w_{RTSH}(x,\vartheta)dx\ ;\ \ 0 \le x \le 1 \tag{22}$$

So from Equation (5)

$$\mathcal{M}_x(t) = \int_0^1 e^{tx}\ \frac{\vartheta^2(\vartheta+x)e^{-\vartheta x}}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}dx$$

$$= \frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\int_0^1 (\vartheta + x)e^{-(\vartheta-t)x}\,dx \tag{23}$$

But $e^{-(\vartheta-t)x} = \sum_{j=0}^{\infty}\frac{[-(\vartheta-t)x]^j}{j!}$ (24)

Substitute Equation (24) in Equation (23)

$$\mathcal{M}_x(t) = \frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\sum_{j=0}^{\infty}\frac{[-(\vartheta-t)]^j}{j!}\int_0^1 x^j(\vartheta+x)\,dx$$

then

$$\mathcal{M}_x(t) = \frac{\vartheta^2}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\sum_{j=0}^{\infty}\frac{[-(\vartheta-t)]^j}{j!}\left(\frac{\vartheta}{j+1}+\frac{1}{j+2}\right) \tag{25}$$

5. Mean time to failure $MTTE$

Let T denote the lifetime of a component so

$$MTTE = \int_0^1 S_{(RTSH)}(x)\,dx$$

$$= \int_0^1 \frac{-[(\vartheta^2+\vartheta+1]e^{-\vartheta}-[\vartheta^2+\vartheta x+1]e^{-\vartheta x}}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}dx \tag{26}$$

$$= \frac{1}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\int_0^1\left(-[(\vartheta^2+\vartheta+1]e^{-\vartheta}-[\vartheta^2+\vartheta x+1]e^{-\vartheta x}\right)dx$$

$$= \frac{1}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\left[-[(\vartheta^2+\vartheta+1]e^{-\vartheta}-\int_0^1\left(-[\vartheta^2+\vartheta x+1]e^{-\vartheta x}\right)dx\right] \tag{27}$$

Recall that $e^{-\vartheta x} = \sum_{j=0}^{\infty}\frac{(-\vartheta x)^j}{j!}$,then

$$MTTE = \frac{1}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\left[-[(\vartheta^2+\vartheta+1]e^{-\vartheta}-\sum_{j=0}^{\infty}\frac{(-\vartheta)^j}{j!}\int_0^1\left(-(x)^j[\vartheta^2+\vartheta x+1]\right)dx\right]$$

$$MTTE = \frac{1}{(\vartheta^2+1)-[(\vartheta^2+\vartheta+1]e^{-\vartheta}}\left[-[(\vartheta^2+\vartheta+1]e^{-\vartheta}+\sum_{j=0}^{\infty}\frac{(-\vartheta)^j}{j!}\frac{[\vartheta^2+\vartheta x+1]}{j+1}\right] \tag{28}$$

6. Conclusion

A new lifetime truncated distribution named, Right truncated Shanker distribution was presented to study the shanker distribution when the random variable x belong to the interval [0,1] by driving the probability density function and cumulative distribution function The statistical properties including, hazard rate function, survival function, moments, variance and coefficients of skewness, and generating function, have been provided in addition to Mean time to failure to know more about the behavior of this function.

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Conflict of Interest

Bayda Atiya, the manager, declared that she was the one at IHJPAS when submitting the manuscript. The editor-in-chief of IHJPAS confirms that (Bayda Atiya) was excluded from any decisions made regarding this paper.

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