Ibn Al-Haitham Journal for Pure and Applied Sciences
Journal homepage: jih.uobaghdad.edu.iq
PISSN: 1609-4042, EISSN: 2521-3407
IHJPAS. 2025, 38(4)

# On $R_{I-}$ Separation Axioms

**Asmaa Rheem Kadhim**[1*], **R. B. Esmaeel**[2] **and Abdelaziz E. Radwan**[3]

[1,2]Department of Mathematics, College of Education for Pure Science (Ibn Al-Haitham), University of Baghdad, Baghdad, Iraq.
[3]Department of Mathematics, College Faculty of science, University of Ain shams, Cairo, Egypt.
*Corresponding Author.

Received: 5 August 2024 Accepted: 19 November 2024 Published: 20 October 2025
**doi.org/10.30526/38.4.4032**

**Abstract**

The purpose of this research is to present the new set that is named $R_{I-}$ open set, where the set was defined and the general properties And the basic concepts and properties of the set were explained, such as the open set, the intersection, and the union. also In this paper the separation axioms were defined and studied in ideal topological spaces in a new way. and discuss some properties. Also discuss the relationship of our definition with other definitions and prove some results in the context of separation axioms in ideal topological space on this set, where the axioms of the type $\mathcal{T}_0$- space, $\mathcal{T}_1$-space and $\mathcal{T}_2$ – space were defined and the basic concepts and generalizations of these axioms on the set were studied. In addition the relationships between these concepts and their converses on this set under study were discussed, and illustrative examples and proofs of those properties were provided for that. Additionally, a diagram illustrating these concepts is presented.

**Keywords:** Ideal topological space, R – open set, $R_I$ –T0– space, $R_I$ –T1– pace, $R_I$ –T2 – Space.

## 1.Introduction

The idea of the ideal is presented by(1). A topological space $(\mathbb{X}, T)$ in ideal I is a non-empty collection subsets of $\mathbb{X}$ that satisfies the subsequent condition:

i. Let B ⊆ A and A ∈ I, then B ∈ I. (heredity property).

ii. Let A and B are both in I therefor A ∪ B ∈ I. (additively property).

Several classes of ideals have been introduced (2-4).

$I_{\emptyset}$ : The ideal on $\mathbb{X}$ symbolized by {ø} (trivial ideal) and the improper ideal I is called P(X).

$I_f$ : The ideal is comprised of every finite subset thereof.

$I_C$ : The ideal comprising every countable subset of $\mathbb{X}$.

$I_{\mathcal{A}}$: The principle ideal produced by any set $\mathcal{M}$ from the topological space $(\mathcal{W},\mathcal{T})$. Expressed as $I_{\mathcal{M}}$ = P($\mathcal{M}$)= { $\mathcal{B} \subseteq \mathcal{W}$ : $\mathcal{B} \subseteq \mathcal{M}$}. Also Vaidyananthswamy in 1945 (5), A study submitted about An operator assigned $(\,)^*: \mathcal{P}(\mathcal{K}) \to \mathcal{P}(\mathcal{K})$ explanation of a local function of $\mathcal{F}$ via $\mathcal{T}$ *describe which are* as follows: $\mathcal{S} \subseteq \mathcal{W}, \mathcal{S}^*(\mathcal{F},\mathcal{T})$={$\mathscr{k} \in \mathcal{W}$ :$\mathcal{S} \cap \mathcal{M} \notin$

$\mathcal{F}$ for each $\mathcal{M} \in \mathcal{T}(\mathcal{W})$ so $\mathcal{T}$ ($\mathcal{W}$ = $\{\mathcal{M} \in \mathcal{T}; k \in \mathcal{M}\}$, Operator $\psi^*(\ )$ for a topology $\mathcal{T}^*(\mathcal{F},\mathcal{T})$ of the Kuratowski closure (6), called the * topology, finer than $\mathcal{T}$, was presented as follows: $\psi^*(\mathcal{S}) = \mathcal{S} \cup \mathcal{S}^*(\mathcal{F},\mathcal{T})$ also $\mathcal{T}^*$ $(\mathcal{F},\mathcal{T}) = (\mathcal{S} \subseteq \mathcal{W} : \psi^*(\mathcal{W} - \mathcal{S})$ $= (\mathcal{W} - \mathcal{S})\}$also $\mathcal{B}$ *is* the collection $(\mathcal{F},\mathcal{T})=$ $\{\mathcal{S}$ - $\mathcal{B})$; $\mathcal{S} \in \mathcal{T}$ and $\mathcal{B} \in \mathcal{F}\}$ is a basis for $\mathcal{T}^*(\mathcal{T},\mathcal{F})$ (7-9). The simple $\mathcal{S}^*$ write for $\mathcal{S}^*$ $(\mathcal{T},\mathcal{F})$ , $\mathcal{T}^*$ $(\mathcal{F},\mathcal{T})$. When there is no chance for confusion, the concept $(\mathcal{W},\mathcal{T},\mathcal{F})$ *it* will be Symbolize to the topological space $(\mathcal{W},\mathcal{T})$ with an ideal $\mathcal{F}$ on $\mathcal{W}$ thats no assumption of Separation properties, it is named the ideal topological space to as short. Each element in $\mathcal{T}^*$ named $\mathcal{T}^*$*are an* open set. If ($\mathcal{W}$ - $\mathcal{S}$ ) is $\mathcal{T}^*$ *an* open set, So $\mathcal{S}$ called $\mathcal{T}^*$ Closed, therefore, *it is closed in* $(\mathcal{W},\mathcal{T}^*)$. The subset $\mathcal{D}$ of the space $(\mathcal{W},\mathcal{T},\mathcal{F})$ *is a* $\mathcal{T}^*$ - Closed if $\mathcal{S}^* \subseteq \mathcal{S}$.

In the ideal topological space $(\mathcal{W},\mathcal{T},\mathcal{F})$ a subset $\mathcal{S}$ *is* called dense if $CL^*(\mathcal{S}) = \mathcal{W}$. Note that in $(\mathcal{W},\mathcal{T},\mathcal{F})$ if $\mathcal{F}=\{\emptyset\}$, so $\mathcal{T} = \mathcal{T}^*(\mathcal{F},\mathcal{T})$ . Let $\mathcal{S} \subseteq \mathcal{W}$,

*The* $int^*(\mathcal{A})$ (respectively) the $cl^*(\mathcal{A})$ it denoted (interior, respectively, closure of $\mathcal{A}$) in $(\mathcal{W},\mathcal{T}^*)$(10-11).

Additionally, Jankovic and Hamlet presented a study on the topological properties using the concept of the ideal (12-14). Using the concept of the ideal, some researchers studied a new type of separation axioms (15-17). Furthermore, studies continued, and after nano soft topology was defined, many types of open sets were known on the ideal topological space. Moreover, the definition of soft ideal topological spaces was extended to include soft topological properties. Also nano soft (18-22), although more, application on the ideal topological spaces. And separation axioms were offered like connectedness (23), and many properties of the nano ideal spaces were studied. Also a new classes Sets are defined, and many of the properties of these sets were discussed, so other research was developed on these sets and their topological properties based on the concept of the ideal. (24-27).

## 2.Methodology

### 2.1. R − Open Set

In this subsection, a new set will be defined and named R- open in the space $(\mathcal{W},\mathcal{T},\mathrm{I})$

#### 2.1.1. Definition

Let $(\mathcal{W},\mathcal{T},\mathrm{I})$ be an ideal topological space, a subset $\mathcal{Y}$ of $\mathcal{W}$ is said to be R −open set if $\exists$ H $\in \mathcal{T} \setminus \{\mathcal{W}, \emptyset\}$ such that, $\mathcal{Y} - \mathrm{CL}(\mathcal{Y} \cap \mathrm{H}) \in \mathrm{I}$, the complement of the R − open set is called R −closed set. [The symbol R − O refes to R- open set, and the symbol R-C refers to an R-closed set].

#### 2.1.2. Proposition

In any ideal topological space $(\mathcal{W},\mathcal{T},\mathrm{I})$, every open proper subset of $\mathcal{W}$ is an R-open set.

**proof:**

Suppose $\mathcal{K}$ is open proper $\subseteq \mathcal{W}$, If $\boldsymbol{\mathcal{K}} = \emptyset$, then for any nonempty open proper subset H Of $\mathcal{W}$, $\emptyset - \mathrm{CL}(\emptyset \cap \mathrm{H}) = \emptyset \in \mathrm{I}$. If $\boldsymbol{\mathcal{K}} \neq \emptyset$, then $\exists$ H $\in \tau \setminus \{\mathcal{W},\emptyset\}$ when $\mathrm{H} = \boldsymbol{\mathcal{K}}$ such that $\boldsymbol{\mathcal{K}} - \mathrm{CL}(\boldsymbol{\mathcal{K}} \cap \mathrm{H}) = \boldsymbol{\mathcal{K}} - \mathrm{CL}(\boldsymbol{\mathcal{K}} \cap \boldsymbol{\mathcal{K}}) = \boldsymbol{\mathcal{K}} - \mathrm{Cl}(\boldsymbol{\mathcal{K}}) = \emptyset \in \mathrm{I}$.

#### 2.1.3. Remark

In any ideal topological space $(\mathcal{W},\mathcal{T},\mathrm{I})$, $\mathcal{W}$ does not be *an* R − open set.

**Example**

Suppose $\mathcal{W} = \{k_1, k_2, k_3\}$, $\mathcal{T} = \{\mathcal{W}, \{k_1\}, \{k_2, k_3\}, \emptyset\}$ & $I = \{\emptyset, \{k_2\}\}$ .Notice that $\mathcal{W}$ is Not R − O Set, since the only nonempty open proper subsets of $\mathcal{W}$ are $\{k_1\}$,$\{k_2,k_3\}$ and $\mathcal{W} - \mathrm{CL}(\mathcal{W} \cap \{k_1\}) = \{k_2,k_3\} \notin$ I. Also $\mathcal{W} - \mathrm{CL}(\mathcal{W} \cap \{k_2,k_3\} = \{k_1\} \notin$ I.

#### 2.1.4. Remark

In the ideal topological space $(\mathcal{W},\mathcal{T},\mathrm{I})$, every R − O set cannot be an open set.

**Example**

In the example of Remark 2.3, $\{\hbar_2\}$ is an $\mathrm{R}-\mathrm{O}$ set but not an open set .

**2.1.5. Remark**

For any two $\mathrm{R}-$ open sets, the intersection needs not to be $\mathrm{R}-\mathrm{O}$.

**Example**

Let $\mathcal{W}=\{\ell_{1,}\ell_{2,}\ell_{3,}\ell_{4,}\}, \mathcal{T}=\{\mathcal{W},\{\ell_{1,}\ell_{2,}\},\varnothing\}, \mathrm{I}=\{\varnothing,\{\ell_{2,}\}\}$. Notice that $\{\ell_{1,}\ell_{3}\},\{\ell_{2,}\ell_{3}\}$ $\mathrm{R}-\mathrm{O}$ sets, but $\{\ell_{1,}\ell_{3}\}\{\cap\{\ell_{2,}\ell_{3}\}=\{\ell_{3}\}$ cannot $\mathrm{R}-$ open.

**2.1.6. Remark**

A union for any two $\mathrm{R}-$ open sets cannot $\mathrm{R}-$ open.

**For example**

Suppose that $\mathcal{W}=\{\ell_{1,}\ell_{2,}\ell_{3}\}$, $\mathcal{T}=\{\mathcal{W},\{\ell_1\},\{\ell_{2,}\ell_3\},\varnothing\}, \mathrm{I}=\{\varnothing,\ell_1\}$ .

Since $\{\ell_1\},\{\ell_{2,}\}$ are two $\mathrm{R}-$ open sets and $\{\ell_1\}\cup\{\ell_2\}=\{\ell_{1,}\ell_2\}$ cannot be $\mathrm{R}-$ open sets.

**2.1.7. Theorem**

If the space $(\mathcal{W},\mathcal{T},\mathrm{I})$ has no proper open subset whose union equals to $\mathcal{W}$, then the union of any two $\mathrm{R}-\mathrm{O}$ sets is R – O.

**proof:**

Assume that $\mathcal{A}_1$ ,$\mathcal{A}_2$ are two $\mathrm{R}-$ O sets, since there is $\mathcal{H}_1,\mathcal{H}_2\in\tau\setminus\{\mathcal{W},\emptyset\}$ implies $\mathcal{A}_{1,}-\mathrm{CL}(\mathcal{A}_1\cap\mathcal{H}_1)\in\mathrm{I}$ and $\mathrm{A}_2-\mathrm{CL}(\mathcal{A}_2\cap\mathcal{H}_2)\in\mathrm{I}$, it follows from conditions of ideal that $[\mathcal{A}_1-\mathrm{CL}(\mathcal{A}_1\cap\mathcal{H}_1)]\cup[\mathcal{A}_2-\mathrm{CL}(\mathcal{A}_2\cap\mathcal{H}_2)]\in I$, hence. $(\mathcal{A}_1\cup\mathcal{A}_2)-[\mathrm{CL}\,(\mathcal{A}_{1,}\cap\mathcal{H}_1)\cup\mathrm{CL}\,(\mathcal{A}_2\cap\mathcal{H}_2)]\subseteq[(\mathcal{A}_{1,}-\mathrm{CL}(\mathcal{A}_{1,}\cap\mathcal{H}_1)\cup[\mathcal{A}_2-\mathrm{CL}\,(\mathcal{A}_2\cap\mathcal{H}_2)]$, then $(\mathcal{A}_1\cup\mathcal{A}_2)-[CL(\mathcal{A}_1\cap\mathcal{H}_1)\cup CL(\mathcal{A}_2\cap\mathcal{H}_2)]\in I$, but $(\mathcal{A}_1\cap\mathcal{H}_1)\cup(\mathcal{A}_2\cap\mathcal{H}_2)\subseteq(\mathcal{A}_1\cup\mathcal{A}_2)\cap(\mathcal{H}_1\cup\mathcal{H}_2)$, then, $\mathrm{CL}\,[(\mathcal{A}_1\cap\mathcal{H}_1)\cup(\mathcal{A}_2\cap\mathcal{H}_2)\subseteq CL\,[(\mathcal{A}_1\cup\mathcal{A}_2)\cap(\mathcal{H}_1\cup\mathcal{H}_2)]$, so $(\mathcal{A}_1\cup\mathrm{A}_2)-$
$\mathrm{CL}\,[(\mathcal{A}_1\cup\mathcal{A}_2)\cap(\mathcal{H}_1\cup\mathrm{H}_2)]\subseteq(\mathcal{A}_1\cup\mathcal{A}_2)-\mathrm{CL}\,[(\mathcal{A}_1\cap\mathcal{H}_1)\cup(\mathcal{A}_2\cap\mathcal{H}_2)]$. That Subsequently $(\mathcal{A}_1\cup\mathcal{A}_2)-\mathrm{CL}\,[\mathcal{A}_1\cup\mathcal{A}_2)\cap(\mathcal{H}_1\cup\mathcal{H}_{2)}]\in\mathrm{I}$. then $\mathcal{A}_1\cup\mathcal{A}_2$ is an $\mathrm{R}-\mathrm{O}$.

**2.1.8. Remark**

In the context of the space $(\mathcal{W},\mathcal{T},\mathrm{I})$, the set R - O is a union for every R-open set; we will denote it $\mathrm{R_I O}(\mathcal{W})$. such that $\mathrm{R_I O}\,(\mathcal{W})=\{\mu\subseteq\mathcal{W}\colon\mu\text{ be an }\mathrm{R}-\mathrm{O}\text{ set}\}\cup\{\mathcal{W}\}$, respectively $\mathrm{R_I C}(\mathcal{W})=\{\mathcal{A}\subseteq\mathcal{W}\colon\mathcal{A}^c\text{ be an R}-\text{open set}\}\cup\{\phi\}$.

**2.1.9. Remark**

Assume the space $(\mathcal{W},\mathcal{T},\mathrm{I})$, it is easy to achieve that $\mathcal{T}\subseteq\mathrm{R_I O}(\mathcal{W})$.

## 3. Separation axioms

In this section, using a set under study, R-open a new kind of separation axioms will be defined and the relationship among these concepts will be studied.

**3.1. Definition**

Consider the space $(\mathcal{W},\mathcal{T},\mathrm{I})$, then $(\mathcal{W},\mathcal{T},\mathrm{I})$ is named an $R_I-\mathcal{T}_0-$ space if and only if each of the two different points $\mathfrak{R}_{1\neq}\,\mathfrak{R}_2$ *in* $\mathcal{W}$ all contain to an $\mathrm{R_I}-$ open set while excluding the other.

**Example**

Consider the space $(\mathcal{W},\mathcal{T},\mathrm{I})$; $\mathcal{W}=\{\mathfrak{R}_1,\mathfrak{R}_2,\mathfrak{R}_3\}, \mathcal{T}=\{\mathcal{W},\varnothing,\{\mathfrak{R}_1\}\}$
$\mathrm{I}=\{\varnothing,\{\mathfrak{R}_2\}\}$ , $R_IO(\mathcal{W})=\{\mathcal{W},\varnothing,\{\mathfrak{R}_1\},\{\mathfrak{R}_2\},\{\mathfrak{R}_1,\mathfrak{R}_2\},\{\mathfrak{R}_1,\mathfrak{R}_3\}\}$, Notice that $(\mathcal{W},\mathcal{T},\mathrm{I})$ is $R_I-\mathcal{T}_0$ space since $\mathfrak{R}_{1\neq}\,\mathfrak{R}_2$ , $\exists$ R- open set $\{\mathfrak{R}_1\}\in R_IO(\mathcal{W})$ Such that $\mathfrak{R}_1\in$

$\{\mathfrak{R}_1\}$ and $\mathfrak{R}_2 \notin \{\mathfrak{R}_1\}$ , $Also\, \mathfrak{R}_2 \neq \mathfrak{R}_3$ , $\exists$ R- open set $\{\mathfrak{R}_2\} \in R_I O(\mathcal{W})$ such that $\mathfrak{R}_2 \in \{\mathfrak{R}_2\}\, and\; \mathfrak{R}_3 \notin \{\mathfrak{R}_2\}$ hence $(\mathcal{W}, \mathcal{T}$, I) is $R_I - \mathcal{T}_0 -$ space.

**3.2. Theorem**

Suppose the space $(\mathcal{W}, \mathcal{T})$ is $\mathcal{T}_0$ – space, so $(\mathcal{W}, \mathcal{T}$, I ) Be an $R_I - \mathcal{T}_0 -$ space If, for Every two different elements $\mathfrak{R}_1 \neq \mathfrak{R}_2 \in (\mathcal{W}, \mathcal{T})$.

**Proof:**

Consider $(\mathcal{W}, \mathcal{T}$ ) to be the $,\mathcal{T}_0$ –space, then for each two different $elements\, \mathfrak{R}_1 \neq \mathfrak{R}_2 \in \mathcal{W}$ $\exists$ μ $\in \mathcal{T}$ such that $\mathfrak{R}_1 \in$ μ but $\mathfrak{R}_2 \notin$ μ , Since $,\mathcal{T} \subseteq R_I O(\mathcal{W})$ so $\mathfrak{R}_1 \in$ μ but $\mathfrak{R}_2 \notin$ μ, then $(\mathcal{W}, \mathcal{T}$, I) is $R_I - \mathcal{T}_0 -$ space. The converse does not hold true in general for instance.

**Example**

Suppose $\mathcal{W} = \{\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3\}$, $\mathcal{T} = \{\mathcal{W}, \emptyset, \{\mathfrak{R}_1\}, \{\mathfrak{R}_2, \mathfrak{R}_3\}\}$, I = { $\emptyset$ , $\{\mathfrak{R}_1\}$ }. Then, $R_I O(\mathcal{W}) = \mathbb{P}(\mathcal{W})$. Notice that $(\mathcal{W}, T$, I) is $R_I - \mathcal{T}_0 -$ space but not $\mathcal{T}_0 -$ space since $\mathfrak{R}_2 \neq \mathfrak{R}_3$ and there is no open set μ. That belongs to $\mathfrak{R}_2$ but does not contain $\mathfrak{R}_3$ .

**3.3. Theorem**

$(\mathcal{W}, \mathcal{T}$, I) is an $R_I - \mathcal{T}_0 -$ space if and only if for each two different components $\mathcal{B}_1 \neq \mathcal{B}_2$ there exists an $R_I -$ C set consisting of one and not containing the other.

**Proof:**

Suppose $\mathcal{B}_1$ & $\mathcal{B}_2$ $are$ two distinct elements in $\mathcal{W}$, and since $\mathcal{W}$ is an $R_I - T_0 -$ Space, then there exists an $R_I -$open set containing one of them, such that $\mu^c$ is an $R_I -$ closed set includes the other. In contrast, suppose that $\mathcal{B}_1$ & $\mathcal{B}_2$ are distinct elements. In $\mathcal{W}$ and since it contains an $R_I -$ closed set μ containing one and not containing the other. Therefore, $\mu^c$ is an $R_I -$ open set that comprises only one of them, hence $\mathcal{W}$ is an $R_I - T_0 -$ space.

**3.4. Definition**

Suppose $(\mathcal{W}, \mathcal{T}$, I) is an ideal topological space, so $(\mathcal{W}, \mathcal{T}$, I) is named an $R_I - \mathcal{T}_1 -$space if and only if for each pair of the point $\mathcal{B}_1$ & $\mathcal{B}_2$ ; $\mathcal{B}_1 \neq \mathcal{B}_2 \in \mathcal{W}$, there exist two sets U & V that are $R_I O(\mathcal{W})$ sets Therefor $\mathcal{B}_1 \in$ (U $- V$) and $\mathcal{B}_2 \in$ (V – U).

**Example**

Let $\mathcal{W}$ be a space ; $\mathcal{W} = \{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3\}$, $\mathcal{T} = \{\mathcal{W}, \emptyset, \{\mathcal{B}_1\}, \{\mathcal{B}_2, \mathcal{B}_3\}\}$ And I = { ø, $\{\mathcal{B}_2\}, \{\mathcal{B}_3\}, \{\mathcal{B}_2, \mathcal{B}_3\}\}$, $so\; R_I O(\mathcal{W}) = \mathcal{P}(\mathcal{W})$ .such that ( $\mathcal{W}, \mathcal{T}$, I) be an $R_I - \mathcal{T}_1 -$ space.

**3.5. Proposition**

Let $(\mathcal{W}, \mathcal{T})$ be a $\mathcal{T}_1 -$ space such that $(\mathcal{W}, \mathcal{T}$, I ) is an $R_I - \mathcal{T}_1 -$ space.

**proof:**

Assume $(\mathcal{W}, \mathcal{T})$ is a $\mathcal{T}_1 -$ space so there exists U & V ( two open sets) belong to $\mathcal{T}$ and for any two distinct elements $\mathcal{B}_1\; and\; \mathcal{B}_2 \in \mathcal{W}$ implies $\mathcal{B}_1 \in$ (U-V), also $\mathcal{B}_2 \in$ (V-U) since $\mathcal{T} \subseteq R_I O(\mathcal{W})$ So U ,V $\in R_I O(\mathcal{W})$, $\exists$ U ,V $\in R_I O(\mathcal{W})$ and $\mathcal{B}_1 \in$ (U-V) and $\mathcal{B}_2 \in$ (V-U), then $(\mathcal{W}, \mathcal{T}$, I) is $R_I - \mathcal{T}_1 -$ space. But the opposite does not hold in (3.8) in general. Consider the instance (3.4).

**3.6. Proposition**

Consider $(\mathcal{W}, \mathcal{T}$, I) to be an $R_I - \mathcal{T}_1 -$ space so $(\mathcal{W}, \mathcal{T}$, I) is an $R_I - \mathcal{T}_0 -$ space.

**Proof:**

Consider $\mathcal{B}_1$ & $\mathcal{B}_2$ $to$ be two different components $\in \mathcal{W}$, since $(\mathcal{W}, \mathcal{T}$, I) is an $R_I - \mathcal{T}_1 -$ space so U, V are two $R_I O(\mathcal{W})$ sets, hence $\mathcal{B}_1 \in$ (U-V), $\mathcal{B}_2 \in$ (V-U). Therefore, $\exists$ $R_I O(\mathcal{W})$ set U which includes only one of them, so $(\mathcal{W}, \mathcal{T}$ , I) is an $R_I - \mathcal{T}_0 -$ space

While the inverse of (3.9) does not hold for instance.

**Example**

Assume $that\ \mathcal{W} = \{\mathbb{Q}_1, \mathbb{Q}_2, \mathbb{Q}_3\}$, $\mathcal{T} = \{\mathcal{W}, \emptyset, \{\mathbb{Q}_1\}\}$, I = $\{\emptyset, \{\mathbb{Q}_2\}\}$, $R_I O(\mathcal{W}) = \{\mathcal{W}, \emptyset, \{\mathbb{Q}_1\}, \{\mathbb{Q}_2\}, \{\mathbb{Q}_1, \mathbb{Q}_2\}, \{\mathbb{Q}_1, \mathbb{Q}_3\}\}$, such that ($\mathcal{W}$, $\mathcal{T}$, I) is an $R_I - T_0 -$ space but not an $R_I - \mathcal{T}_1 -$ space since $\mathbb{Q}_1 \neq \mathbb{Q}_3 \in \mathcal{W}$ and so $\nexists$ U ,V $\in R_I\, O(\mathcal{W})$, then $\mathbb{Q}_1 \in$ (U-V) and $\mathbb{Q}_3 \in$ (V-U).

**3.7. Theorem**

Suppose ($\mathcal{W}$, $\mathcal{T}$, I) is a space ; ($\mathcal{W}$, $T$, I) is an $R_I - \mathcal{T}_1 -$ space if and only if for each element $\mathcal{B}_1 \neq \mathcal{B}_2$, so there are two $R_I -$ C sets: $\mathcal{F}_1$ and $\mathcal{F}_2$, implying that $\mathcal{B}_1$ belongs to ($\mathcal{F}_1$ - $\mathcal{F}_2$), also $\mathcal{B}_2$ belongs to ($\mathcal{F}_2$ - $\mathcal{F}_1$).

**Proof**

Consider $\mathcal{B}_1$ & $\mathcal{B}_2$ are two different constituents in $\mathcal{W}$, also ($\mathcal{W}$, $\mathcal{T}$, I) is an $R_I - \mathcal{T}_1 -$ space. Consequently, there are two $R_I -$O sets: U and V, therefore $\mathcal{B}_1 \in$ (U-V) & $\mathcal{B}_2 \in$ (V-U) and there exists $R_I -$C sets ($\mathcal{W}$ -U) and ($\mathcal{W}$ -V). $\mathcal{B}_1 \in$ ($\mathcal{W}$ -V) - ($\mathcal{W}$ -U) , $\mathcal{B}_2 \in$ ($\mathcal{W}$ -U) - ($\mathcal{W}$ -V) then ($\mathcal{W}$ -V) = $\mathcal{F}_1$ & ($\mathcal{W}$ -U) = $\mathcal{F}_2$, such that there are two $R_I C -$ sets: $\mathcal{F}_1$ , $\mathcal{F}_2$ in order to satisfy $\mathcal{B}_1 \in (\mathcal{F}_1 \cap \mathcal{F}_2^c)$. Also $\mathcal{B}_2 \in (\mathcal{F}_2 \cap \mathcal{F}_1^c)$ such that $\mathcal{B}_1 \in (\mathcal{F}_1 - \mathcal{F}_2)$. Conversely, let $\mathcal{B}_1$& $\mathcal{B}_2$ be two different components $\in \mathcal{W}$ so thus, two $R_I$c $-$ Sets $\mathcal{F}_1$ & $\mathcal{F}_2$ are achieved: $\mathcal{B}_1\ belongs\ to\ (\mathcal{F}_1 \cap \mathcal{F}_2^c)$. $\mathcal{B}_2\ belongs\ to\ (\mathcal{F}_2 \cap \mathcal{F}_1^c)$, then there exists $R_I O -$ sets ($\mathcal{W} - \mathcal{F}_1$) and ($\mathcal{W} - \mathcal{F}2$ ), when $\mathcal{B}_1 \in (\mathcal{W} -$F$_2) - (\mathcal{W} - \mathcal{F}_1)$, $\mathcal{B}_2\ belongs\ to\ (\mathcal{W} - \mathcal{F}_1) - (\mathcal{W} - \mathcal{F}2)$ such that ($\mathcal{W} - \mathcal{F}2$) = U and ($\mathcal{W} - \mathcal{F}_1$) = V.

**3.8. Proposition**

Assume { $x$ } is an $R_I -$ C set, for each $\mathcal{Z} \in \mathcal{W}$. Hence ($\mathcal{W}$, $\mathcal{T}$, I) is an $R_I - \mathcal{T}_1 -$ space.

**Proof:**

Consider $\mathcal{Z}_1$ & $\mathcal{Z}_2$ are two different components belong to $\mathcal{W}$, since { $\mathcal{Z}_1$ },{$\mathcal{Z}_2$ } are $R_I -$C sets, then ($\mathcal{W}$ -{ $\mathcal{Z}_1$ }). Also, ($\mathcal{W}$ -{$\mathcal{Z}_2$ } ) is $R_I -$ O sets such that there are $R_I -$ O sets, U & V, since U = ($\mathcal{W}$ -{ $\mathcal{Z}_2$ }). Also, V = ($\mathcal{W}$ -{ $\mathcal{Z}_1$} ); therefore, $\mathcal{Z}_1 \in$ (U – V) , $\mathcal{Z}_2 \in$ (V – U). So ($\mathcal{W}$ , $\mathcal{T}_1$, I) is an $R_I - \mathcal{T}_1$ space.

**3.9. Definition**

The ideal topological space($\mathcal{W}$, $\mathcal{T}$, I) is said to be an $R_I - \mathcal{T}_2 -$space if for each pair of different point $\mathfrak{R}_1 \neq \mathfrak{R}_2 \in \mathcal{W}$, there are two disjoint $R_I O(\mathcal{W})$ sets U and V, whenever $\mathfrak{R}_1 \in U\ and\ \mathfrak{R}_2 \in$ V and U$\cap$V = $\emptyset$.

**3.10. Proposition**

Suppose ($\mathcal{W}$ , $\mathcal{T}$ ) is $\mathcal{T}_2$ – space, such that ($\mathcal{W}$ , $T$, I) will be $an\ R_I - \mathcal{T}_2 -$ space.

**Proof:**

Suppose $\mathfrak{R}_1$ & $\mathfrak{R}_2$ are two different elements belong to $\mathcal{W}$ , and since ($\mathcal{W}$ , $\mathcal{T}$) is $a\ \mathcal{T}_2$– space such that it is two open sets U and V , therefore $\mathfrak{R}_1 \in U\ and\ \mathfrak{R}_2 \in$ V and U$\cap$V =$\emptyset$ since $\mathcal{T} \subseteq R_I O(\mathcal{W})$ such that U,V are $R_I O(\mathcal{W})$ sets satisfying the condition $\mathfrak{R}_1 \in U\ and\ \mathfrak{R}_2 \in$ V, and U$\cap$V = $\emptyset$ such that ($\mathcal{W}$, $\mathcal{T}$, I) will be $a\ R_I - \mathcal{T}_2$– space.

**3.11. Proposition**

When the space ($\mathcal{W}$, $\mathcal{T}$, I) is an $R_I - \mathcal{T}_2 -$ space implying an $R_I - \mathcal{T}_1 -$ space.

**Proof:**

Consider $\mathfrak{R}_1$ & $\mathfrak{R}_2$ $to$ be two different elements $\in \mathcal{W}$; $\mathfrak{R}_1 \neq \mathfrak{R}_2$ since ($\mathcal{W}$,$\mathcal{T}$,I) is $R_I - \mathcal{T}_2$ – space. U and V are $R_I O(\mathcal{W})$ sets, therefore $\mathfrak{R}_1 \in$ U and $\mathfrak{R}_2 \in$ V & U $\cap$ V = $\phi$, so there exists $R_I O(\mathcal{W})$ sets U and V in order to satisfy $\mathfrak{R}_1 \in$ (U $-$ V) and $\mathfrak{R}_2 \in$ (V $-$ U). So ($\mathcal{W}$, $\mathcal{T}$, I) is an $R_I - \mathcal{T}_{1_1} -$ space. In Proposition (3.15), the opposite meaning is not achieved as explained by the below example.

**Example**

In the $(\mathcal{W},\mathcal{T}$ , I), let $\mathcal{W}$ $=\{\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3\}$, $\mathcal{T}=\{\mathcal{W},\emptyset,\{\mathcal{E}_1\},\{\mathcal{E}_3\},\{\mathcal{E}_1,\mathcal{E}_3\}\}$

I = { ø }, $R_I O(\mathcal{W})=\{\mathcal{W},\emptyset,\{\mathcal{E}_1\},\{\mathcal{E}_3\},\{\mathcal{E}_1,\mathcal{E}_2\},\{\mathcal{E}_1,\mathcal{E}_3\},\{\mathcal{E}_2,\mathcal{E}_3\}\}$

Notice that $(\mathcal{W},\mathcal{T}$, I) is an $R_I-\mathcal{T}_1$ – space, while it is not an $R_I-\mathcal{T}_2-$ space.

**3.12. Remark**

Suppose $(\mathcal{W},\mathcal{T})$ is a $\mathcal{T}_j$ – space for each $j$ = {0, 1, 2}, such that the ideal topological space $(\mathcal{W},\mathcal{T}$, I ) is *an* $R_I$–$\mathcal{T}_j$ – space, ∀ $j$ = {0, 1, 2}. Notice that the opposite in the remark (3.17) is generally false as illustrated in the below example.

**Example**

Consider $\mathcal{W}=\{\mathfrak{R}_1,\mathfrak{R}_2,\mathfrak{R}_3\}$, $\mathcal{T}=\{\mathcal{W},\emptyset,\{\mathfrak{R}_3\}\}$, I = P($\mathcal{W}$) & $R_I O(\mathcal{W})$ = P($\mathcal{W}$). So it is evident that the space $(\mathcal{W},\mathcal{T}$, I) is an $R_I-\mathcal{T}_j$– space for each $j$ = {0, 1 ,2}. Hence, it is not a $\mathcal{T}_j$– space for each $j$ = {0, 1 ,2}. The following diagram elucidates the interrelationships between the preceding concepts:

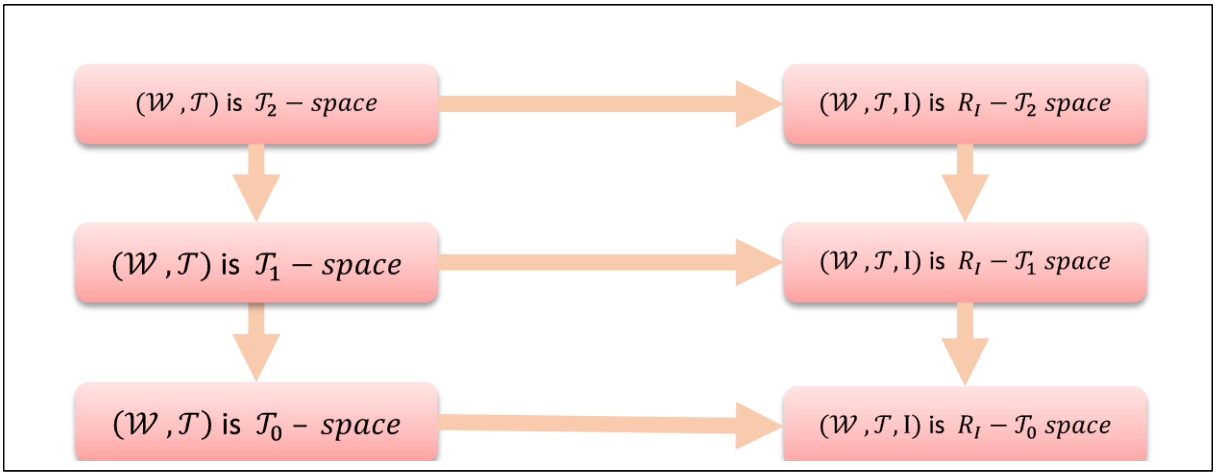


**Diagram 1** .The Relationships among $\mathcal{T}_j$– space & $R_I-\mathcal{T}_j$– spaces.

## 4.Conclusion

In this article, we created and introduced a set [namely an $R_I$-open] by using the ideal topological space and also studied many properties of this set. The separation of axioms and relationships between these axioms were also studied, and this has been reinforced with examples and the converse of these properties has also been discussed. In addition, we will work on other different topological properties of this set, such as convergence and compactness, using the set under study.


## Acknowledgments

Our researcher extends his Sincere thanks to the editor and members of the preparatory committee of the Ibn AL-Haitham Journal of Pure and Applied Sciences .


## Conflict of Interest

There are no conflicts of interest.


## Funding

There is no funding for the article


## References


1. Kuratowski K. Topology. New York: Acad. Press. 1933; I.
2. Dontchev, Julian, Maximilian Ganster, and David Rose. Ideal resolvability.Topology and its Applications 1999: 93.1 ;1-16, https://doi.org/10.1016/S0166-8641(97)00257-5 .
3. Mohammed, M. W., A. A. Mohammed. Some Classes in Ideal Topological Spaces.Journal of

Education & Science 2023: 32.2. https://doi.org/10.33899/edusj.2023.137766.1318
4. Auda, Ghufran H., Hula M. Salih. New supra open sets with ideal in supra topological space. AIP Conference Proceedings. 2023;Vol. 2834. No. 1. AIP Publishing,. https://doi.org/10.1063/5.0163559
5. Vaidyanathaswamy, R. The localisation theory in set-topology. Proceedings of the Indian Academy of Sciences-Section A. 1944:Vol. 20. Springer India.
6. Al-Saadi, H., A. Al-Omari. Some operators in ideal topological spaces. Missouri Journal of Mathematical Sciences 2018: 30.1; 59-71. https://doi.org/10.35834/mjms/1534384955
7. Mandal, Dhananjoy, M. N. Mukherjee. Certain new classes of generalized closed sets and their applications in ideal topological spaces. Filomat 2015: 29.5; 1113-1120.
8. Atay, A., F. Eren. KURATOWSKI CLOSURE OPERATORS IN SOFT IDEAL TOPOLOGICAL SPACES. Acta Universitatis Apulensis: Mathematics-Informatics 2023: 74. https://doi.org/ I 10.17114/j.aua
9. Tunç, Ayşe Nur, Sena Özen Yıldırım. On a topological operator via local closure function. Turkish Journal of Mathematics and Computer Science 2023: 15.2; 227- https://doi.org/10.47000/tjmcs.1195540
10. Viplavanjali, K. S., et al. Some Kinds of Separation Axioms in Kasaj Topological Spaces. Indian Journal of Science And Technology 2023:16.41;3575-3582.https://doi.org/10.17485/IJST/v16i41.1509
11. Karim, MA Abdel, and A. I. Nasir. "Separation Axioms via Ĭ Semi g Open Sets." Ibn AL-Haitham Journal For Pure and Applied Science, 2020: 33.1; 157-161. https://doi.org/10.30526/33.1.2382
12. Mohammad, Muna Jabbar, Faik Jameel Hasan Mayah. On Pre-δ-Separation Axioms in Ideal Topological Spaces.Journal of Wasit for science and medicine 2023:16.1;7-12. https://doi.org/10.31185/jwsm.342
13. Jackson, S., T. G. Denosha, P. Mariappan. Separation Axioms through Nano JD open set. Indian Journal of Science and Technology 2024: 17;9-13. https://doi.org/10.17485/IJST/v17sp1.252
14. Yalaz, Ferit, and Aynur Keskin KaymakcÄs. A New Separation Axiom in Ideal Topological Spaces.Conference Proceeding Science and Technology: International ONLINE Conference on Mathematical Advances and Applications (ICOMAA-2022), Istanbul. Vol. 5. No. 1. 2022.
15. Saeed, Sufyan G., R. B. Esmaeel. Separation axioms via αg Ị-open set. Journal of Physics: Conference Series. 2020: Vol.1591.No.1.IOP Publishing, https://iopscience.iop.org/article/10.1088/1742-6596/1591/1/012099
16. Dontchev,Julian. On Hausdorff Spaces via Topological Ideals and I-irresolute Functions. Annals of the New York Academy of Sciences 1995: 767.1; 28-38. https://doi.org/10.1111/j.1749-6632.1995.tb55891.x
17. Banerjee, Amar Kumar, Jagannath Pal. New separation axioms in generalized bitopological spaces. Mathematical Sciences, 2020: 14.2 ;185-192. https://doi.org/10.1007/s40096-020-00330-z
18. Mohammad, R. J. Esmaeel. R. B.On Separation Axioms With Soft-J-Semi-g-Open Sets. IOP Conference Series: Materials Science and Engineering 2020:Vol. 871. No. 1. IOP Publishing, https://doi.org/10.1088/1757-899X/871/1/012052
19. Esmaeel, R. B.,R. J. Mohammad.On Nano soft-$\mathcal{J}$-semi-g-closed sets. Journal of Physics: Conference Series. 2020:Vol. 1591. No. 1. IOP Publishing. https://doi.org/ 10.1088/1742-6596/1591/1/012071
20. Renuka, J. Joycy, J. Arul Jesti. Some Separation Axioms In Nano Ideal Topological Spaces. Journal of Namibian Studies: History Politics Culture, 2023: 35;2502-2508. https://doi.org/10.59670/jns.v35i.4029
21. Karim, MA Abdel, A. I. Nasir. Some Game via Ĩ-Semi-g-Separation Axioms. Baghdad Science Journal , 2020: 17.3 https://doi.org/10.21123/bsj.2020.17.3.0861
22. Mahmood, S. I. On generalized regular continuous functions in topological spaces. Ibn Al-Haitham Journal for pure and applied science 2017: 25.3; 377-376.
23. Nasir, NA Jabbar AI, N. A. Jabbar. Some Types of Compactness in Bitopological Spaces. Ibn Al-

Haitham Journal For Pure and Applied Sciences 2010:23.1.

24. Mahmoud, Balqees K., Yousif Y. Yousif. CutPoints and Separations in Alpha-Connected Topological Spaces. Iraqi Journal of Science 2021: 24;3091-3096. https://doi.org/10.24996/ijs.2021.62.9.24
25. Al-talkany, Yiezi K. Mahdi, Suadud H. Al-ismaily. On separation axioms and continuity with respect to some types of sets in ideal topological space.International Journal of Pure and Applied Mathematics 2018: 119.10; 409-422.
26. Al-shami, Tareq M. Soft somewhat open sets: Soft separation axioms and medical application to nutrition. Computational and Applied Mathematics 2022: 41.5; 216. https://doi.org/10.1007/s40314-022-01919-x
27. El-Atik, Abd El-Fattah A., Hanan Z. Hassan. Some nano topological structures via ideals and graphs. Journal of the Egyptian Mathematical Society 2020: 28.1; 41. DOI https://doi.org/10.1186/s42787-020-00093-5