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$\tilde{s}\boldsymbol{\beta}_s$- open sets and $\tilde{s}\boldsymbol{\beta}_s$- continuous in soft topological spaces

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Abstract

Our work introduces the notion of soft β_s-open set. This class of $\tilde{s}\beta_s$- open sets is not directly comparable to the categories of soft open, soft closed sets and soft semi-closed. We demonstrate that the category of $\tilde{s}\beta_s$-open sets lies strictly between the class of $\tilde{s}\beta_c$-open sets and $\tilde{s}\beta$-open sets. Additionally, we investigate the connections that exist between $\tilde{s}\beta_s$-open sets along with other kinds of soft sets. Moreover, we provide several criteria that are sufficient for determining the equivalence between $\tilde{s}\beta_s$-open sets and every one of $\tilde{s}\beta_c$-open sets and $\tilde{s}\beta$-open sets. Also, according to our findings, the family of $\tilde{s}\beta_s$-open sets is a supra soft topology. Furthermore, we make it clear the correlation that exists between the respective categories of $\tilde{s}\beta$-open sets in a soft topological space and in its soft topological subspace. Finally, the class of $\tilde{s}\beta_s$-continuous function is introduced, it is main properties are studied and derive some of the properties of these soft functions under the soft composition of soft functions.

Keywords: soft β_s- open, soft $\beta -$ open, soft semi-closed, soft β_s-continuous function.

1. Introduction

The concept of first using soft set theory by (1) was presented as a novel tool in mathematics for dealing with several forms of ambiguity in complicated scientific problems and solving these problems. Soft sets research and its characteristics were used to several areas of mathematics, including theory, probability, operations research, algebra, mathematical analysis, etc. In 2011, the main concepts of soft topologies were investigated by (2). They presented the ideas of a soft open set and showed that a soft topological space produces a parameterized family of topological spaces, soft closed set, soft interior point, soft closure and soft separation axioms. A multitude of contributions has been made to the research of topological principles in soft environments after soft topology was developed (3-6).

Athar Kharal, A. and Ahmad, B. (7) explained what soft mapping is in the context of soft classes and investigated several properties of soft set images and inverse images. Chen (8) introduced and investigated soft semi-open sets and their properties. Mahanta and Das (9) also introduced and defined soft semi-open sets and soft semi-continuous functions. The authors of (10) introduced the concept of soft β-open subsets.

However, this work introduces a novel kind of soft open sets (respectively soft functions) called $\tilde{s}\beta_s$- open sets (resp.; $\tilde{s}\beta_s$-continuous functions), which are strictly placed between the soft classes of $\tilde{s}\beta_c$- open sets and $\tilde{s}\beta$- open sets (resp.; $\tilde{s}\beta_c$-continuous functions and soft β-continuous functions). Some of $\tilde{s}\beta_s$-continuous functions basic properties and relationships with some other types of soft functions are given.

Throughout this work, $\mathfrak{X}$ will be a nonempty initial universal set and $\mathcal{L}$ will be a set of parameters. $(\tilde{\mathfrak{X}}, \tilde{\mathfrak{T}}, \mathcal{L})$ and $(\tilde{y}, \tilde{\mu}, \mathrm{N})$ or basically $\tilde{\mathfrak{X}}$ and $\tilde{y}$ show soft topological spaces (simply, STS), the family of all soft clopen sets (resp.; semi closed) in $\tilde{\mathfrak{X}}$ is denoted by $\tilde{s}CO(\tilde{\mathfrak{X}})$(resp.; $\tilde{s}sC(\tilde{\mathfrak{X}})$). A pair $(F, \mathcal{L})$ is known as a soft set over $\mathfrak{X}$, in which F is a function Therefore $F.\mathcal{L} \to P(\mathfrak{X})$. The collection of soft sets $(F, \mathcal{L})$ over a universal set $\mathfrak{X}$ with the parameter set $\mathcal{L}$ is indicated by $\tilde{s}\mathrm{P}(\tilde{\mathfrak{X}})_{\mathcal{L}}$.

2.Preliminaries

2.1. Definition (11) For any two soft subsets $(F, \mathcal{L})$ and $(G, \mathcal{L})$ over a common universe $\mathfrak{X}$, we say that $(F, \mathcal{L})$ is a soft subset of $(G, \mathcal{L})$, indicated as $(F, \mathcal{L}) \tilde{\sqsubseteq} (G, \mathcal{L})$, if $F(\ell) \sqsubseteq G(\ell) \cdot \forall \ell \in \mathcal{L}$.

2.2. Definition (12) The soft complement of a soft set $(F, \mathcal{L})$ is indicated by $(F, \mathcal{L})^c$ or $\tilde{\mathfrak{X}} \backslash (F, \mathcal{L})$ and is described as $(F, \mathcal{L})^c = (F^c, \mathcal{L})$ in which $F^c.\mathcal{L} \to P(\mathfrak{X})$ is a function Provided by $F^c(\ell) = \mathfrak{X} \backslash F(\ell), \forall \ell \in \mathcal{L}$.

2.3. Definition (13) a soft set $(F, \mathcal{L})$ is said to be.

1. A null soft set indicated by $\tilde{\emptyset}$ if $\forall \ell \in \mathcal{L}.$, $F(\ell) = \emptyset$.
2. An absolute soft set indicated by $\tilde{\mathfrak{X}}$ if $\forall \ell \in \mathcal{L}, F(\ell) = \mathfrak{X}$.
3. A soft point indicated by $\mathfrak{x}_\ell$, if

$$F(\ell_*) = \begin{cases} \mathfrak{x} & \ell_* = \ell \\ \emptyset & \ell_* \in \mathcal{L} \backslash \{\ell\} \end{cases}$$

We say that $\mathfrak{x}_\ell \in (F, \mathcal{L})$ if $\mathfrak{x} \in F(\ell)$.

2.4. Definition (13) Let $(F, \mathcal{L})$ and $(G, \mathcal{L})$ be soft sets.

1. $(H, \mathcal{L}) = (F, \mathcal{L}) \tilde{\sqcup} (G, \mathcal{L})\, where, H(\ell) = F(\ell) \sqcup G(\ell) \cdot \forall \ell \in \mathcal{L}$.
2. $(H, \mathcal{L}) = (F, \mathcal{L}) \tilde{\sqcap} (G, \mathcal{L})\, where, H(\ell) = F(\ell) \sqcap G(\ell) \cdot \forall \ell \in \mathcal{L}$.

2.5. Definition (14) Let $\tilde{\mathfrak{T}} \tilde{\sqsubseteq} \tilde{s}P(\tilde{\mathfrak{X}})_{\mathcal{L}}$, then $\tilde{\mathfrak{T}}$ is described as a soft topology on $\tilde{\mathfrak{X}}$ if

1. $\tilde{\emptyset}, \tilde{\mathfrak{X}} \tilde{\in} \tilde{\mathfrak{T}}$.
2. $\forall\, (F, \mathcal{L}), (G, \mathcal{L}) \tilde{\in} \tilde{\mathfrak{T}}$, then$(F, \mathcal{L}) \tilde{\sqcap} (G, \mathcal{L}) \tilde{\in} \tilde{\mathfrak{T}}$.
3. If $(F_\lambda, \mathcal{L}) \tilde{\in} \tilde{\mathfrak{T}}, \forall \lambda \in \Lambda$, then $\tilde{\sqcup}_{\lambda \in \Lambda} (F_\lambda, \mathcal{L}) \tilde{\in} \tilde{\mathfrak{T}}$.

The triple $(\tilde{\mathfrak{X}}, \tilde{\mathfrak{T}}, \mathcal{L}))$ (simply, $\tilde{\mathfrak{X}}$) is known as a STS over $\tilde{\mathfrak{X}}$. $(F, \mathcal{L})$ is soft open set if $(F, \mathcal{L}) \tilde{\in} \tilde{\mathfrak{T}}$. $(G, \mathcal{L})$ is soft closed set if $(G, \mathcal{L}) \tilde{\in} \tilde{\mathfrak{T}}^c$.

The triple $(\tilde{W}, \tilde{\mathfrak{T}}_{\tilde{W}}, \mathcal{L})$is a soft subspace of $(\tilde{\mathfrak{X}}, \tilde{\mathfrak{T}}, \mathcal{L})$ where $\mathrm{W} \sqsubseteq \mathfrak{X}$, $\tilde{\mathfrak{T}}_W = \{F_{\mathrm{W}}, \mathcal{L}) = \tilde{W} \tilde{\sqcap} (F, \mathcal{L}) . (F, \mathcal{L}) \in \tilde{\mathfrak{T}}\}$ is known as "the soft relative topology" on $\tilde{W}$, and $F_W(\ell) = \tilde{W} \tilde{\sqcap} F(\ell), \forall \ell \in \mathcal{L}$.

2.6. Definition (14) Let $\tilde{\mu}$ be a collection of soft subsets over a $\tilde{\mathfrak{X}}$ with a fixed set of parameters $\mathcal{L}$.Then $\tilde{\mu}$ is described as a soft supra topology on $\tilde{\mathfrak{X}}$ with a fixed set of $\mathcal{L}$ if

1. $\tilde{\emptyset}, \tilde{X} \tilde{\in} \tilde{\mu}$.
2. The union of any number of soft sets in $\tilde{\mu}$ belonges to $\tilde{\mu}$.

The triple $(\tilde{\mathfrak{X}}, \tilde{\mu}, \mathcal{L})$ is called supra soft topological space and the elements of $\tilde{\mu}$ are called supra open sets. The soft complement of any soft supra open set is called supra soft closed.

2.7.Definition $\tilde{s}int(F, \mathcal{L})$ is a soft interior of the soft subset $(F, \mathcal{L})$ and is defined as follows. $\tilde{s}int(F, \mathcal{L}) = \tilde{\sqcup} \{(G, \mathcal{L}). (G, \mathcal{L}) \tilde{\sqsubseteq} (F, \mathcal{L}), (G, \mathcal{L}) \tilde{\in} \tilde{\mathfrak{T}}\}$.

2.8. Definition $\tilde{s}cl(F,\mathcal{L})$ is a soft closure of the soft subset $(F,\mathcal{L})$ and is defined as follows. $\tilde{s}cl(F,\mathcal{L}) = \tilde{\sqcap}\{(G,\mathcal{L}).(F,\mathcal{L}) \tilde{\sqsubseteq} (G,\mathcal{L}),(G,\mathcal{L}) \tilde{\in} \tilde{T}^c\}$.

2.9. Definition (15) A soft subset $(F,\mathcal{L})$ of a STS $(\tilde{X},\tilde{T},\mathcal{L})$ become a soft pre-open (resp., soft semi-open ,soft α-open, soft β-open, soft b-open ,and soft regular-open) if $(F,\mathcal{L}) \sqsubseteq \tilde{s}int(\tilde{s}cl(F,\mathcal{L}))$(resp.,$(F,\mathcal{L}) \sqsubseteq \tilde{s}cl(\tilde{s}int(F,\mathcal{L}))$, $(F,\mathcal{L}) \sqsubseteq \tilde{s}int\left(\tilde{s}cl\left(\tilde{s}int(F,\mathcal{L})\right)\right)$, $(F,\mathcal{L}) \sqsubseteq \tilde{s}cl\left(\tilde{s}int\left(\tilde{s}cl(F,\mathcal{L})\right)\right)$, $(F,\mathcal{L}) \sqsubseteq \tilde{s}int(\tilde{s}cl(F,\mathcal{L})) \tilde{\sqcup} \tilde{s}cl(\tilde{s}int(F,\mathcal{L}))$, and $(F,\mathcal{L}) = \tilde{s}int(\tilde{s}cl(F,\mathcal{L}))$).The family of all soft pre-open (resp., soft semi-open ,soft α-open, soft β-open, soft b-open, and soft regular-open) subsets of $\tilde{X}$ is indicated by $\tilde{s}PO(\tilde{X})$(resp., $\tilde{s}SO(\tilde{X}), \tilde{s}\alpha O(\tilde{X}), \tilde{s}\beta O(\tilde{X}), \tilde{s}bO(\tilde{X})$, and $\tilde{s}RO(\tilde{X})$).

2.10. Definition A soft subset $(F,\mathcal{L})$ of STS $(\tilde{X},\tilde{T},\mathcal{L})$ is recognized as a soft S_c-open (13) (resp., soft ξ open (14), soft δ-open (14),soft P_c-open (15), soft β_c-open (16) and soft b_c-open (17)) set, if $(F,\mathcal{L}) \tilde{\in} \tilde{s}SO(\tilde{X})$ (resp., $\tilde{T}, \tilde{s}P(\tilde{X})_{\mathcal{L}}$, $\tilde{s}PO(\tilde{X}), \tilde{s}\beta O(\tilde{X})$, and $\tilde{s}bO(\tilde{X})$) and$\forall$ $x_\alpha \tilde{\in} (F,\mathcal{L})$, $\exists$ a soft closed (resp., soft semi-closed, soft regular open, soft closed, soft closed and soft closed) subset $(K,\mathcal{L})$ of $\tilde{X}$ s.t. $x_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} (F,\mathcal{L})$. The family of all soft S_c-open (resp., soft ξ-open, soft δ-open ,soft P_c-open, soft β_c-open and soft b_c-open) subsets of $\tilde{X}$ is indicated by $\tilde{s}S_cO(\tilde{X})$ (resp., $\tilde{s}\xi O(\tilde{X}), \tilde{s}\delta O(\tilde{X}), \tilde{s}P_cO(\tilde{X}), \tilde{s}\beta_cO(\tilde{X})$ and $\tilde{s}b_cO(\tilde{X})$).

2.11. Definition (15,16) A STS $(\tilde{X},\tilde{T},\mathcal{L})$ is recognized as.

1. Soft extremally disconnected space (simply, SEDS), if $\tilde{s}cl(F,\mathcal{L}) \tilde{\in} \tilde{T}, \forall(F,\mathcal{L}) \tilde{\in} \tilde{T}$.
2. Soft locally indiscrete, if $(F,\mathcal{L}) \tilde{\in} \tilde{T}$,then $(F,\mathcal{L}) \tilde{\in} \tilde{T}^c$.
3. Soft T_1-space , if $x_\alpha, x_\varepsilon \tilde{\in} \tilde{s}P(\tilde{X})$ such that $x_\alpha \neq x_\varepsilon$, there are soft open sets $(F_1,\mathcal{L})$ and $(F_2,\mathcal{L})$ s.t $x_\alpha \tilde{\in} (F_1,\mathcal{L}), x_\varepsilon \tilde{\notin} (F_1,\mathcal{L})$ and $x_\varepsilon \tilde{\epsilon} (F_2,\mathcal{L}), x_\alpha \tilde{\notin} (F_2,\mathcal{L})$.

2.12. Definition (15) Let $(\tilde{X},\tilde{T},\mathcal{L})$ be a STS. The soft θ-interior of a soft subset $(F,\mathcal{L}) \in \tilde{s}P(\tilde{X})_{\mathcal{L}}$ is the union of all soft open sets over $\tilde{X}$ whose soft closures are located within $(F,\mathcal{L})$, and is indicated by $\tilde{s}\theta int(F,\mathcal{L})$. The soft subset $(F,\mathcal{L})$ is known as soft θ- open if $\tilde{s}\theta int(F,\mathcal{L}) = (F,\mathcal{L})$. The complement of a soft θ-open set is called soft θ-closed.

2.13. Definition (14) Suppose that $\tilde{s}P(\tilde{X})_E$ and $\tilde{s}P(\tilde{y})_K$ be soft categories. Suppose $u.X \to y$ and $p.E \to K$ be functions. Next, a soft function $f_{pu}.\tilde{s}P(\tilde{X})_E \to \tilde{s}P(\tilde{y})_K$ is described as the following.

1. For a soft set $(F,\mathcal{L})$ in $\tilde{s}P(\tilde{X})_E$, $(f_{pu}(F,\mathcal{L}),N)$, $N = p(\mathcal{L}) \sqsubseteq K$ is a soft set in $\tilde{s}P(\tilde{y})_K$ provided by

$$f_{pu}(F,\mathcal{L})(\beta) = \begin{cases} u\left(\cup_{\alpha\in p^{-1}(\beta)\cap\mathcal{L}}\left(F(\alpha)\right)\right) & ,p^{-1}(\beta)\cap\mathcal{L} \neq \emptyset \\ \emptyset & otherwise \end{cases}$$

For $\beta \in N \sqsubseteq K$. $f_{pu}((F,\mathcal{L}),N)$ is called a soft image of a soft set $(F,\mathcal{L})$. If $N = K$, Afterward, we'll write $(f_{pu}(F,\mathcal{L}),K)$ as $f_{pu}(F,\mathcal{L})$.

2. For a soft set (G,C) in $\tilde{s}P(\tilde{y})_K$, where $C \sqsubseteq K$. $(f_{pu}^{-1}(G,C),D)$, $D = p^{-1}(C)$ is a soft set in $\tilde{s}P(\tilde{X})_E$ provided by

$$f_{pu}^{-1}(G,C)(\alpha) = \begin{cases} u^{-1}\left(G\left(p(\alpha)\right)\right) & p(\alpha) \in C \\ \emptyset & otherwise \end{cases}$$

for $\alpha \in D \sqsubseteq E$,$(f_{pu}^{-1}(G,C),D)$ is known as a soft inverse image of a soft set (G,C). We'll write $(f_{pu}^{-1}(G,C),E)$ as $f_{pu}^{-1}(G,C)$.

The soft function f_{pu} is called surjective if p and u are surjective. The soft function f_{pu} is called injective if p and u are injective functions.

2.14. Definition (15) Suppose that $\mathfrak{f}_{pu}.\left(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L}\right) \to (\tilde{\mathrm{y}},\tilde{\mu},\mathrm{N})$ be a soft mapping and $W \sqsubseteq \mathrm{X}$. Then the restriction of $\mathfrak{f}_{pu}$to $\tilde{s}P(\widetilde{W})_{\mathcal{L}}$ is the soft mapping $\mathfrak{f}_{pu}|\tilde{s}P(\widetilde{W})_{\mathcal{L}}$ from $\tilde{s}P(\widetilde{W})_{\mathcal{L}}$to $\tilde{s}P(\widetilde{\mathrm{y}})_{\mathrm{N}}$ which is defined by the functions $u|W.W \to \mathrm{y}$ and $p.\mathcal{L} \to \mathrm{N}$ where $u|W$ is the restriction of u to W.

2.15. Definition (18)A soft function $\tilde{f}_{pu}.\left(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L}\right) \to (\tilde{\mathrm{y}},\tilde{\mu},\mathrm{N})$ is known as.

1. Soft continuous, if $\tilde{f}_{pu}^{-1}(G,\mathrm{N}) \tilde{\in} \tilde{\mathrm{T}}, \forall (G,\mathrm{N}) \tilde{\in} \tilde{\mu}$.
2. Soft perfectly continuous, if $\tilde{f}_{pu}^{-1}(G,\mathrm{N})$ is a soft clopen set in $\tilde{\mathrm{X}}, \forall (G,\mathrm{N}) \tilde{\in} \tilde{\mu}$.
3. Soft RC-continuous, if $\tilde{f}_{pu}^{-1}(G,\mathrm{N}) \tilde{\in} \tilde{s}RC(\tilde{\mathrm{X}}), \forall (G,\mathrm{N}) \tilde{\in} \tilde{\mu}$.
4. Soft regular continuous, if $\tilde{f}_{pu}^{-1}(G,\mathrm{N}) \tilde{\in} \tilde{s}RO\left(\tilde{\mathrm{X}}\right), \forall (G,\mathrm{N}) \tilde{\in} \tilde{\mu}$.
5. $\tilde{s}S_c$-continuous, if $\tilde{f}_{pu}^{-1}(G,\mathrm{N}) \tilde{\in} \tilde{s}S_cO(\tilde{\mathrm{X}}), \forall (G,\mathrm{N}) \tilde{\in} \tilde{\mu}$.
6. Soft β_c-continuous, if $\tilde{f}_{pu}^{-1}(G,\mathrm{N}) \tilde{\in} \tilde{s}\beta_cO(\tilde{\mathrm{X}}), \forall (G,\mathrm{N}) \tilde{\in} \tilde{\mu}$.
7. Soft ξ-continuous, if $\tilde{f}_{pu}^{-1}(G,\mathrm{N}) \tilde{\in} \tilde{s}\xi O(\tilde{\mathrm{X}}), \forall (G,\mathrm{N}) \tilde{\in} \tilde{\mu}$.
8. $\tilde{s}p_c$-continuous, if $\tilde{f}_{pu}^{-1}(G,\mathrm{N}) \tilde{\in} \tilde{s}P_cO(\tilde{\mathrm{X}}), \forall (G,\mathrm{N}) \tilde{\in} \tilde{\mu}$.
9. Soft irresolute, $\tilde{f}_{pu}^{-1}(G,\mathrm{N}) \tilde{\in} \tilde{s}SO(\tilde{\mathrm{X}}), \forall (G,\mathrm{N}) \tilde{\in} \tilde{s}SO(\tilde{\mathrm{y}})$.
10. Soft open. if $\tilde{f}_{pu}(F,\mathcal{L}) \tilde{\in} \tilde{\mu}, \forall (F,\mathcal{L}) \tilde{\in} \tilde{\mathrm{T}}$.

2.16. Proposition (16) A STS $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ is T_1-space iff $\forall\, \tilde{\mathrm{x}}_\ell \tilde{\in} \tilde{s}P(\tilde{\mathrm{X}})$ is soft closed set.

2.17. Proposition (16) Let $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ be a STS, then $\tilde{s}\beta O(\tilde{\mathrm{X}})$ forms a soft supra topology.

2.18. Proposition (19) If $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ be a SEDS, then

1. $\tilde{s}\beta O\left(\tilde{\mathrm{X}}\right) \tilde{\sqsubseteq} \tilde{s}pO\left(\tilde{\mathrm{X}}\right)$.
2. $\tilde{s}RO\left(\tilde{\mathrm{X}}\right) \tilde{\sqsubseteq} \tilde{s}C\left(\tilde{\mathrm{X}}\right)$

2.19. Proposition (10) If $(F,\mathcal{L}) \tilde{\in} \tilde{\mathrm{T}}$ and $(K,\mathcal{L}) \tilde{\in} \tilde{s}\beta O\left(\tilde{\mathrm{X}}\right)$ in a STS $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$, then $(F,\mathcal{L}) \tilde{\sqcap} (K,\mathcal{L}) \tilde{\in} \tilde{s}\beta O\left(\tilde{\mathrm{X}}\right)$.

2.20. Proposition (14) Let $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ be a STS, $\mathrm{x}_\ell \tilde{\in} \tilde{s}\xi O\left(\tilde{\mathrm{X}}\right)$ iff $\mathrm{x}_\ell \tilde{\in} \tilde{s}RO\left(\tilde{\mathrm{X}}\right)$.

2.21. Proposition (15) Let $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ be a soft regular space iff $\forall\ \mathrm{x}_\alpha \in \tilde{\mathrm{X}}$ and $\forall\ \mathrm{x}_\alpha \tilde{\in} (F,\mathcal{L}), \exists\, (G,\mathcal{L}) \tilde{\in} \tilde{\mathrm{T}}$ s.t. $\mathrm{x}_\alpha \tilde{\in} (G,\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}cl(G,\mathcal{L}) \tilde{\sqsubseteq} (F,\mathcal{L})$.

2.22. Proposition (18) Let $(\widetilde{W},\tilde{\mathrm{T}}_W,\mathcal{L})$ be a soft subspace of $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ and $(F,\mathcal{L}) \tilde{\sqsubseteq} \widetilde{W}$ then, $\tilde{s}int_{\widetilde{W}}(F,\mathcal{L}) = \tilde{s}int(F,\mathcal{L})$ if $\widetilde{W} \tilde{\in} \tilde{\mathrm{T}}$.

2.23. Proposition (16) Let $(\widetilde{W},\tilde{\mathrm{T}}_W,\mathcal{L})$ be a soft subspace of $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ and $(F,\mathcal{L}) \tilde{\sqsubseteq} \widetilde{W}$ s.t. $\widetilde{W} \tilde{\in} \tilde{s}\beta O\left(\tilde{\mathrm{X}}\right)$. If $(F,\mathcal{L}) \tilde{\in} \tilde{s}\beta O\left(\tilde{\mathrm{X}}\right)$, then $(F,\mathcal{L}) \tilde{\in} \tilde{s}\beta O\left(\widetilde{W}\right)$.

3. $\tilde{s}\boldsymbol{\beta}_s$- Open Sets.

In this section, we study a new type of soft sets called $\tilde{s}\beta_s$- open set in soft topological spaces. Some properties of this type of soft set are given, and the relationships with a few other kinds of soft sets are introduced.

3.1. Definition A soft set $(F,\mathcal{L})$ of STS $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ is called $\tilde{s}\beta_s$-open if $(F,\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\tilde{\mathrm{X}})$ $\forall\, \mathrm{x}_\alpha \tilde{\in} (F,\mathcal{L}), \exists\, (K,\mathcal{L}) \tilde{\in} \tilde{s}s0C(\tilde{\mathrm{X}})$ s.t. $\mathrm{x}_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} (F,\mathcal{L})$.The family of all $\tilde{s}\beta_s$-open subsets of $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ is indicated by $\tilde{s}\beta_sO(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ or $\tilde{s}\beta_sO(\tilde{\mathrm{X}})$.

3.2. Proposition A soft set $(F,\mathcal{L})$ of STS $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ is $\tilde{s}\beta_s$-open iff $(F,\mathcal{L}) = \sqcup\left(K_{\lambda,}\mathcal{L}\right)$ where $(F,\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\tilde{\mathrm{X}})$ and $(K_\lambda,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{\mathrm{X}})\ \forall\, \lambda \tilde{\in} \Lambda$.

Proof: Obvious.

Corollary A soft set $(F,\mathcal{L})$ of STS $(\tilde{\mathrm{X}},\tilde{\mathrm{T}},\mathcal{L})$ is $\tilde{s}\beta_s$-open. If $(F,\mathcal{L}) \tilde{\in} \tilde{s}\beta O\left(\tilde{\mathrm{X}}\right) \tilde{\sqcap} \tilde{s}sC\left(\tilde{\mathrm{X}}\right)$.

Proof: directly from definition 3.1.

Remark

1- $\tilde{s}\beta_s O(Ҳ) \tilde{\sqsubseteq} \tilde{s}\beta O(Ҳ)$.But $\tilde{s}\beta O(Ҳ) \tilde{\sqsubseteq} \tilde{s}\beta_s O(Ҳ)$ is not true in general.

2- $\tilde{s}sC(Ҳ)$is incomparable with $\tilde{s}\beta_s O(Ҳ)$.

3- $\widetilde{\mathcal{T}}$ is incomparable with $\tilde{s}\beta_s O(Ҳ)$.

4- $\widetilde{\mathcal{T}}^c$ is incomparable with $\tilde{s}\beta_s O(Ҳ)$.

The examples that follow demonstrate the earlier point.

Example Consider $Ҳ = \{x_1, x_2\}$, and $\mathcal{L} = \{\ell_1, \ell_2\}$,with the two soft topological $\widetilde{\mathcal{T}}_1 = \{\widetilde{\emptyset}, \widetilde{Ҳ}, (\mathcal{F}_1,\mathcal{L}), (\mathcal{F}_3,\mathcal{L})\}$ and $\widetilde{\mathcal{T}}_2 = \{\widetilde{\emptyset}, \widetilde{Ҳ}, (\mathcal{F}_1,\mathcal{L}), (\mathcal{F}_5,\mathcal{L}), (\mathcal{F}_8,\mathcal{L})\}$ where $\widetilde{\emptyset} = \{(\ell_1, \emptyset), (\ell_2, \emptyset)\}, \widetilde{Ҳ} = \{(\ell_1, \{(x_1, x_2), (\ell_2, \{(x_1, x_2)\}, (\mathcal{F}_1,\mathcal{L}) = \{(\ell_1, \{x_1\}), (\ell_2, \emptyset)\}, (\mathcal{F}_2,\mathcal{L}) = \{(\ell_1, \{x_2\}), (\ell_2, \emptyset)\}, (\mathcal{F}_3,\mathcal{L}) = \{(\ell_1, \{x_1, x_2\}), (\ell_2, \emptyset)\}, (\mathcal{F}_4,\mathcal{L}) = \{(\ell_1, \emptyset), (\ell_2, \{x_1\})\}, (\mathcal{F}_5,\mathcal{L}) = \{(\ell_1, \emptyset), (\ell_2, \{x_2\})\}, (\mathcal{F}_6,\mathcal{L}) = \{(\ell_1, \emptyset), (\ell_2, \{x_1, x_2\})\}, (\mathcal{F}_7,\mathcal{L}) = \{(\ell_1, \{x_1\}), (\ell_2, \{x_1\})\}, (\mathcal{F}_8,\mathcal{L}) = \{(\ell_1, \{x_1\}), (\ell_2, \{x_2\})\}, (\mathcal{F}_9,\mathcal{L}) = \{(\ell_1, \{x_1\}), (\ell_2, \{x_1, x_2\})\}, (\mathcal{F}_{10},\mathcal{L}) = \{(\ell_1, \{x_2\}), (\ell_2, \{x_1\})\}, (\mathcal{F}_{11},\mathcal{L}) = \{(\ell_1, \{x_2\}), (\ell_2, \{x_2\})\}, (\mathcal{F}_{12},\mathcal{L}) = \{(\ell_1, \{x_2\}), (\ell_2, \{x_1, x_2\})\}, (\mathcal{F}_{13},\mathcal{L}) = \{(\ell_1, \{x_1, x_2\}), (\ell_2, \{x_1\})\}, (\mathcal{F}_{14},\mathcal{L}) = \{(\ell_1, \{x_1, x_2\}), (\ell_2, \{x_2\})\}$ are all soft subset over $\widetilde{Ҳ}$.

$(\widetilde{Ҳ}, \widetilde{\mathcal{T}_1}, \mathcal{L})$ and $(\widetilde{Ҳ}, \widetilde{\mathcal{T}_2}, \mathcal{L})$ are two STSs over $Ҳ$.

1- $(\mathcal{F}_1,\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\widetilde{Ҳ}, \widetilde{\mathcal{T}}_1, \mathcal{L})$. But $(\mathcal{F}_1,\mathcal{L}) \tilde{\notin} \tilde{s}\beta_s O(\widetilde{Ҳ}, \widetilde{\mathcal{T}}_1, \mathcal{L})$.

2- $(\mathcal{F}_{10},\mathcal{L}) \tilde{\in} \tilde{s}sC(\widetilde{Ҳ}, \widetilde{\mathcal{T}}_2, \mathcal{L})$. But $(\mathcal{F}_{10},\mathcal{L}) \tilde{\notin} \tilde{s}\beta_s O(\widetilde{Ҳ}, \widetilde{\mathcal{T}}_2, \mathcal{L})$.
While $(\mathcal{F}_8,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\widetilde{Ҳ}, \widetilde{\mathcal{T}}_2, \mathcal{L})$. But $(\mathcal{F}_8,\mathcal{L}) \tilde{\notin} \tilde{s}sC(\widetilde{Ҳ}, \widetilde{\mathcal{T}}_2, \mathcal{L})$.

3- $(\mathcal{F}_1,\mathcal{L}) \tilde{\in} \widetilde{\mathcal{T}}_1$. But $(\mathcal{F}_1,\mathcal{L}) \tilde{\notin} \tilde{s}\beta_s O(\widetilde{Ҳ}, \widetilde{\mathcal{T}}_1, \mathcal{L})$. While
$(\mathcal{F}_3,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\widetilde{Ҳ}, \widetilde{\mathcal{T}}_2, \mathcal{L})$. But $(\mathcal{F}_3,\mathcal{L}) \tilde{\notin} \widetilde{\mathcal{T}}_2$.

4- $(\mathcal{F}_{10},\mathcal{L}) \tilde{\in} \widetilde{\mathcal{T}}_2{}^c$. But $(\mathcal{F}_{10},\mathcal{L}) \tilde{\notin} \tilde{s}\beta_s O(\widetilde{Ҳ}, \widetilde{\mathcal{T}}_2, \mathcal{L})$. While
$(\mathcal{F}_3,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\widetilde{Ҳ}, \widetilde{\mathcal{T}}_2, \mathcal{L})$. But $(\mathcal{F}_3,\mathcal{L}) \tilde{\notin} \widetilde{\mathcal{T}}_2{}^c$.

The following diagram shows the relations between $\tilde{s}\beta O(Ҳ)$, $\tilde{s}s_c O(Ҳ)$, $\tilde{s}p_c O(Ҳ)$, $\tilde{s}b_c O(Ҳ)$, $\tilde{s}\beta_c O(Ҳ)$ *and* $\tilde{s}\beta_s O(Ҳ)$

$$\begin{array}{lllllll} \tilde{s}s_c O(Ҳ) & & & & & & \\ & \searrow & & & & & \\ & & \tilde{s}b_c O(Ҳ) \rightarrow & \tilde{s}\beta_c O(Ҳ) \rightarrow & \tilde{s}\beta_s O(Ҳ) \rightarrow & \tilde{s}\beta O(Ҳ) \\ & \nearrow & & & & & \\ \tilde{s}p_c O(Ҳ) & & & & & & \end{array}$$

3.3. Proposition For any soft set $(\mathcal{F},\mathcal{L})$ of STS $(\widetilde{Ҳ}, \widetilde{\mathcal{T}}, \mathcal{L})$ the folowing statetment are hold.

1. $(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\widetilde{Ҳ})$, if $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}s_c O(\widetilde{Ҳ})$.
2. $(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\widetilde{Ҳ})$, if $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}p_c O(\widetilde{Ҳ})$.
3. $(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\widetilde{Ҳ})$, if $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}b_c O(\widetilde{Ҳ})$.
4. $(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\widetilde{Ҳ})$, if $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\beta_c O(\widetilde{Ҳ})$.
5. $(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\widetilde{Ҳ})$, if $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}RO(\widetilde{Ҳ})$.
6. $(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\widetilde{Ҳ})$, if $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}RC(\widetilde{Ҳ})$.
7. $(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\widetilde{Ҳ})$, if $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\xi O(\widetilde{Ҳ})$.
8. $(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\widetilde{Ҳ})$, if $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\theta O(\widetilde{Ҳ})$.
9. $(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\widetilde{Ҳ})$, if $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\delta O(\widetilde{Ҳ})$.

10. $(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$, if $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}CO(\tilde{\mathfrak{X}})$.

The following example illustrates above Proposition.

Example Consider the STS $(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L})$ in Example, let $(\mathcal{F}_{1,}\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L})$ which is not in $\tilde{s}s_c O(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L}), \tilde{s}p_c O(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L}), \tilde{s}b_c O(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L})$, and $\tilde{s}\beta_c O(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L})$. Also $(\mathcal{F}_{3,}\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L})$, but $(\mathcal{F}_{3,}\mathcal{L}) \tilde{\notin} \tilde{s}RO(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L}), \tilde{s}\xi O(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L}), \tilde{s}\delta O(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L})$ and $\tilde{s}CO(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}}_2,\mathcal{L})$.

3.4.Proposition If a space $(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L})$ is a soft T_1-space, subsequently $\tilde{s}\beta_s O(\tilde{\mathfrak{X}}) = \tilde{s}\beta_c O(\tilde{\mathfrak{X}}) = \tilde{s}\beta O(\tilde{\mathfrak{X}})$.

Proof: $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\tilde{\mathfrak{X}})$. If $(\mathcal{F},\mathcal{L}) = \tilde{\emptyset}$, then $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$. If $(\mathcal{F},\mathcal{L}) \neq \tilde{\emptyset}$, then for each $\mathfrak{x}_\alpha \tilde{\in} (\mathcal{F},\mathcal{L})$, then by Proposition 2.13 $\{\mathfrak{x}_\alpha\} \tilde{\in} \tilde{\mathfrak{T}}^c$. Hence, $\mathfrak{x}_\alpha \tilde{\in} \{\mathfrak{x}_\alpha\} \tilde{\sqsubseteq} (\mathcal{F},\mathcal{L})$. Therefore, $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\beta_c O(\tilde{\mathfrak{X}})$.since $\tilde{\mathfrak{T}}^c \tilde{\sqsubseteq} \tilde{s}sC(\tilde{\mathfrak{X}})$then $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$. Hence $\tilde{s}\beta O(\tilde{\mathfrak{X}}) \tilde{\sqsubseteq} \tilde{s}\beta_c O(\tilde{\mathfrak{X}}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$, but $\tilde{s}\beta_c O(\tilde{\mathfrak{X}}) \tilde{\sqsubseteq} \tilde{s}\beta O(\tilde{\mathfrak{X}})$ and $\tilde{s}\beta_s O(\tilde{\mathfrak{X}}) \tilde{\sqsubseteq} \tilde{s}\beta O(\tilde{\mathfrak{X}})$ generally, therefore $\tilde{s}\beta_s O(\tilde{\mathfrak{X}}) = \tilde{s}\beta_c O(\tilde{\mathfrak{X}}) = \tilde{s}\beta O(\tilde{\mathfrak{X}})$.

Corollary If $(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L})$is soft T_1-space, then $\tilde{\mathfrak{T}} \tilde{\sqsubseteq} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$and $\tilde{s}\alpha O(\tilde{\mathfrak{X}}) \tilde{\sqsubseteq} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$.

The following findings demonstrate that any union of $\tilde{s}\beta_s$-open sets of $(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L})$ is $\tilde{s}\beta_s -$open.

3.5. Proposition Arbitrary soft union of soft β_s-open sets in a STS $(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L})$ is a soft β_s-open set.

Proof: Let $\{(\mathcal{F}_\lambda,\mathcal{L}).\lambda \in \Lambda\}$ be a family of soft β_s-open sets in STS $(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L})$.Then $\{(\mathcal{F}_\lambda,\mathcal{L}).\lambda \in \Lambda\} \tilde{\in} \tilde{s}\beta O(\tilde{\mathfrak{X}})$ and by Proposition 2.14, $\tilde{\sqcup}\{(\mathcal{F}_\lambda,\mathcal{L}).\lambda \in \Lambda\} \tilde{\in} \tilde{s}\beta O(\tilde{\mathfrak{X}})$. Let $\mathfrak{x}_\alpha \tilde{\in} \tilde{\sqcup}\{(\mathcal{F}_\lambda,\mathcal{L}).\lambda \in \Lambda\}$, hence $\mathfrak{x}_\alpha \in (\mathcal{F}_\lambda,\mathcal{L})$ for some $\lambda \in \Lambda$. Since $(\mathcal{F}_\lambda,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})\ \forall\lambda$, so there is $(K,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{\mathfrak{X}})$ s.t. $\mathfrak{x}_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} (\mathcal{F}_\lambda,\mathcal{L}) \tilde{\sqsubseteq} \tilde{\sqcup}\{(\mathcal{F}_\lambda,\mathcal{L}).\lambda \in \Lambda\}$, so $\mathfrak{x}_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} \tilde{\sqcup}\{(\mathcal{F}_\lambda,\mathcal{L}).\lambda \in \Lambda\}$. Therefore, $\tilde{\sqcup}\{(\mathcal{F}_\lambda,\mathcal{L}).\lambda \in \Lambda\} \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$.

The soft intersection of two soft β_s-open sets need not be a soft β_s-open sets in general as an exam:

Example If $(\mathcal{F},\mathcal{L}),(G,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$, then $(\mathcal{F},\mathcal{L}) \cap (G,\mathcal{L})$ need not be $\tilde{s}\beta_s$ open set. If we consider the STS $(\tilde{\mathfrak{X}},\widetilde{\mathfrak{T}_2},\mathcal{L})$ in Example 3.5, let $(\mathcal{F}_{7,}\mathcal{L}),(\mathcal{F}_{12,}\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}},\widetilde{\mathfrak{T}_2},\mathcal{L})$. Now we have $(\mathcal{F}_{7,}\mathcal{L}) \sqcap (\mathcal{F}_{12,}\mathcal{L}) = (\mathcal{F}_{4,}\mathcal{L})$

It is clear that $(\mathcal{F}_{4,}\mathcal{L}) \tilde{\notin} \tilde{s}\beta_s O(\tilde{\mathfrak{X}},\widetilde{\mathfrak{T}_2},\mathcal{L})$ and hence $(\mathcal{F}_{7,}\mathcal{L}) \sqcap (\mathcal{F}_{12,}\mathcal{L}) \tilde{\notin} \tilde{s}\beta_s O(\tilde{\mathfrak{X}},\widetilde{\mathfrak{T}_2},\mathcal{L})$.

3.6. Proposition If $\tilde{s}\beta O(\tilde{\mathfrak{X}})$is a soft topology on $\tilde{\mathfrak{X}}$, then $\tilde{s}\beta_s O(\tilde{\mathfrak{X}})$ is also a soft topology on $\tilde{\mathfrak{X}}$.

Proof: This is sufficient evidence to demonstrate whether the intersection of two $\tilde{s}\beta_s -$open subsets is $\tilde{s}\beta_s -$open. Suppose $(\mathcal{F},\mathcal{L}),(G,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$, then $(\mathcal{F},\mathcal{L}),(G,\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\tilde{\mathfrak{X}})$. Since $\tilde{s}\beta O(\tilde{\mathfrak{X}})$ is a soft topology on $\tilde{\mathfrak{X}}$, so $(\mathcal{F},\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\tilde{\mathfrak{X}})$. Let $\mathfrak{x}_\alpha \tilde{\in} (\mathcal{F},\mathcal{L}) \sqcap (G,\mathcal{L})$, then $\mathfrak{x}_\alpha \tilde{\in} (\mathcal{F},\mathcal{L})$ and $\mathfrak{x}_\alpha \tilde{\in} (G,\mathcal{L})$, so there exist $(H,\mathcal{L}),(K,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{\mathfrak{X}})$. s.t. $\mathfrak{x}_\alpha \tilde{\in} (H,\mathcal{L}) \tilde{\sqsubseteq} (\mathcal{F},\mathcal{L})$ and $\mathfrak{x}_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} (G,\mathcal{L})$ which implies that $\mathfrak{x}_\alpha \tilde{\in} (H,\mathcal{L}) \sqcap (K,\mathcal{L}) \tilde{\sqsubseteq} (\mathcal{F},\mathcal{L}) \sqcap (G,\mathcal{L})$, then $(H,\mathcal{L}) \sqcap (K,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{\mathfrak{X}})$. Thus, $(\mathcal{F},\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$.

3.7. Proposition A STS $(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L})$ is SEDS if $\tilde{s}\beta_s O(\tilde{\mathfrak{X}}) \tilde{\sqsubseteq} PO(\tilde{\mathfrak{X}})$.

Proof: By Remark 3.4(1) and Proposition 2.18(1),the prove only if part will follow. Conversely, let $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{\mathfrak{T}}$. Then, $\tilde{s}cl(\mathcal{F},\mathcal{L}) = \tilde{s}cl(\tilde{s}int(\mathcal{F},\mathcal{L}))$. That is, $\tilde{s}cl(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}RC(\tilde{\mathfrak{X}})$, so by Proposition 3.6(6), $\tilde{s}cl(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$. By hypothesis, $\tilde{s}cl(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}PO(\tilde{\mathfrak{X}})$. That is,

$\tilde{s}cl(\digamma,\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}int\left(\tilde{s}cl\left(\tilde{s}cl(\digamma,\mathcal{L})\right)\right)$,Then $\tilde{s}cl(\digamma,\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}int(\tilde{s}cl(\digamma,\mathcal{L}))$,but $\tilde{s}int(\tilde{s}cl(\digamma,\mathcal{L})) \tilde{\sqsubseteq} \tilde{s}cl(\digamma,\mathcal{L})$. Hence, $\tilde{s}cl(\digamma,\mathcal{L}) = \tilde{s}int(\tilde{s}cl(\digamma,\mathcal{L}))$. This means that, $\tilde{s}cl(\digamma,\mathcal{L}) \tilde{\in} \tilde{T}$. Thus, $(\tilde{X},\tilde{T},\mathcal{L})$ is a SEDS.

3.8. Proposition A $(\tilde{X},\tilde{T},\mathcal{L})$ is SEDS iff $\tilde{s}\beta_s O(\tilde{X}) \tilde{\sqsubseteq} \tilde{s}\alpha O(\tilde{X})$.

3.9. Proposition If a space $(\tilde{X},\tilde{T},\mathcal{L})$ is a soft locally indiscrete space, then $\tilde{s}\beta_s O(\tilde{X}) = \tilde{s}\beta_c O(\tilde{X}) = \tilde{s}sO(\tilde{X})$.

Corollary Let $(\tilde{X},\tilde{T},\mathcal{L})$ is a soft locally indiscrete space, then.

1. $\tilde{s}\beta_s O(\tilde{X}) = \tilde{T}$.
2. $\tilde{s}\beta_s O(\tilde{X}) = \tilde{s}\alpha O(\tilde{X})$.

3.10. Proposition A subset $(\digamma,\mathcal{L})$ of $(\tilde{X},\tilde{T},\mathcal{L})$ is $\tilde{s}\beta_s$- open iff for each $x_\alpha \tilde{\in} (\digamma,\mathcal{L})$,$\exists$ $(K,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$ s.t. $x_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} (\digamma,\mathcal{L})$.

Proof: Assume that $(\digamma,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$. Then for each $x_\alpha \tilde{\in} (\digamma,\mathcal{L})$, take $(\digamma,\mathcal{L}) = (K,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$ containing x_α s.t. $x_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} (\digamma,\mathcal{L})$. Conversely, suppose that $\forall\, x_\alpha \tilde{\in} (\digamma,\mathcal{L})$, there exists $(K,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$ s.t. $x_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} (\digamma,\mathcal{L})$. Thus $(\digamma,\mathcal{L}) = \sqcup \{x_\alpha\} \tilde{\sqsubseteq} \sqcup (K,\mathcal{L}) \tilde{\sqsubseteq} (\digamma,\mathcal{L})$ for each $x_\alpha \tilde{\in} (\digamma,\mathcal{L})$, this implies that $(\digamma,\mathcal{L}) = \sqcup (K,\mathcal{L})$, therefore $(\digamma,\mathcal{L})$ is $\tilde{s}\beta_s$-open set.

3.11.Proposition If $(\tilde{X},\tilde{T},\mathcal{L})$ is soft regular space, then $\tilde{T} \tilde{\sqsubseteq} \tilde{s}\beta_s O(\tilde{X})$.

Proof: Let $(\digamma,\mathcal{L}) \tilde{\in} \tilde{T}$, then $(\digamma,\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\tilde{X})$. Since $\tilde{X}$ is regular, then by Proposition 2.21, for each $x_\alpha \tilde{\in} (\digamma,\mathcal{L})$,$\exists$ $(G,\mathcal{L}) \tilde{\in} \tilde{T}$ s.t. $x_\alpha \tilde{\in} (G,\mathcal{L}) \tilde{\sqsubseteq} \tilde{s}cl(G,\mathcal{L}) \tilde{\sqsubseteq} (\digamma,\mathcal{L})$. So that, $x_\alpha \tilde{\in} \tilde{s}cl(G,\mathcal{L}) \tilde{\sqsubseteq} (\digamma,\mathcal{L})$ .Since $\tilde{s}cl(G,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{X})$, then $(\digamma,A) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$.

3.12.Proposition Let $(\tilde{X},\tilde{T},\mathcal{L})$ be a SEDS. If $(\digamma,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$ and $(G,\mathcal{L}) \tilde{\in} \tilde{s}RO(\tilde{X})$. Then $(\digamma,\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$.

Proof: Let $(\digamma,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$ and $(G,\mathcal{L}) \tilde{\in} \tilde{s}RO(\tilde{X})$ then, $(\digamma,\mathcal{L}) \tilde{\in} \tilde{T}$ then by Proposition 2.19 $(\digamma,\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\tilde{X})$. If $x_\alpha \tilde{\in} (\digamma,\mathcal{L}) \sqcap (G,\mathcal{L})$ then, since $(\digamma,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$, $\exists (K,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{X})$ s.t. $x_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} (\digamma,\mathcal{L})$ and so, $x_\alpha \tilde{\in} (K,\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\sqsubseteq} (\digamma,\mathcal{L}) \sqcap (G,\mathcal{L})$. Since $(G,\mathcal{L}) \tilde{\in} \tilde{s}RO(\tilde{X})$ in a SEDS so by Proposition 2.18(2), $(G,\mathcal{L}) \tilde{\in} \tilde{s}C(\tilde{X})$ then it is soft semi-closed and hence $(K,\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{X})$. Therefore, $(\digamma,\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$.

3.13.Proposition Let $\tilde{X}$ be a SEDS. If $(\digamma,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$ and $(G,\mathcal{L}) \tilde{\in} \tilde{s}\xi O(\tilde{X})$. Then $(\digamma,\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$.

Proof: by Proposition 2.20 and Proposition 3.19.

3.14. Proposition Let $(\tilde{X},\tilde{T},\mathcal{L})$ be a STS and $(\digamma,\mathcal{L}),(G,\mathcal{L}) \tilde{\in} \tilde{s}P(\tilde{X})_{\mathcal{L}}$. If $(\digamma,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$ and $(G,\mathcal{L}) \tilde{\in} \tilde{s}CO(\tilde{X})$, then $(\digamma,\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$.

Proof: Let $(\digamma,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$ and $(G,\mathcal{L}) \tilde{\in} \tilde{s}CO(\tilde{X})$, then $(G,\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\tilde{X})$. Then by Proposition 2.19 $(\digamma,\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\tilde{X})$. After that, let's $x_\alpha \tilde{\in} (\digamma,\mathcal{L}) \sqcap (G,\mathcal{L})$, therefore $x_\alpha \tilde{\in} (\digamma,\mathcal{L})$ and $x_\alpha \tilde{\in} (G,\mathcal{L})$, so $\exists\, (K,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{X})$ such that $x_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} (\digamma,\mathcal{L})$. Since $(G,\mathcal{L}) \tilde{\in} \tilde{s}C(\tilde{X})$ then $(G,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{X})$ implying that $(K,\mathcal{L}) \cap (G,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{X})$. Therefore, $x_\alpha \tilde{\in} (K,\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\sqsubseteq} (\digamma,\mathcal{L}) \sqcap (G,\mathcal{L})$. Thus, $(\digamma,\mathcal{L}) \sqcap (G,\mathcal{L})$ is $\tilde{s}\beta_s O(\tilde{X})$.

Corollary Suppose $\tilde{X}$ be a STS, let $(\digamma,\mathcal{L}),(G,\mathcal{L}) \tilde{\sqsubseteq} \tilde{X}$. If $(\digamma,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$,$(G,\mathcal{L}) \tilde{\in} \tilde{s}\alpha O(\tilde{X})$ and $(G,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{X})$, then $(\digamma,\mathcal{L}) \sqcap (G,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$.

3.15. Proposition Let $(\tilde{W},\tilde{T}_W,\mathcal{L})$be a soft subspace of a soft space $(\tilde{X},\tilde{T},\mathcal{L})$ and $\tilde{W} \tilde{\in} \tilde{T}$. If $(\digamma,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{X})$, then $(\digamma,\mathcal{L}) \tilde{\cap} \tilde{W} \tilde{\in} \tilde{s}sC(\tilde{W})$.

Proof: Since $\widetilde{W} \mathrel{\tilde\in} \widetilde{\mathcal{T}}$ and by Proposition 2.22 then $\tilde{s}int_{\widetilde{W}}(\mathcal{F},\mathcal{L}) = \tilde{s}int(\mathcal{F},\mathcal{L})$ for all $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\sqsubseteq} \widetilde{W}$. Hence, we obtain $\tilde{s}int_{\widetilde{W}}(\tilde{s}cl_{\widetilde{W}}((\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W})) = \tilde{s}int(\tilde{s}cl((\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W}) \tilde\sqcap \widetilde{W} \mathrel{\tilde\sqsubseteq} \tilde{s}int(\tilde{s}cl(\mathcal{F},\mathcal{L}) \tilde\sqcap \tilde{s}cl(\widetilde{W})) \tilde\sqcap \widetilde{W}$ $\mathrel{\tilde\sqsubseteq} \tilde{s}int(\tilde{s}cl(\mathcal{F},\mathcal{L})) \tilde\sqcap \tilde{s}int\left(\tilde{s}cl(\widetilde{W})\right) \tilde\sqcap \widetilde{W} \mathrel{\tilde\sqsubseteq} \tilde{s}int(\tilde{s}cl(\mathcal{F},\mathcal{L})) \tilde\sqcap \widetilde{W}.since\ (\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}sC(\tilde{\mathfrak{X}})$,

then $\tilde{s}int(\tilde{s}cl(\mathcal{F},\mathcal{L})) \mathrel{\tilde\sqsubseteq} (\mathcal{F},\mathcal{L})$.

Thus, $\tilde{s}int_{\widetilde{W}}\left(\tilde{s}cl_{\widetilde{W}}\left((\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W}\right)\right) \mathrel{\tilde\sqsubseteq} (\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W}$. Therefore, $(\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W} \mathrel{\tilde\in} \tilde{s}sC(\widetilde{W})$.

3.16.Proposition Let $(\widetilde{W}, \widetilde{\mathcal{T}}_W, \mathcal{L})$ be a soft subspace of a soft space $(\tilde{\mathfrak{X}}, \widetilde{\mathcal{T}}, \mathcal{L})$ *and* $\widetilde{W} \mathrel{\tilde\in} \widetilde{\mathcal{T}}$(resp., soft clopen). If $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$,then $(\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W} \mathrel{\tilde\in} \tilde{s}\beta_s(\widetilde{W})$.

Proof: since $\widetilde{W} \mathrel{\tilde\in} \widetilde{\mathcal{T}}$, *and* $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta O(\tilde{\mathfrak{X}})$ by Proposition 2.19 $(\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W} \mathrel{\tilde\in} \tilde{s}\beta O(\tilde{\mathfrak{X}})$ Since $(\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W} \mathrel{\tilde\sqsubseteq} \widetilde{W}$and $\widetilde{W} \mathrel{\tilde\in} \widetilde{\mathcal{T}}$ then $\widetilde{W} \mathrel{\tilde\in} \tilde{s}\beta O(\tilde{\mathfrak{X}})$ then by Proposition 2.20 $(\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W} \mathrel{\tilde\in} \tilde{s}\beta(\widetilde{W})$.But $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta_s(\tilde{\mathfrak{X}})$ then $\forall\, \mathfrak{x}_\alpha \mathrel{\tilde\in} (\mathcal{F},\mathcal{L})$, $\exists$ (G, $\mathcal{L}$) $\mathrel{\tilde\in} \tilde{s}sC(\tilde{\mathfrak{X}})$ s.t. $\mathfrak{x}_\alpha \mathrel{\tilde\in}$ (G,$\mathcal{L}$) $\mathrel{\tilde\sqsubseteq} (\mathcal{F},\mathcal{L})$. Hence $\mathfrak{x}_\alpha \mathrel{\tilde\in}$ (G,$\mathcal{L}$) $\tilde\sqcap \widetilde{W} \mathrel{\tilde\sqsubseteq} (\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W}$,since $\widetilde{W} \mathrel{\tilde\in} \widetilde{\mathcal{T}}$ and (G, $\mathcal{L}$) $\mathrel{\tilde\in} \tilde{s}sC(\tilde{\mathfrak{X}})$,then by Proposition 3.23 (G, $\mathcal{L}$)$\tilde\sqcap \widetilde{W} \mathrel{\tilde\in} \tilde{s}sC(\widetilde{W})$ therefor $(\mathcal{F},\mathcal{L}) \tilde\sqcap \widetilde{W} \mathrel{\tilde\in} \tilde{s}\beta_s(\widetilde{W})$.

Shown by the example following, the converse of Proposition 3.24 is incorrect, if $\widetilde{W} \mathrel{\tilde\notin} \widetilde{\mathcal{T}}$.

Example Examine $\mathfrak{X} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3\}$, $W = \{\mathfrak{x}_1, \mathfrak{x}_2\}$,and $\mathcal{L} = \{\ell_1, \ell_2\}$. We consider $\widetilde{\mathcal{T}} = \{(\mathcal{F}_i, \mathcal{L}).\ i = 1,2,\dots,6\} \sqcup \{\tilde{\mathfrak{X}}, \tilde{\emptyset}\}$ where

$$\mathcal{F}_1(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_1\} & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_3\} & \text{if } \mathfrak{x} = \ell_2\end{cases} \qquad \mathcal{F}_2(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_2\} & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_1\} & \text{if } \mathfrak{x} = \ell_2\end{cases}$$

$$\mathcal{F}_3(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_3\} & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_2\} & \text{if } \mathfrak{x} = \ell_2\end{cases} \qquad \mathcal{F}_4(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_1, \mathfrak{x}_2\} & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_1, \mathfrak{x}_3\} & \text{if } \mathfrak{x} = \ell_2\end{cases}$$

$$\mathcal{F}_5(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_1, \mathfrak{x}_3\} & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_2, \mathfrak{x}_3\} & \text{if } \mathfrak{x} = \ell_2\end{cases} \qquad \mathcal{F}_6(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_2, \mathfrak{x}_3\} & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_1, \mathfrak{x}_2\} & \text{if } \mathfrak{x} = \ell_2\end{cases}$$

Then $\tilde{s}\beta_s O(\mathfrak{X}) = \widetilde{\mathcal{T}}$, and $\widetilde{\mathcal{T}}_{\widetilde{W}} = \{\tilde{\emptyset}, \widetilde{W}, (H_i, \mathcal{L}).\ i = 1,2,\dots,6\}$ where

$$H_1(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_1\} & \text{if } \mathfrak{x} = \ell_1\\ \emptyset & \text{if } \mathfrak{x} = \ell_2\end{cases} \qquad H_2(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_2\} & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_1\} & \text{if } \mathfrak{x} = \ell_2\end{cases}$$

$$H_3(\mathfrak{x}) = \begin{cases}\emptyset & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_2\} & \text{if } \mathfrak{x} = \ell_2\end{cases} \qquad H_4(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_1, \mathfrak{x}_2\} & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_1\} & \text{if } \mathfrak{x} = \ell_2\end{cases}$$

$$H_5(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_1\} & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_2\} & \text{if } \mathfrak{x} = \ell_2\end{cases} \qquad H_6(\mathfrak{x}) = \begin{cases}\{\mathfrak{x}_2\} & \text{if } \mathfrak{x} = \ell_1\\ \{\mathfrak{x}_1, \mathfrak{x}_2\} & \text{if } \mathfrak{x} = \ell_2\end{cases}$$

So $(H_3, \mathfrak{x}) \mathrel{\tilde\in} \tilde{s}\beta_s O(\widetilde{W})$ but $(H_3, \mathfrak{x}) \mathrel{\tilde\notin} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$.

3.17. Proposition Let $(\widetilde{W}, \widetilde{\mathcal{T}}_W, \mathcal{L})$ be a subspace of a space $(\tilde{\mathfrak{X}}, \widetilde{\mathcal{T}}, \mathcal{L})$ and $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\sqsubseteq} \widetilde{W}$. If $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta_s O(\widetilde{W})$ and $\widetilde{W} \mathrel{\tilde\in} \tilde{s}CO(\tilde{\mathfrak{X}})$, then $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$.

Proof: Let $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta_s O(\widetilde{W})$, then $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta O(\widetilde{W})$ and for $\mathfrak{x}_\alpha \mathrel{\tilde\in} (\mathcal{F},\mathcal{L})$, there is $(K,\mathcal{L}) \mathrel{\tilde\in} \tilde{s}sC(\widetilde{W})$ s.t. $\mathfrak{x}_\alpha \mathrel{\tilde\in} (K,\mathcal{L}) \mathrel{\tilde\sqsubseteq} (\mathcal{F},\mathcal{L})$. Since $\widetilde{W} \mathrel{\tilde\in} \widetilde{\mathcal{T}}$, and $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\sqsubseteq} \widetilde{W}$ then by proposition 2.22 $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\sqsubseteq} \tilde{s}cl_{\widetilde{W}}(\tilde{s}int_{\widetilde{W}}(\tilde{s}cl_{\widetilde{W}}(\mathcal{F},\mathcal{L}))) = \tilde{s}cl(\tilde{s}int(\tilde{s}cl(\mathcal{F},\mathcal{L})))$, $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta O(\tilde{\mathfrak{X}})$.since $(K,\mathcal{L}) \mathrel{\tilde\in} \tilde{s}sC(\widetilde{W})$, $\exists\, (G,\mathcal{L}) \mathrel{\tilde\in} \tilde{s}sC(\tilde{\mathfrak{X}})$ s.t. $(K,\mathcal{L}) = (G,\mathcal{L}) \tilde\sqcap \widetilde{W}$ but $\widetilde{W} \mathrel{\tilde\in} \widetilde{\mathcal{T}}^c$ then it is soft semi-closed therefore $(K,\mathcal{L}) \mathrel{\tilde\in} \tilde{s}sC(\tilde{\mathfrak{X}})$

The following corollary is derived directly from Proposition 3.26. and Proposition 3.24.

Corollary Let $(\tilde{\mathfrak{X}}, \widetilde{\mathcal{T}}, \mathcal{L})$ be a STS, and $(\mathcal{F},\mathcal{L}), \widetilde{W}$ be soft subsets of $\tilde{\mathfrak{X}}$ s.t. $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\sqsubseteq} \widetilde{W} \mathrel{\tilde\sqsubseteq} \tilde{\mathfrak{X}}$ and $\widetilde{W} \mathrel{\tilde\in} \tilde{s}CO(\tilde{\mathfrak{X}})$. Therefore $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta_s O(\widetilde{W})$ iff $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}})$.

Corollary Let $(\mathcal{F},\mathcal{L}) \mathrel{\tilde\in} \tilde{s}\beta_s O(\tilde{\mathfrak{X}}), (\mathcal{F},\mathcal{L}) \mathrel{\tilde\sqsubseteq} \widetilde{W}$and $\widetilde{W} \mathrel{\tilde\in} \tilde{s}CO(\tilde{\mathfrak{X}})$, then $(\mathcal{F},\mathcal{L}) \sqcap \widetilde{W} \mathrel{\tilde\in} \tilde{s}\beta_s O(\widetilde{W})$.

Proof: Let $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$, then $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\beta O(\tilde{X})$. Since $\widetilde{W} \tilde{\in} \tilde{T}$, by Proposition 2.19, $(\mathcal{F},\mathcal{L}) \sqcap \widetilde{W} \quad \tilde{\in} \tilde{s}\beta O(\tilde{X})$. Since $\widetilde{W} \tilde{\sqsubseteq} \tilde{s}\beta O(\tilde{X})$ then by Proposition 2.14 in (16) , $(\mathcal{F},\mathcal{L}) \sqcap \widetilde{W} \tilde{\in} \tilde{s}\beta O(\widetilde{W})$. And $\forall\, x_\alpha \tilde{\in} (\mathcal{F},\mathcal{L})$, there is $(K,\mathcal{L}) \tilde{\in} \tilde{s}sC(\tilde{X})$ s.t. $x_\alpha \tilde{\in} (K,\mathcal{L}) \tilde{\sqsubseteq} (\mathcal{F},\mathcal{L})$. Hence, $x_\alpha \tilde{\in} (K,\mathcal{L}) \sqcap \widetilde{W} \tilde{\sqsubseteq} (\mathcal{F},\mathcal{L}) \sqcap \widetilde{W}$ by Proposition 3.23. $(K,\mathcal{L}) \sqcap \widetilde{W} \tilde{\in} \tilde{s}sC(\widetilde{W})$ and therefore, $(\mathcal{F},\mathcal{L}) \sqcap \widetilde{W} \tilde{\in} \tilde{s}\beta_s O(\widetilde{W})$.

4.$\tilde{s}\beta_s$-Continuity

In this section we define the concept of soft $\tilde{s}\beta_s$-continuous function by using soft β_s-open sets. We then explain various properties of this concept as well as compare it to several other forms of soft continuous functions.

4.1.Definition 4.1. A soft function $f_{pu}.\,(\tilde{X},\tilde{T},\mathcal{L}) \to (\tilde{y},\tilde{\mu},N)$ is called $\tilde{s}\beta_s$-continuous at a soft point $x_\alpha \tilde{\in} \tilde{s}P(\tilde{X})_{\mathcal{L}}$, if $(G,N) \tilde{\in} \tilde{\mu}$ containing $f_{pu}(x_\alpha)$, $\exists$ $(\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$ containing x_α s.t. $f_{pu}(\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} (G,N)$. And it is called $\tilde{s}\beta_s -$continuous function, if f_{pu} is $\tilde{s}\beta_s -$continuous at every soft point of $\tilde{X}$**.**

4.2. Proposition 4.2. A soft function $f_{pu}.\,(\tilde{X},\tilde{T},\mathcal{L}) \to (\tilde{y},\tilde{\mu},N)$
is $\tilde{s}\beta_s$-continuous iff the inverse image $f_{pu}^{-1}(G,N) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$,$\forall (G,N) \tilde{\in} \tilde{\mu}$.

Proof: Suppose f_{pu} is $\tilde{s}\beta_c$-continuous and let $(G,N) \tilde{\in} \tilde{\mu}$. We show that $f_{pu}^{-1}((G,N)) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$. If $f_{pu}^{-1}((G,N)) = \tilde{\emptyset}$, then $f_{pu}^{-1}((G,N)) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$. If $f_{pu}^{-1}((G,N)) \neq \tilde{\emptyset}$, then $\exists\, x_\alpha \tilde{\in} f_{pu}^{-1}((G,N))$ which implies that $f_{pu}(x_\alpha) \tilde{\in} (G,N)$. Since f_{pu} is $\tilde{s}\beta_s$ $-$continuous, $\exists$ $x_\alpha \tilde{\in} (\mathcal{F},\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$ s.t. $f_{pu}((\mathcal{F},\mathcal{L})) \tilde{\sqsubseteq} (G,N)$. This implies that $x_\alpha \tilde{\in} (\mathcal{F},\mathcal{L}) \tilde{\sqsubseteq} f_{pu}^{-1}((G,N))$. This shows that $f_{pu}^{-1}((G,N)) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})$.

Conversely, let $x_\alpha \tilde{\in} \tilde{s}P(\tilde{X})_{\mathcal{L}}$ and $f_{pu}(x_\alpha) \tilde{\in} (G,N) \tilde{\in} \tilde{\mu}$, Then $x_\alpha \tilde{\in} f_{pu}^{-1}((G,N)) \tilde{\in} \tilde{s}\beta_s(\tilde{X})$ *and* $(\mathcal{F},\mathcal{L}) = f_{pu}^{-1}((G,N))$
s.t. $f_{pu}((\mathcal{F},\mathcal{L})) = f_{pu}(f_{pu}^{-1}((G,N))) \tilde{\sqsubseteq} (G,N)$. Hence, f_{pu} is $\tilde{s}\beta_s$-continuous.

4.3. Proposition Let $f_{pu}.\,(\tilde{X},\tilde{T},\mathcal{L}) \to (\tilde{y},\tilde{\mu},N)$ be a soft function

1. f_{pu} is $\tilde{s}\beta -$ continuous, if f_{pu} is a $\tilde{s}\beta_s$-continuous.
2. f_{pu} is $\tilde{s}\beta_s$-continuous, if f_{pu} is a $\tilde{s}s_c -$continuous (resp., $\tilde{s}p_c$-continuous, $\tilde{s}b_c -$ continuous, $\tilde{s}\beta_c -$ continuous, soft regular continuous, $\tilde{s}RC$ $-$continuous, $\tilde{s}\xi -$ continuous, $\tilde{s}\theta -$ continuous, $\tilde{s}\delta -$ continuous and soft perfectly continuous)

Proof: By $\tilde{s}\beta_s O(\tilde{X}) \tilde{\sqsubseteq} \tilde{s}\beta O(\tilde{X})$ and Proposition 4.2. we get prove (1).
And by Proposition 3.6. and Proposition 4.2. Prove (2) will follow.
The following examples illustrate the previous remark.

Example Consider the soft topology $\tilde{T}_1 = \{\tilde{\emptyset}, \tilde{X}, (\mathcal{F}_{1,}\mathcal{L}), (\mathcal{F}_{3,}\mathcal{L})\}$ in Example 3.5. Then the identity function $I.\,(\tilde{X},\tilde{T}_1,\mathcal{L}) \to (\tilde{X},\tilde{T}_1,\mathcal{L})$ is $\tilde{s}\beta$-continuous but not $\tilde{s}\beta_s$-continuous since $(\mathcal{F}_{1,}\mathcal{L}) \tilde{\in} \tilde{T}_1$, but $I^{-1}(\mathcal{F}_{1,}\mathcal{L}) = (\mathcal{F}_{1,}\mathcal{L}) \tilde{\notin} \tilde{s}\beta_s O(\tilde{X})$.

1. Consider the soft topology $\tilde{T}_2 = \{\tilde{\emptyset}, \tilde{X}, (\mathcal{F}_{1,}\mathcal{L}), (\mathcal{F}_{5,}\mathcal{L}), (\mathcal{F}_{8,}\mathcal{L})\}$ in Example 3.5. Then the identity function $I.\,(\tilde{X},\tilde{T}_2,\mathcal{L}) \to (\tilde{X},\tilde{T}_2,\mathcal{L})$ is $\tilde{s}\beta_s$-continuous but is not $\tilde{s}s_c -$ continuous, $\tilde{s}p_c$-
continuous, $\tilde{s}b_c -$ continuous, $\tilde{s}\beta_c -$ continuous, soft regular continuous,
$\tilde{s}RC$ $-$continuous, $\tilde{s}\theta -$ continuous, and soft perfectly continuous. Since $(\mathcal{F}_{8,}\mathcal{L}) \tilde{\in} \tilde{T}_2$,then $I^{-1}(\mathcal{F}_{8,}\mathcal{L}) \tilde{\in} \tilde{s}\beta_s O(\tilde{X})butI^{-1}(\mathcal{F}_{8,}\mathcal{L})$ is not $\tilde{s}s_c -$ open, $\tilde{s}p_c$-open, $\tilde{s}b_c -$ open, $\tilde{s}\beta_c -$ open, $\tilde{s}R -$ open, $\tilde{s}RC$ $-$open, $\tilde{s}\theta -$ open, and soft clopen.

2. Consider the soft topology $\widetilde{\daleth}_1 = \{\widetilde{\emptyset}, \widetilde{X}, (\mathcal{F}_{1,}\mathcal{L}), (\mathcal{F}_{3,}\mathcal{L})\}$ and $\widetilde{\daleth}_2 = \{\widetilde{\emptyset}, \widetilde{X}, (\mathcal{F}_{1,}\mathcal{L}), (\mathcal{F}_{5,}\mathcal{L}), (\mathcal{F}_{8,}\mathcal{L})\}$ in Example 3.5. then $I.(\tilde{X}, \widetilde{\daleth}_2, \mathcal{L}) \to (\tilde{X}, \widetilde{\daleth}_1, \mathcal{L})$ is $\tilde{s}\beta_s$-continuous but is not $\tilde{s}\xi -$ continuous, $\tilde{s}\delta -$ continuous since $(\mathcal{F}_{3,}\mathcal{L}) \widetilde{\in} \widetilde{\daleth_1}$, but $I^{-1}(\mathcal{F}_{3,}\mathcal{L}) = (\mathcal{F}_{3,}\mathcal{L})$ is not $\tilde{s}\xi -$ open and $\tilde{s}\delta -$open.

4.4.Proposition A soft function $f_{pu}.(\tilde{X}, \widetilde{\daleth}, \mathcal{L}) \to (\widetilde{y}, \tilde{\mu}, N)$ is $\tilde{s}\beta_s$- continuous iff f_{pu} is a soft β-continuous and $\forall\, x_\alpha \widetilde{\in} \tilde{X}$ and $\forall\, (\mathcal{F}, N) \widetilde{\in} \tilde{\mu}$ containing $f_{pu}(x_\alpha)$, $\exists (K, \mathcal{L}) \widetilde{\in} \tilde{s}sC(\tilde{X})$ s.t $f_{pu}((K, \mathcal{L})) \widetilde{\sqsubseteq} (\mathcal{F}, N)$.

Proof: let $f_{pu}.(\tilde{X}, \widetilde{\daleth}, \mathcal{L}) \to (\widetilde{y}, \tilde{\mu}, N)$ be a $\tilde{s}\beta_s$ −continuous function and also let $x_\alpha \widetilde{\in} \tilde{s}P(\tilde{X})$ and $(\mathcal{F}, N) \widetilde{\in} \tilde{\mu}$ containing $f_{pu}(x_\alpha)$. By assumption, $\exists\, x_\alpha \widetilde{\in} (G, \mathcal{L}) \widetilde{\in} \tilde{s}\beta_s O(\tilde{X})$ s.t. $f_{pu}((G, \mathcal{L})) \widetilde{\sqsubseteq} (\mathcal{F}, N)$. Since $(G, \mathcal{L}) \widetilde{\in} \tilde{s}\beta_s O(\tilde{X})$ $\exists (K, \mathcal{L}) \widetilde{\in} \tilde{s}sC(\tilde{X})$ s.t $x_\alpha \widetilde{\in} (K, \mathcal{L}) \widetilde{\sqsubseteq} (G, \mathcal{L})$. This implies that $f_{pu}(K, \mathcal{L}) \widetilde{\sqsubseteq} (\mathcal{F}, N)$. Therefore, f_{pu} is $\tilde{s}\beta_s$- continuous. Then f_{pu} is soft β-continuous.

Conversely, suppose $(\mathcal{F}, N) \widetilde{\in} \tilde{\mu}$. We have to show that $f_{pu}^{-1}((\mathcal{F}, N)) \widetilde{\in} \tilde{s}\beta_s O(\tilde{X})$. Since f_{pu} is soft β-continuous, then $f_{pu}^{-1}((\mathcal{F}, N)) \widetilde{\in} \tilde{s}\beta O(\tilde{X})$. Let $x_\alpha \widetilde{\in} f_{pu}^{-1}((\mathcal{F}, N))$. Then $f_{pu}(x_\alpha) \widetilde{\in} (\mathcal{F}, N)$. By hypothesis $\exists\, x_\alpha \widetilde{\in} (K, \mathcal{L}) \widetilde{\in} \tilde{s}sC(\tilde{X})$ s.t. $f_{pu}((K, \mathcal{L})) \widetilde{\sqsubseteq} (\mathcal{F}, N)$, which implies that $x_\alpha \widetilde{\in} (K, \mathcal{L}) \widetilde{\sqsubseteq} f_{pu}^{-1}((\mathcal{F}, N))$, therefore $f_{pu}^{-1}((\mathcal{F}, N)) \widetilde{\in} \tilde{s}\beta_s O(\tilde{X})$ .Hence by Proposition 4.2 f_{pu} is $\tilde{s}\beta_s$-continuous.

4.5. Proposition If $\tilde{X}$ is soft T_1 -space in a soft function $f_{pu}.(\tilde{X}, \widetilde{\daleth}, \mathcal{L}) \to (\widetilde{y}, \tilde{\mu}, N)$, then The following properties are equivalent.

1. f_{pu} is $\tilde{s}\beta$-continuous.
2. f_{pu} is $\tilde{s}\beta_c$-continuous.
3. f_{pu} is $\tilde{s}\beta_s$-continuous.

Proof: by Proposition 3.8

4.6.Proposition If a soft function $f_{pu}.(\tilde{X}, \widetilde{\daleth}, \mathcal{L}) \to (\widetilde{y}, \tilde{\mu}, N)$, is soft continuous, and $\tilde{X}$ is soft T_1- space, then f_{pu} is $\tilde{s}\beta_s$-continuous.

4.7.Proposition If $f_{pu}.(\tilde{X}, \widetilde{\daleth}, \mathcal{L}) \to (\widetilde{y}, \tilde{\mu}, N)$, is soft continuous, and $\tilde{X}$ is soft regular space, then f_{pu} is $\tilde{s}\beta_s$-continuous.

Proof: By Proposition 4.2 and Proposition 3.18 we get the prove.

4.8. Proposition If $f_{pu}.(\tilde{X}, \widetilde{\daleth}, \mathcal{L}) \to (\widetilde{y}, \tilde{\mu}, N)$, is soft β_s −continuous, and $\tilde{X}$ is SEDS, then f_{pu} is $\tilde{s}p$-continuous (resp., $\tilde{s}\alpha$ −continuous).

Proof: By Proposition 4.2 and Proposition 3.13(resp., Proposition 3.14) we get the prove.

4.9.Proposition If $\tilde{X}$ is soft locally indiscrete space in a soft function $f_{pu}.(\tilde{X}, \widetilde{\daleth}, \mathcal{L}) \to (\widetilde{y}, \tilde{\mu}, N)$, then the following properties are equivalent.

1. f_{pu} is $\tilde{s}s$-continuous.
2. f_{pu} is $\tilde{s}\beta_c$-continuous.
3. f_{pu} is $\tilde{s}\beta_s$-continuous.

Proof: by Proposition 3.15.

Corollary Let $f_{pu}.(\tilde{X}, \widetilde{\daleth}, \mathcal{L}) \to (\widetilde{y}, \tilde{\mu}, N)$ be a soft function s.t. $(\tilde{X}, \widetilde{\daleth}, \mathcal{L})$ is a soft locally indiscrete space. Then

(1) f_{pu} is a soft β_s-continuous function, iff f_{pu} is a soft continuous function.

(2) f_{pu} is a soft β_s-continuous function, iff f_{pu} is a soft α-continuous function.

Proof: (1) and (2) direct by using corollary 3.16 and by Proposition 4.2.

5. Properties and Comparisons

5.1.Proposition Let $\mathfrak{f}_{pu}.(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ be $\tilde{s}\beta_s$ −continuous function. If $\widetilde{W} \tilde{\in} \tilde{\mathfrak{T}}$ (resp., soft clopen), then $\mathfrak{f}_{pu}|\tilde{s}p(\widetilde{W})_{\mathcal{L}}.(\widetilde{W},\tilde{\gamma},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ is $\tilde{s}\beta_s$-continuous function in the subspace $\widetilde{W}$.

Proof: Let $(F,\mathrm{N}) \tilde{\in} \tilde{\mu}$. Since $\mathfrak{f}_{pu}$ is $\tilde{s}\beta_s$-continuous function, then by Proposition 4.2 $\mathfrak{f}_{pu}^{-1}(F,\mathrm{N})$ is $\tilde{s}\beta_s$O$(\tilde{\mathfrak{X}})$. Since $\widetilde{W} \tilde{\in} \tilde{\mathfrak{T}}$, then by Proposition 3.24 $(\mathfrak{f}_{pu}|\tilde{s}P(\widetilde{W})_{\mathcal{L}})^{-1}(F,\mathrm{N}) = \mathfrak{f}_{pu}^{-1}((F,\mathrm{N})) \sqcap \widetilde{W} \tilde{\in} \tilde{s}\beta_sO(\widetilde{W})$. This shows that $\mathfrak{f}_{pu}|\tilde{s}P(\widetilde{W})_{\mathcal{L}}.(\widetilde{W},\tilde{\gamma},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ is $\tilde{s}\beta_s$-continuous function.

5.2.Proposition Let $\mathfrak{f}_{pu}.(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ be $\tilde{s}\beta_s$-continuous function. If $\widetilde{W} \tilde{\in} \tilde{s}RO(\tilde{\mathfrak{X}})$then $\mathfrak{f}_{pu}|\tilde{s}P(\widetilde{W})_{\mathcal{L}}).(\widetilde{W},\tilde{\gamma},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ is $\tilde{s}\beta_s$-continuous function in the subspace $\widetilde{W}$.

Proof: Since every soft regular open is soft open, this is a direct result of Proposition 4.2 and Proposition 5.1.

5.3.Proposition Let $\mathfrak{f}_{pu}.(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ and $g_{qv}.(\tilde{y},\tilde{\mu},\mathrm{N}) \to (\widetilde{W},\tilde{\gamma},T)$ be two soft functions, then the following properties hold.
(1) If $\mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$ −continuous and g_{qv} is a soft continuous, then $g_{qv} \circ \mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$ −continuous.
(2) If $\mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$ −irresolute and g_{qv} is a $\tilde{s}\beta_s$ −continuous, then $g_{qv} \circ \mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$ −continuous.

Proof: (1) Let $(F,T) \tilde{\in} \tilde{\gamma}$. Then $g_{qv}^{-1}(F,T) \tilde{\in} \tilde{\mu}$ by continuity of g_{qv}. Since $\mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$-continuous, $\mathfrak{f}_{pu}^{-1}(g_{qv}^{-1}(F,T)) \tilde{\in} \tilde{s}\beta_s(\tilde{\mathfrak{X}})$ and hence, $(g_{qv} \circ \mathfrak{f}_{pu})^{-1}(F,T) = \mathfrak{f}_{pu}^{-1}(g_{qv}^{-1}(F,T)) \tilde{\in} \tilde{s}\beta_s(\tilde{\mathfrak{X}})$. Therefore, $g_{qv} \circ \mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$-continuous.
(2) Let $(F,T) \tilde{\in} \tilde{\gamma}$. Since g_{qv} is a $\tilde{s}\beta_s$-continuous, $g_{qv}^{-1}(F,T) \tilde{\in} \tilde{s}\beta_s(\tilde{y})$. Since $\mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$-irresolute, $\mathfrak{f}_{pu}^{-1}(g_{qv}^{-1}(F,T)) \tilde{\in} \tilde{s}\beta_s(\tilde{\mathfrak{X}})$ and hence, $(g_{qv} \circ \mathfrak{f}_{pu})^{-1}(F,\mathrm{T}) = \mathfrak{f}_{pu}^{-1}(g_{qv}^{-1}(F,T)) \tilde{\in} \tilde{s}\beta_s(\tilde{\mathfrak{X}})$. Therefore, $g_{qv} \circ \mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$-continuous.

5.4.Proposition Let $\mathfrak{f}_{pu}.(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ and $g_{qv}.(\tilde{y},\tilde{\mu},\mathrm{N}) \to (\widetilde{W},\tilde{\gamma},T)$ be two soft functions. If $\mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$-open and surjective and $g_{qv} \circ \mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$-continuous, then g_{qv} is a $\tilde{s}\beta_s$-continuous.

Proof: Let $(F,T) \tilde{\in} \tilde{\gamma}$. Since $g_{qv} \circ \mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$-continuous, $(g_{qv} \circ \mathfrak{f}_{pu})^{-1}(F,T) = \mathfrak{f}_{pu}^{-1}(g_{qv}^{-1}(F,T)) \tilde{\in} \tilde{s}\beta_s(\tilde{\mathfrak{X}})$. Since $\mathfrak{f}_{pu}$ is a $\tilde{s}\beta_s$-open and surjective, then $\mathfrak{f}_{pu}\left(\mathfrak{f}_{pu}^{-1}(g_{qv}^{-1}(F,T))\right) = g_{qv}^{-1}(F,T) \tilde{\in} \tilde{s}\beta_s(\tilde{y})$. Hence, g_{qv} is a $\tilde{s}\beta_s$-continuous.

5.5.Proposition A function $\mathfrak{f}_{pu}.(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ is $\tilde{s}\beta_s$-continuous. If $\forall\, \mathfrak{x}_\alpha \tilde{\in} \tilde{s}P(\tilde{\mathfrak{X}})_{\mathcal{L}}$, $\exists$ a soft clopen set $(F,\mathcal{L})$ of $\tilde{\mathfrak{X}}$ containing $\mathfrak{x}_\alpha$ s.t. $\mathfrak{f}_{pu}|(F,\mathcal{L}); (F,\mathcal{L}) \to \tilde{y}$ is $\tilde{s}\beta_s$-continuous.

Proof: Let $\mathfrak{x}_\alpha \tilde{\in} \tilde{s}P(\tilde{\mathfrak{X}})_{\mathcal{L}}$, then by hypothesis, $\exists$ $(F,\mathcal{L}) \tilde{\in} \tilde{s}CO(\tilde{\mathfrak{X}})$ containing $\mathfrak{x}_\alpha$ s.t. $\mathfrak{f}_{pu}|(F,\mathcal{L}); (F,\mathcal{L}) \to \tilde{y}$ is $\tilde{s}\beta_s$-continuous. Let $(G,\mathrm{N}) \tilde{\sqsubseteq} SP(\tilde{y})_{\mathrm{N}}$ containing $\mathfrak{f}_{pu}(\mathfrak{x}_\alpha)$, $\exists$ a $\tilde{s}\beta_s$-open subset $(H,\mathcal{L})$ of $(F,\mathcal{L})$ containing $\mathfrak{x}_\alpha$ s.t. $\left(\mathfrak{f}_{pu}|(F,\mathcal{L})\right)(H,\mathcal{L}) \tilde{\sqsubseteq} (G,\mathrm{N})$. Since $(F,\mathcal{L}) \tilde{\in} \tilde{s}CO(\tilde{\mathfrak{X}})$. By Proposition 3.26 $(H,\mathcal{L}) \tilde{\in} \tilde{s}\beta_s(\tilde{\mathfrak{X}})$ and hence $\mathfrak{f}_{pu}((H,\mathcal{L})) \tilde{\sqsubseteq} (G,\mathrm{N})$. This implies that $\mathfrak{f}_{pu}$ is $\tilde{s}\beta_s$-continuous.

5.6.Proposition If $\tilde{\mathfrak{X}} = (F,\mathcal{L}) \sqcup (G,\mathcal{L})$, where $(F,\mathcal{L}),(G,\mathcal{L}) \tilde{\in} \tilde{s}CO(\tilde{\mathfrak{X}})$ and $\mathfrak{f}_{pu}.(\tilde{\mathfrak{X}},\tilde{\mathfrak{T}},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ is a function s.t. both $\mathfrak{f}_{pu}|(F,\mathcal{L})$ and $\mathfrak{f}_{pu}|(G,\mathcal{L})$ are both $\tilde{s}\beta_s$-continuous, then $\mathfrak{f}_{pu}$ is $\tilde{s}\beta_s$-continuous.

Proof: Let $(H,\mathrm{N}) \tilde{\in} \tilde{\mu}$. Then $\mathfrak{f}_{pu}^{-1}((H,\mathrm{N})) = (\mathfrak{f}_{pu}|(F,\mathcal{L}))^{-1}((H,\mathrm{N})) \sqcup (\mathfrak{f}_{pu}|(G,\mathcal{L}))^{-1}((H,\mathrm{N}))$. Since $\mathfrak{f}_{pu}|(F,\mathcal{L})$ and $\mathfrak{f}_{pu}|(G,\mathcal{L})$ are $\tilde{s}\beta_s$-continuous. Then by Proposition 4.2 $(\mathfrak{f}_{pu}|(F,\mathcal{L}))^{-1}((H,\mathrm{N}))$ and $(\mathfrak{f}_{pu}|(G,\mathcal{L}))^{-1}((H,\mathrm{N}))$ are $\tilde{s}\beta_s$-open sets in $(F,\mathcal{L})$ and $(G,\mathcal{L})$ respectively. Since $(F,\mathcal{L}),(G,\mathcal{L}) \tilde{\in} \tilde{s}CO(\tilde{X})$, then by Corollary 3.26 $(\mathfrak{f}_{pu}|(F,\mathcal{L}))^{-1}((H,\mathrm{N}))$, $(\mathfrak{f}_{pu}|(G,\mathcal{L}))^{-1}((H,\mathrm{N})) \tilde{\in} \tilde{s}\beta_s(\tilde{X})$. Since the union of two $\tilde{s}\beta_s$-open sets are $\tilde{s}\beta_s$- open. Hence $\mathfrak{f}_{pu}^{-1}((H,\mathrm{N})) \tilde{\in} \tilde{s}\beta_s(\tilde{X})$. Therefore, by Proposition 4.2 $\mathfrak{f}_{pu}$ is $\tilde{s}\beta_s$-continuous.

In general, if $\tilde{X} = \sqcup\{(K_\alpha,\mathcal{L});\ \alpha \tilde{\in} \Delta\}$, where each $(K_\alpha,\mathcal{L})$ is a soft clopen set and $\mathfrak{f}_{pu}.(\tilde{X},\tilde{T},\mathcal{L}) \to (\tilde{y},\tilde{T}_{y},\mathrm{N})$ is a function s.t. $\mathfrak{f}_{pu}|(K_\alpha,\mathcal{L})$ is $\tilde{s}\beta_s$-continuous for each α, then $\mathfrak{f}_{pu}$ is $\tilde{s}\beta_s$-continuous.

5.7.Proposition Let $\tilde{X} = (F_1,\mathcal{L}) \sqcup (F_2,\mathcal{L})$, where $(F_1,\mathcal{L}),(F_2,\mathcal{L}) \tilde{\in} \tilde{s}CO(\tilde{X})$. Let $\mathfrak{f}_{pu}.(F_1,\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ and $g_{pu}.(F_2,\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ be a $\tilde{s}\beta_s$-continuous. If $\mathfrak{f}_{pu}(x_\alpha) = g_{pu}(x_\alpha)$, $\forall\, x_\alpha \tilde{\in} (F_1,\mathcal{L}) \sqcap (F_2,\mathcal{L})$. Then the function $h_{pu}.(F_1,\mathcal{L}) \sqcup (F_2,\mathcal{L}) \to \tilde{y}$ s.t.

$$h_{pu}(x_\alpha) = \begin{cases} \mathfrak{f}_{pu}(x_\alpha) & \text{if } x_\alpha \tilde{\in} (F_1,\mathcal{L}) \\ g_{pu}(x_\alpha) & \text{if } x_\alpha \tilde{\in} (F_2,\mathcal{L}) \end{cases}$$

Is $\tilde{s}\beta_s$-continuous.

Proof: Let (G,N) be a soft open set of $\tilde{y}$. Now $h_{pu}^{-1}((G,\mathrm{N})) = \mathfrak{f}_{pu}^{-1}((G,\mathrm{N})) \sqcup g_{pu}^{-1}((G,\mathrm{N}))$. Since $\mathfrak{f}_{pu}$ is $\tilde{s}\beta_s$-continuous, then by Proposition 4.2, $\mathfrak{f}_{pu}^{-1}(G,\mathrm{N})$ is $\tilde{s}\beta_s$-open set in $(F_1,\mathcal{L})$. But $(F_1,\mathcal{L})$ is soft clopen set in $\tilde{X}$. Then by Corollary 3.26, $\mathfrak{f}_{pu}^{-1}(G,\mathrm{N})$ is $\tilde{s}\beta_s$-open set in $\tilde{X}$. Similarly, $g_{pu}^{-1}(G,\mathrm{N})$ is $\tilde{s}\beta_s$-open set in $(F_2,\mathcal{L})$, and hence $\tilde{s}\beta_s$-open set in $\tilde{X}$. Since the union of two $\tilde{s}\beta_s$-open set is $\tilde{s}\beta_s$-open. Therefore, $h_{pu}^{-1}((G,\mathrm{N})) = \mathfrak{f}_{pu}^{-1}(G,\mathrm{N}) \sqcup g_{pu}^{-1}(G,\mathrm{N})$ is $\tilde{s}\beta_s$-open set in $\tilde{X}$. Hence by Proposition 4.2 h_{pu} is $\tilde{s}\beta_s$-continuous.

5.8. Proposition Let $\mathfrak{f}_{pu}.(\tilde{X},\tilde{T},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ be $\tilde{s}\beta_s$-continuous. If $\tilde{y}$ is a soft clopen subset of a STS $\tilde{W}$, then $\mathfrak{f}_{pu}.(\tilde{X},\tilde{T},\mathcal{L}) \to (\tilde{W},\tilde{\gamma},\mathrm{N})$ is $\tilde{s}\beta_s$-continuous.

Proof: Suppose $x_\alpha \tilde{\in} \tilde{s}P(\tilde{X})_{\mathcal{L}}$ and (G,N) be any soft open set of $\tilde{W}$ containing $\mathfrak{f}_{pu}(x_\alpha)$, then $(G,\mathrm{N}) \sqcap \tilde{y}$ is a soft open set in $\tilde{y}$. But $\mathfrak{f}_{pu}(x_\alpha) \tilde{\in} \tilde{s}P(\tilde{y})_{\mathrm{N}}$ $\forall\, x_\alpha \tilde{\in} \tilde{s}P(\tilde{X})_{\mathcal{L}}$, then $\mathfrak{f}_{pu}(x_\alpha) \tilde{\in} (G,\mathrm{N}) \sqcap \tilde{y}$. Since $\mathfrak{f}_{pu}.(\tilde{X},\tilde{T},\mathcal{L}) \to (\tilde{y},\tilde{\mu},\mathrm{N})$ is $\tilde{s}\beta_s$-continuous, then $\exists$ a $\tilde{s}\beta_s$-open set $(F,\mathcal{L})$ containing x_α s.t. $\mathfrak{f}_{pu}((F,\mathcal{L})) \tilde{\sqsubseteq} ((G,N) \sqcap \tilde{y}) \tilde{\sqsubseteq} (G,\mathrm{N})$. Therefore $\mathfrak{f}_{pu}.(\tilde{X},\tilde{T},\mathcal{L}) \to (\tilde{W},\tilde{\gamma},\mathrm{N})$ is $\tilde{s}\beta_s$-continuous.

6. Conclusion

Through the current research work, we have continued to investigate the properties of soft beta open sets in soft topological spaces. $\tilde{s}\beta_s$-open sets is defined as a new kind of soft sets which is a stronger form of soft β -open sets and weaker than each of $\tilde{s}\beta_c$-open, soft clopen sets and some other. We presented and examined $\tilde{s}\beta_s$-continuous functions in a STS. We also identified some characterizations and basic properties of these soft functions as well as their relationships to certain other categories of soft functions and documented with some illustrative examples.

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Conflict of Interest

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