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Study of Cubic Q-ideals in Q-algebras

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Abstract
The concept of cubic ideals is an important generalization in an algebraic structure. In this work, we introduce the notion of a cubic Q-sub-algebra and a cubic Q-ideal within a Q-algebra. Some properties of a cubic Q-ideal with a cubic BCK-ideal are studied, and a few results of these ideals are discussed. Such that every cubic Q-ideal is a cubic BCK-ideal, and the opposite is not true. Also, the concept of the level set of a Q-algebra as a cubic set is defined, and some important results associated with it have been shown. The relationship between a cubic Q-ideal and a Q-ideal with a cubic Q-sub-algebra and a Q-sub-algebra by the level set is proved. Finally, the image and the inverse image of cubic Q-ideals under a homomorphism are introduced, and a few theorems are proven, such that the image and the inverse image of a cubic Q-ideal are also cubic Q-ideals by the homomorphism mapping.
Keywords: Q-algebra, cubic Q-ideal, cubic BCK-ideal, homomorphism map.

1. Introduction

The two new classes of abstract algebras, BCK and BCI-algebras, were introduced in 1966. BCI-algebra is a generalization of BCK-algebra. Various Mathematicians have studied these algebras extensively, and as a result, a lot of literature has emerged. So, many mathematical applications papers presented this structure in different ways.[1–4]
The structure of the BCH-algebra was studied, and it was shown that the class of BCI-algebra is a subclass of BCH-algebra. And then, it has been proven that the algebraic structure of a BH-algebra is a generalization of the BCH-algebra. After that, A Q-algebra was introduced as a generalization of BCK/BCI/BCH-algebras and several research papers have studied this structure using various algebraic concepts, including the idea of fuzzy Q-ideal, two notions of atoms and the strong Atoms for Q-ideals, new types of pseudo ideals and intuitionistic fuzzy pseudo ideals and a neutrosophic Q-ideal on Q-algebras [5–10].
The concept of cubic ideals is an important generalization in algebraic structures. Cubic sets of BCK/BCI-algebras and some of the relationships between cubic subalgebras and cubic ideals are applied. Also, cubic KU-algebras and several significant results were introduced by some authors. Following this, the idea of cubic ideals of semigroups in KU-algebra and various types of ideals related to this framework are introduced. Additionally, two concepts, the cubic implicative and intuitionistic implicative ideals, were studied[11–14]. Also, many papers studied a cubic set in different ways, including the study of cubic dual ideals in BCK-Algebras and also the study of cubic intuitionistic fuzzy Topology, cubic ideals in semihypergroups, cubic ideals of TM-algebras, and cubic on B-algebras.[15–19]
In this paper, a cubic Q-ideal on a Q-algebra was introduced. Some properties related to a cubic Q-ideal and a BCK-ideal are studied, and new results of these ideals are discussed. Such that every Q-ideal is a BCK-ideal, and the opposite is not true. The concept of a cubic Q-sub-algebra

is defined, and some important results associated with it have been proven. Also, the level set of a cubic Q-ideal is determined, and the relationship between a cubic Q-ideal and a Q-ideal by level set is studied. Finally, a new definition of cubic Q-ideals under a homomorphism is introduced, and a few theorems are discussed, such that the image and the inverse image of a homomorphism mapping for a cubic Q-ideal are a cubic Q-ideal in $\aleph'$ and $\aleph$, respectively.

2.Methodology and Methods

In this section, we review essential definitions, examples, and theoretical results that form the foundation of our study. These preliminary concepts will be used to develop and prove the main results regarding cubic Q-ideals and cubic Q-subalgebras in Q-algebras.

2.1. Definition A BCK-algebra is defined as a non-empty set $\aleph$ with a binary relation $*$ and a constant 0 satisfying: For all $\theta, \lambda, \epsilon \in \aleph$

BCI_1 : $[(\theta * \lambda) * (\theta * \epsilon)] * (\epsilon * \lambda) = 0$,

BCI_2 : $(\theta * (\theta * \lambda)) * \lambda = 0$,

BCI_3 : $\theta * \theta = 0$,

BCI_4 : $\theta * \lambda = 0$ and $\lambda * \theta = 0$ implies $\theta = \lambda$ and

BCI_5 : $0 * \theta = 0$.

For example, if $\aleph = \{0,1,2\}$ is a set and $*$ is a binary operation defined as follows:

Table 1. A BCK-algebra

$*$	**0**	**1**	**2**
0	0	0	2
1	1	0	0
2	2	2	0

Then$(\aleph, *, 0)$ is a BCK-algebra.[20]

2.2. Definition In a BCK-algebra $(\aleph, *, 0)$, the non-empty subset ψ of $\aleph$ is called a BCK-ideal if it satisfies the following conditions: For any $\theta, \lambda \in \aleph$

I. $0 \in \psi$

II. $\theta * \lambda \in \psi$ and $\lambda \in \psi$ imply $\theta \in \psi$.[20]

2.3. Definition The structure of a Q-algebra $(\aleph, *, 0)$ is a nonempty subset $\aleph$ and a constant "0" with a binary operation "$*$" the following is achieved:

I. $\theta * \theta = 0$

II. $\theta * 0 = \theta$,

III. $(\theta * \lambda) * \epsilon = (\theta * \epsilon) * \lambda, \forall \theta, \lambda, \epsilon \in \aleph$.

In a Q-algebra $(\aleph, *, 0)$, a binary operation $\leq$ is defined by $\theta \leq \lambda$ if and only if $\theta * \lambda = 0$.[7]

2.4. Example Let $\aleph = \{0, z, x\}$ be a set as shown in the table below.

Table 2. A Q-algebra

$*$	**0**	**z**	***x***
0	0	0	0
z	z	0	0
x	*x*	0	0

Then $(\aleph, *, 0)$ is a Q-algebra.[7]

2.5. Proposition If $(\aleph, *, 0)$ is a Q-algebra, then $\left(\theta * (\theta * \lambda)\right) * \lambda = 0$ for any $\theta, \lambda \in \aleph$.[21]

2.6. Theorem Every Q-algebra $\aleph$ satisfying the conditions

(i) $[(\theta * \lambda) * (\theta * \epsilon)] = (\epsilon * \lambda)$,

(ii) $\left(\theta * (\theta * \lambda)\right) * \lambda = 0$,

(iii) $(\theta * \lambda) * \theta = 0$, for any $\theta, \lambda \in \aleph$ is a BCK-algebra.[22]

2.7. Definition In a Q-algebra $(\aleph, *, 0)$, a non-empty subset $\mathcal{L}$ of $\aleph$ is called a Q-subalgebra if $\theta * \lambda \in \mathcal{L}$ whenever $\theta, \lambda \in \mathcal{L}$.[7]

2.8. Proposition Let $(\aleph, *, 0)$ be Q-algebra. Then every BCK-ideal is a Q-subalgebra.[23]

2.9. Definition In Q-algebra $(\aleph, *, 0)$, the non-empty subset ψ of $\aleph$ is called a Q-ideal of $\aleph$ if,

i) $0 \in \psi$,
ii) $\forall\, \theta, \daleth, \epsilon \in \aleph$, $((\theta * \daleth) * \epsilon) \in \psi$, $\daleth \in \psi$ imply $\theta * \epsilon \in \psi$.[7]

2.10. Example Let $\aleph = \{0, l, m, n, k\}$ be a set defined by the table below.

Table 3. A Q-ideal

*	**0**	**l**	**m**	**n**	**k**
0	0	0	0	0	k
l	l	0	0	l	k
m	m	m	0	0	k
n	n	0	n	0	k
k	k	k	k	k	0

Then $(\aleph, *, 0)$ is a Q-algebra and it is clear that the set $\psi = \{0, l, m, n\}$ is a Q-ideal of $\aleph$.[7]

2.11. Proposition In a Q-algebra $(\aleph, *, 0)$. Every Q-ideal of $\aleph$ is a BCK-ideal.[7]

2.12. Definition In a BCK-algebra $(\aleph, *, 0)$. A fuzzy set π of $\aleph$ is called a fuzzy BCK-ideal, If it satisfies
(i) $\pi(0) \geq \pi(\theta)$, for any $\theta \in \aleph$
(ii) $\pi(\theta) \geq \min\{\pi(\theta * \daleth), \pi(\daleth)\}$, for any $\theta, \daleth$ in $\aleph$.[24]

2.13. Definition In a Q-algebra $(\aleph, *, 0)$. A fuzzy set π of $\aleph$ is called a fuzzy Q-ideal, If it satisfies
i- $\pi(0) \geq \pi(\theta)$, for any $\theta \in \aleph$
ii- $\pi(\theta * \epsilon) \geq \min\{\pi((\theta * \daleth) * \epsilon), \pi(\daleth)\}$, for any $\theta, \daleth, \epsilon$ in $\aleph$.[22]

2.14. Proposition In a Q-algebra$(\aleph, *, 0)$. Every fuzzy Q-ideal of $\aleph$ is a fuzzy BCK-ideal.[22]

2.15. Proposition If π is a fuzzy Q-ideal of Q-algebra $\aleph$, then $\theta \leq \daleth$ implies $\pi(\theta) \geq \pi(\daleth)$.[22]

2.16. Definition An interval-valued fuzzy set $V =< \theta, \tilde{\pi}(\theta) >$, where $\tilde{\pi}: \aleph \to D[0,1]$, the degree of membership of the element θ with defined as $\tilde{\pi} = \{(\theta, [\pi^L(\theta), \pi^U(\theta)]): \theta \in \aleph\}$, where π^L and π^U are two fuzzy sets and $\pi^L(\theta) \leq \pi^U(\theta)$, for all $\theta \in \aleph$. All closed sub-intervals of $[0,1]$ is denoted by $D[0,1]$ so, $\tilde{\pi}(\theta) \in D[0,1]$ and $D[0,1] = \{[m^L, m^U]: m^L \leq m^U \ for\ m^L, m^U \in [0,1]\}$. Two elements $M_1 = [a^L, a^U]$ and $M_2 = [b^L, b^U]$ in $D[0,1]$ are defined
$rmin(M_1, M_2) = [\min(a^L, b^L), \min(a^U, b^U)]$ and
$rmax(M_1, M_2) = [\max(a^L, b^L), \max(a^U, b^U)]$.
Also,
I. $M_1 \geq M_2 \Leftrightarrow a^L \geq b^L\ and\ a^U \geq b^U$,
II. $M_1 \leq M_2 \Leftrightarrow a^L \leq b^L\ and\ a^U \leq b^U$,
III. $M_1 = M_2 \Leftrightarrow a^L = b^L\ and\ a^U = b^U$.[25,26]

2.17.Definition Let $\aleph$ be a non-empty set. The structure $\varpi = \{\langle \theta, \tilde{\pi}_\varpi(\theta), \alpha_\varpi(\theta) \rangle: \theta \in \aleph\}$ is called a cubic set in $\aleph$, such that $\tilde{\pi}_\varpi(\theta) = [\pi^L_\varpi(\theta), \pi^U_\varpi(\theta)]$ is an interval-valued fuzzy set, where $\tilde{\pi}_\varpi : \aleph \to D[0,1]$ and $\alpha_\varpi: \aleph \to [0,1]$ is a fuzzy set.[11]

2.18.Definition The level subset of $\varpi = \{\langle \theta, \tilde{\pi}_\varpi(\theta), \alpha_\varpi(\theta) \rangle: \theta \in \aleph\}$ is defined as follows
$U(\varpi, \tilde{t}, r) = \{\theta \in \aleph: \tilde{\pi}_\varpi(\theta) \geq \tilde{t}, \alpha_\varpi \leq r\}$, for every $[0,0] \leq \tilde{t} \leq [1,1]$ and $r \in [0,1]$.[27-30]

3.Cubic Q-ideals of Q-Algebras

Let $(\aleph, *, 0)$ is a Q-algebra. The structure $\varpi = \{\langle \theta, \tilde{\pi}_\varpi(\theta), \alpha_\varpi(\theta) \rangle: \theta \in \aleph\}$ is a cubic set in $\aleph$, where $\tilde{\pi}_\varpi : \aleph \to D[0,1]$ and $\alpha_\varpi: \aleph \to [0,1]$. For brevity, the cubic set in a Q-algebra is written as follows $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$. Also, we can define the level subset of ϖ as follows
$U(\varpi, \tilde{t}, r) = \{\theta \in \aleph: \tilde{\pi}_\varpi(\theta) \succcurlyeq \tilde{t}, \alpha_\varpi \leq r\}$, for every $[0,0] \preccurlyeq \tilde{t} \preccurlyeq [1,1]$ and $r \in [0,1]$.

3.1. Definition A cubic set $\varpi = \langle \tilde{\pi}_\varpi, \alpha_\varpi \rangle$ in $\aleph$ is named a cubic Q-sub-algebra if
(q_1) $\tilde{\pi}_\varpi(\theta * \daleth) \succcurlyeq rmin\{\tilde{\pi}_\varpi(\theta), \tilde{\pi}_\varpi(\daleth)\}$.
(q_2) $\alpha_\varpi(\theta * \daleth) \leq max\{\alpha_\varpi(\theta), \alpha_\varpi(\daleth)\}$, $\forall \theta, \daleth \in \aleph$.

3.2. Example Let $\aleph = \{0, m, n, v\}$ be a set defined by the table below.

Table 4. A cubic Q-sub-algebra

*	0	**m**	**n**	**v**
0	0	0	0	0
m	m	0	0	0
n	n	0	0	0
v	v	0	0	0

The triple $(\aleph, *, 0)$ is a Q-algebra. Define $\tilde{\pi}_{\varpi}(\theta)$ and $\alpha_{\varpi}(\theta)$ by

$$\tilde{\pi}_{\varpi}(\theta) = \begin{cases} [0.1,0.8] & if \quad \theta = \{0,m,n\} \\ [0.1,0.2] & if \quad \theta = v \end{cases}, \quad \alpha_{\varpi}(\theta) = \begin{cases} 0.1 & if \quad \theta = \{0,m,n\} \\ 0.5 & if \quad \theta = v \end{cases}.$$

We can simply prove that $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-sub-algebra.

3.3. Proposition Let $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ be a cubic Q-sub-algebra, then $\tilde{\pi}_{\varpi}(0) \succcurlyeq \tilde{\pi}_{\varpi}(\theta)$ and $\alpha_{\varpi}(0) \leq \alpha_{\varpi}(\theta), \forall \theta \in \aleph$.

Proof. Since $\theta * \theta = 0$, then $\tilde{\pi}_{\varpi}(0) = \tilde{\pi}_{\varpi}(\theta * \theta) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}(\theta), \tilde{\pi}_{\varpi}(\theta)\} = \tilde{\pi}_{\varpi}(\theta)$ and $\alpha_{\varpi}(0) = \alpha_{\varpi}(\theta * \theta) \leq max\{\alpha_{\varpi}(\theta), \alpha_{\varpi}(\theta)\} = \alpha_{\varpi}(\theta)$.

3.4. Theorem For a cubic set $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ in $\aleph$, the following are equivalent.

(i) $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-sub-algebra of $\aleph$.

(ii) Every nonempty level set of $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a Q-sub-algebra of $\aleph$.

Proof. (i) ⇒(ii) Assume $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-sub-algebra and $\tilde{t} \in D[0,1], r \in [0,1]$, where $U(\varpi, \tilde{t}, r) \neq \emptyset$. Then $\forall \theta, \gimel \in U(\varpi, \tilde{t}, r)$, we have $\tilde{\pi}_{\varpi}(\theta) \succcurlyeq \tilde{t}$, $\tilde{\pi}_{\varpi}(\gimel) \succcurlyeq \tilde{t}$ and $\alpha_{\varpi}(\theta) \leq r$, $\alpha_{\varpi}(\gimel) \leq r$ and since $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-sub-algebra, we have

$\tilde{\pi}_{\varpi}(\theta * \gimel) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}(\theta), \tilde{\pi}_{\varpi}(\gimel)\} = rmin\{\tilde{t}, \tilde{t}\} = \tilde{t}.$

$\alpha_{\varpi}(\theta * \gimel) \leq max\{\alpha_{\varpi}(\theta), \alpha_{\varpi}(\gimel)\} = max\{r, r\} = r,$

So that $\theta * \gimel \in U(\varpi, \tilde{t}, r)$. Hence, $U(\varpi, \tilde{t}, r)$ is a Q-sub-algebra of $\aleph$.

(ii) ⇒(i) let $U(\varpi, \tilde{t}, r)$ be a Q-sub-algebra and $\theta, \gimel \in \aleph$.

Take $\tilde{t} = rmin\{\tilde{\pi}_{\varpi}(\theta), \tilde{\pi}_{\varpi}(\gimel)\}$ and $r = max\{\alpha_{\varpi}(\theta), \alpha_{\varpi}(\gimel)\}$, for any $\theta, \gimel \in \aleph$.

It follows from the hypothesis that $U(\varpi, \tilde{t}, r)$ is Q-sub algebra of $\aleph$ implies: $\theta * \gimel \in U(\varpi, \tilde{t}, r)$, therefore $\tilde{\pi}_{\varpi}(\theta * \gimel) \succcurlyeq \tilde{t} = rmin\{\tilde{\pi}_{\varpi}(\theta), \tilde{\pi}_{\varpi}(\gimel)\}$ and $\alpha_{\varpi}(\theta * \gimel) \leq r = max\{\alpha_{\varpi}(\theta), \alpha_{\varpi}(\gimel)\}$. Hence, $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-sub-algebra of $\aleph$.

3.5. Definition A cubic set $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ of a BCK-algebra $\aleph$ is named a cubic BCK-ideal if:

$(H_1)\tilde{\pi}_{\varpi}(0) \succcurlyeq \tilde{\pi}_{\varpi}(\theta)$ and $\alpha_{\varpi}(0) \leq \alpha_{\varpi}(\theta)$.

(H_2) $\tilde{\pi}_{\varpi}(\theta) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}(\theta * \gimel), \tilde{\pi}_{\varpi}(\gimel)\}$ and $\alpha_{\varpi}(\theta) \leq max\{\alpha_{\varpi}(\theta * \gimel), \alpha_{\varpi}(\gimel)\}, \forall \theta, \gimel \in \aleph$.

3.6. Definition. Let $\aleph$ be a Q-algebra. A cubic set $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ in $\aleph$ is named a cubic Q-ideal if:

$(B_1)\tilde{\pi}_{\varpi}(0) \succcurlyeq \tilde{\pi}_{\varpi}(\theta) \ and \ \alpha_{\varpi}(0) \leq \alpha_{\varpi}(\theta),$

$(B_2)\tilde{\pi}_{\varpi}(\theta * \epsilon) \succcurlyeq rmin\left\{\tilde{\pi}_{\varpi}\left((\theta * \gimel) * \epsilon\right), \tilde{\pi}_{\varpi}(\gimel)\right\} \ and$

$\alpha_{\varpi}(\theta * \epsilon) \leq max\left\{\alpha_{\varpi}\left((\theta * \gimel) * \epsilon\right), \alpha_{\varpi}(\gimel)\right\}, \forall \theta, \gimel, \epsilon \in \aleph$..

3.7. Example Let $\aleph = \{0, m, n, v\}$ in example3.2, define a cubic set $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ in $\aleph$ as follows:

$$\tilde{\pi}_{\varpi}(\theta) = \begin{cases} [0.1,0.9] & if \quad \theta = 0 \\ [0.3,0.5] & if \quad \theta \in \{m,n\} \\ [0.1,0.2] & if \quad \theta \in v \end{cases}, \quad \alpha_{\varpi}(\theta) = \begin{cases} 0.1 & \text{if} \quad \theta = 0 \\ 0.4 & \text{if} \quad \theta \in \{m,n\} \\ 0.8 & \text{if} \quad \theta \in v \end{cases}$$

Then, $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-ideal.

3.8. Proposition for a Q-algebra $\aleph$. If $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-ideal, then $\theta \leq \gimel$ implies $\tilde{\pi}_{\varpi}(\theta) \succcurlyeq \tilde{\pi}_{\varpi}(\gimel)$ and $\alpha_{\varpi}(\theta) \leq \alpha_{\varpi}(\gimel)$.

Proof. Let $\theta \leq \gimel$, then $\theta * \gimel = 0$ and $\theta * 0 = \theta$. Since $\langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-ideal, then

$\tilde{\pi}_{\varpi}(\theta * 0) = \tilde{\pi}_{\varpi}(\theta) \succcurlyeq rmin\left\{\tilde{\pi}_{\varpi}\left((\theta * \gimel) * 0\right), \tilde{\pi}_{\varpi}(\gimel)\right\} \succcurlyeq rmin\{\tilde{\pi}_{\varpi}(0 * 0), \tilde{\pi}_{\varpi}(\gimel)\}$

$\succcurlyeq rmin\{\tilde{\pi}_{\varpi}(0), \tilde{\pi}_{\varpi}(\gimel)\} = \tilde{\pi}_{\varpi}(\gimel)$. And

$\alpha_{\varpi}(\theta * 0) = \alpha_{\varpi}(\theta) \leq max\left\{\alpha_{\varpi}\left((\theta * \gimel) * 0\right), \alpha_{\varpi}(\gimel)\right\} \leq max\{\alpha_{\varpi}(0 * 0), \alpha_{\varpi}(\gimel)\}$

$\leq max\{\alpha_{\varpi}(0), \alpha_{\varpi}(\gimel)\} \leq \alpha_{\varpi}(\gimel)$. So, $\tilde{\pi}_{\varpi}(\theta) \geq \tilde{\pi}_{\varpi}(\gimel)$ and $\alpha_{\varpi}(\theta) \leq \alpha_{\varpi}(\gimel)$.

3.9. Proposition for a Q-algebra $\aleph$. If $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-ideal, then
$\tilde{\pi}_{\varpi}(\theta * (\theta * \gimel)) \succcurlyeq \tilde{\pi}_{\varpi}(\gimel)$, and $\alpha_{\varpi}(\theta * (\theta * \gimel)) \leq \alpha_{\varpi}(\gimel)$.

Proof. By Proposition 2.5 and Proposition 3.8, we get
$\tilde{\pi}_{\varpi}(\theta * (\theta * \gimel)) \succcurlyeq \tilde{\pi}_{\varpi}(\gimel), \alpha_{\varpi}(\theta * (\theta * \gimel)) \leq \alpha_{\varpi}(\gimel)$.

3.10. Proposition Let $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ be a cubic Q-ideal. If $\theta * \gimel \leq \epsilon$, then
$\tilde{\pi}_{\varpi}(\theta) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}(\epsilon), \tilde{\pi}_{\varpi}(\gimel)\}$ and $\alpha_{\varpi}(\theta) \leq max\{\alpha_{\varpi}(\epsilon), \alpha_{\varpi}(\gimel)\}$.

Proof. Let $\theta * \gimel \leq \epsilon$ in $\aleph$, then $(\theta * \gimel) * \epsilon = 0$ and we have
$\tilde{\pi}_{\varpi}(\theta * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * \gimel) * \epsilon), \tilde{\pi}_{\varpi}(\gimel)\}$, if we put $\epsilon = 0$

Then, $\tilde{\pi}_{\varpi}(\theta * 0) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * \gimel) * 0), \tilde{\pi}_{\varpi}(\gimel)\}$

$$\tilde{\pi}_{\varpi}(\theta) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}(\theta * \gimel), \tilde{\pi}_{\varpi}(\gimel)\} \quad \mathbf{(1)}$$

But, $\tilde{\pi}_{\varpi}(\theta * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * \epsilon) * \gimel), \tilde{\pi}_{\varpi}(\epsilon)\}$

$$\tilde{\pi}_{\varpi}(\theta * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * \gimel) * \epsilon), \tilde{\pi}_{\varpi}(\epsilon)\} = rmin\{\tilde{\pi}_{\varpi}(0), \tilde{\pi}_{\varpi}(\epsilon)\} = \tilde{\pi}_{\varpi}(\epsilon) \quad \mathbf{(2)}$$

From **Equations (1) and (2)** we get $\tilde{\pi}_{\varpi}(\theta) \geq rmin\{\tilde{\pi}_{\varpi}(\epsilon), \tilde{\pi}_{\varpi}(\gimel)\}$.
Similarly, we can show that $\alpha_{\varpi}(\theta) \leq max\{\alpha_{\varpi}(\epsilon), \alpha_{\varpi}(\gimel)\}$.

3.11. Theorem In a Q-algebra $\aleph$, every cubic Q-ideal is a cubic BCK-ideal.

Proof. Take $\epsilon = 0$ in (B_2), then $\tilde{\pi}_{\varpi}(\theta) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * \gimel)), \tilde{\pi}_{\varpi}(\gimel)\}$ *and*
$\alpha_{\varpi}(\theta) \leq max\{\alpha_{\varpi}((\theta * \gimel)), \alpha_{\varpi}(\gimel)\}$. Hence, $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic BCK-ideal.

The converse of Theorem 3.11 is not generally true; that is, a cubic BCK-ideal is not necessarily a cubic Q-ideal. The following example illustrates this fact.

3.12. Example Let $\aleph = \{0, f, g, j\}$ be a set and $*$ defined by the table below.

Table 5. A cubic BCK-ideal

$*$	0	***f***	***g***	***j***
0	0	0	0	0
f	f	0	0	f
g	g	g	0	0
j	j	0	j	0

Then $(\aleph, *, 0)$ is a Q-algebra. Define $\tilde{\pi}_{\varpi}(\theta)$, $\alpha_{\varpi}(\theta)$ by

$$\tilde{\pi}_{\varpi}(\theta) = \begin{cases} [0.1,0.8] & if \quad \theta = \{0, f, g\} \\ [0.1,0.3] & if \quad \theta = j \end{cases}, \quad \alpha_{\varpi}(\theta) = \begin{cases} 0.1 & if \quad \theta = \{0, f, g\} \\ 0.8 & if \quad \theta = j \end{cases},$$

Now, $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic BCK-ideal but it is not a cubic Q-ideal since
$\tilde{\pi}_{\varpi}(j * g) \prec rmin\{\tilde{\pi}_{\varpi}((j * f) * g), \tilde{\pi}_{\varpi}(f)\}$ *and* $\alpha_{\varpi}(j * g) > max\{\alpha_{\varpi}((j * f) * g), \alpha_{\varpi}(f)\}$.

3.13. Theorem Let $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ be a cubic set of $\aleph$. Then $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-ideal if and only if a nonempty level set $U(\varpi, \tilde{t}, r)$ of ϖ is a Q-ideal.

Proof. Let $U(\varpi, \tilde{t}, r) \neq \emptyset$ be a Q-ideal, for any $\theta \in \aleph$, then
$\tilde{\pi}_{\varpi}(\theta) = \tilde{t}$, $\forall \tilde{t} \in D[0,1]$, $\alpha_{\varpi}(\theta) = r$, $\forall r \in [0,1]$ so $\theta \in U(\varpi, \tilde{t}, r)$ and since $0 \in U(\varpi, \tilde{t}, r)$, then $\tilde{\pi}_{\varpi}(0) \succcurlyeq \tilde{t} = \tilde{\pi}_{\varpi}(\theta)$, and $\alpha_{\varpi}(0) \leq r = \alpha_{\varpi}(\theta)$, and

Now, we prove the condition (B_2), if not, assume that there exists $c, d, e \in \aleph$ such that
$\tilde{\pi}_{\varpi}(c * e) \prec rmin\{\tilde{\pi}_{\varpi}((c * d) * e)), \tilde{\pi}_{\varpi}(d))\}$
$\alpha_{\varpi}(c * e) > max\{\alpha_{\varpi}((c * d) * e)), \alpha_{\varpi}(d)\}$.

Let take β, and δ as

$$\beta = \frac{1}{2}\{\tilde{\pi}_{\varpi}(c * e) + rmin\{\tilde{\pi}_{\varpi}((c * d) * e)), \tilde{\pi}_{\varpi}(d))\}, where\ \beta \in D[0,1]$$

$$\delta = \frac{1}{2}\{\alpha_{\varpi}(c * e) + max\{\{\alpha_{\varpi}((c * d) * e)), \alpha_{\varpi}(d)\}, where\ \delta \in [0,1]$$

i.e $\tilde{\pi}_{\varpi}(c * e) \prec \beta \prec rmin\{\tilde{\pi}_{\varpi}((c * d) * e)), \tilde{\pi}_{\varpi}(d))\}$, and
$\alpha_{\varpi}(c * \mathrm{e}) > \delta > max\{\alpha_{\varpi}((c * d) * e)), \alpha_{\varpi}(d)\}$, it is followed

$(c * d) * e) \in U(\varpi, \tilde{t}, r)$, $d \in U(\varpi, \tilde{t}, r)$, but $(c * \mathrm{e}) \notin U(\varpi, \tilde{t}, r)$, but the set $U(\varpi, \tilde{t}, r)$ is a Q-ideal. This is a contradiction. So $\tilde{\pi}_{\varpi}(c * e) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((c * d) * e)), \tilde{\pi}_{\varpi}(d))\}$
$\alpha_{\varpi}(c * e) \leq max\{\alpha_{\varpi}((c * d) * e)) , \alpha_{\varpi}(d)\}$, and ϖ is a cubic Q-ideal.
Conversely, Assume that $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-ideal and $\tilde{t} \in D[0,1]$, $r \in [0,1]$ s.t. $U(\varpi; \tilde{t}, r) \neq \emptyset$, Then there exists $\theta \in U(\varpi, \tilde{t}, r)$, then $\tilde{\pi}_{\varpi}(\rho) \succcurlyeq \tilde{t}$ and $\alpha_{\varpi}(\theta) \leq r$. It follows from Definition 3.6 (B_1) that $\tilde{\pi}_{\varpi}(0) \succcurlyeq \tilde{\pi}_{\varpi}(\theta) \succcurlyeq \tilde{t}$ and $\alpha_{\varpi}(0) \leq \alpha_{\varpi}(\rho) \leq r$, hence $0 \in U(\varpi, \tilde{t}, r)$.
Now, let $((\theta * ℷ) * \epsilon) \in U(\varpi, \tilde{t}, r)$, $ℷ \in U(\varpi, \tilde{t}, r)$, Then $\tilde{\pi}_{\varpi}((\theta * ℷ) * \epsilon) \succcurlyeq \tilde{t}$ and $\tilde{\pi}_{\varpi}(ℷ) \succcurlyeq \tilde{t}$, $\alpha_{\varpi}((\theta * ℷ) * \epsilon) \leq r$ and $\alpha_{\varpi}(ℷ) \leq r$. Then from condition (B_2), we get
$\tilde{\pi}_{\varpi}(\theta * \epsilon) \succcurlyeq rmin\{\tilde{\pi}_{\varpi}((\theta * ℷ) * \epsilon), \tilde{\pi}_{\varpi}(ℷ)\} = rmin\{\tilde{t}, \tilde{t}\} = \tilde{t}$ and
$\alpha_{\varpi}(\theta * \epsilon) \leq max\{\alpha_{\varpi}((\theta * ℷ) * \epsilon), \alpha_{\varpi}(ℷ)\} = \max\{r, r\} = r$.
So that $\theta * \epsilon \in U(\varpi, \tilde{t}, r)$. Therefore, $U(\varpi, \tilde{t}, r)$ is a Q-ideal.

4.The image (inverse image) of cubic sets

In this section, we present definitions of the image and the inverse image of a cubic Q-ideal, along with some results on this concept.

4.1. Definition Let $\mathcal{M}: ℵ \to ℵ'$ be a mapping from Q-algebraℵ into Q-algebra $ℵ'$ and $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ be a cubic set of ℵ, then

$$\mathcal{M}(\tilde{\pi}_{\varpi})(ℷ) = B(ℷ) = \begin{cases} \underset{\theta \in \mathcal{M}^{-1}(ℷ)}{rsup} \ \tilde{\pi}_{\varpi}(\theta), & if \ \ \mathcal{M}^{-1}(ℷ) = \{\theta \in ℵ, \mathcal{M}(\theta) = ℷ\} \neq \emptyset \\ 0 & otherwise \end{cases}$$

$$\mathcal{M}(\alpha_{\varpi})(ℷ) = C(ℷ) = \begin{cases} \inf_{\theta \in \mathcal{M}^{-1}(ℷ)} \alpha_{\varpi}(\theta) & if \ \ \mathcal{M}^{-1}(ℷ) = \{\theta \in ℵ, \mathcal{M}(\theta) = ℷ\} \neq \emptyset \\ 1 & otherwise \end{cases}$$

is said to be the image of $\tilde{\pi}_{\varpi}$ and α_{ϖ} under $\mathcal{M}$ and if $\eta = \langle \tilde{\pi}_{\eta}, \alpha_{\eta} \rangle$ is a cubic set in $ℵ'$, then we can define the inverse image for η by $\mathcal{M}^{-1}(\eta) = \langle \theta, \mathcal{M}^{-1}(\tilde{\pi}_{\eta}), \mathcal{M}^{-1}(\alpha_{\eta}) \rangle$, where $\mathcal{M}^{-1}(\tilde{\pi}_{\eta}(\theta)) = \tilde{\pi}_{\eta}(\mathcal{M}(\theta))$ and $\mathcal{M}^{-1}(\alpha_{\eta}(\theta)) = \alpha_{\eta}(\mathcal{M}(\theta))$, for any $\theta \in ℵ$.

4.2. Theorem Let $\mathcal{M}: ℵ \to ℵ'$ be a homeomorphism between Q-algebra ℵ and $ℵ'$. If $\eta = \langle \tilde{\pi}_{\eta}, \alpha_{\eta} \rangle$ is a cubic Q-ideal of $ℵ'$, then the inverse image of η is a cubic Q-ideal of ℵ.

Proof. We have $\mathcal{M}^{-1}\left(\tilde{\pi}_{\eta}(0)\right) = \tilde{\pi}_{\eta}(\mathcal{M}(0)) = \tilde{\pi}_{\eta}(0') \succcurlyeq \tilde{\pi}_{\eta}(\theta') = \tilde{\pi}_{\eta}(\mathcal{M}(\theta)) = \mathcal{M}^{-1}(\tilde{\pi}_{\eta}(\theta))$
And $\mathcal{M}^{-1}\left(\alpha_{\eta}(0)\right) = \alpha_{\eta}(\mathcal{M}(0)) = \alpha_{\eta}(0') \leq \alpha_{\eta}(\theta') = \alpha_{\eta}(\mathcal{M}(\theta)) = \mathcal{M}^{-1}(\alpha_{\eta}(\theta))$
$\forall\, \theta, ℷ, \epsilon \in ℵ$, we have $\mathcal{M}^{-1}\left(\tilde{\pi}_{\eta}(\theta * \epsilon)\right) = \tilde{\pi}_{\eta}(\mathcal{M}(\theta * \epsilon)) = \tilde{\pi}_{\eta}(\mathcal{M}(\theta) *' \mathcal{M}(\epsilon))$

$$\succcurlyeq rmin\left\{\tilde{\pi}_{\eta}\left((\mathcal{M}(\theta) *' \mathcal{M}(ℷ)) *' \mathcal{M}(\epsilon)\right), \tilde{\pi}_{\eta}(\mathcal{M}(ℷ))\right\}$$

$$= rmin\left\{\tilde{\pi}_{\eta}\left(\mathcal{M}((\theta * ℷ) * \epsilon)\right), \tilde{\pi}_{\eta}(\mathcal{M}(ℷ))\right\} = rmin\{\tilde{\pi}_{\varpi}((\theta * ℷ) * \epsilon), \tilde{\pi}_{\varpi}(ℷ)\},$$

$$\mathcal{M}^{-1}\left(\alpha_{\eta}(\theta * \epsilon)\right) = \alpha_{\eta}(\mathcal{M}(\theta * \epsilon)) = \alpha_{\eta}(\mathcal{M}(\theta) *' \mathcal{M}(\epsilon))$$

$$\leq max\left\{\alpha_{\eta}\left((\mathcal{M}(\theta) *' \mathcal{M}(ℷ)) *' \mathcal{M}(\epsilon)\right), \alpha_{\eta}(\mathcal{M}(ℷ))\right\}$$

$$= max\left\{\alpha_{\eta}\left(\mathcal{M}((\theta * ℷ) * \epsilon)\right), \alpha_{\eta}(\mathcal{M}(ℷ))\right\} = max\{\alpha_{\eta}((\theta * ℷ) * \epsilon), \alpha_{\eta}(ℷ)\}.$$

4.3. Theorem Let $\mathcal{M}: ℵ \to ℵ'$ be an epimorphism mapping from Q-algebra ℵ to Q-algebra $ℵ'$, if $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-ideal of ℵ, then $\mathcal{M}(\varpi)$ is a cubic Q-ideal of $ℵ'$.
Proof. For each $\theta', ℷ' \in ℵ'$, $\exists\, \theta, ℷ \in ℵ$ s. t. $\mathcal{M}(\theta) = \theta'$ and $\mathcal{M}(ℷ) = ℷ'$, we have
$\mathcal{M}(\tilde{\pi}_{\varpi})(\theta') = rsup_{\theta \in \mathcal{M}^{-1}(\theta')} \tilde{\pi}_{\varpi}(\theta)$, for some $\theta \in ℵ$, and
$\mathcal{M}(\alpha_{\varpi})(\theta') = inf_{\theta \in \mathcal{M}^{-1}(\theta')} \alpha_{\varpi}(\theta)$, for some $\theta \in ℵ$, and since $\varpi = \langle \tilde{\pi}_{\varpi}, \alpha_{\varpi} \rangle$ is a cubic Q-ideal, then $\tilde{\pi}_{\varpi}(0) \succcurlyeq \tilde{\pi}_{\varpi}(\theta), \alpha_{\varpi}(0) \leq \alpha_{\varpi}(\theta), \forall\, \theta \in ℵ$, so

$\mathcal{M}(\tilde{\pi}_{\varpi})(0') = rsup_{0\in\mathcal{M}^{-1}(0')}\tilde{\pi}_{\varpi}(0) \succcurlyeq rsup_{\theta\in\mathcal{M}^{-1}(\theta')}\tilde{\pi}_{\varpi}(\theta) = \mathcal{M}(\tilde{\pi}_{\varpi})(\theta')$

And $\mathcal{M}(\alpha_{\varpi})(0') = inf_{0\in\mathcal{M}^{-1}(0')}\alpha_{\varpi}(0) \leq inf_{\theta\in\mathcal{M}^{-1}(\theta')}\alpha_{\varpi}(\theta) = \mathcal{M}(\alpha_{\varpi})(\theta')$

For any $\theta', ƛ', \epsilon' \in \aleph'$, let $\theta_0 \in \mathcal{M}^{-1}(\theta'), ƛ_0 \in \mathcal{M}^{-1}(ƛ')$, and $\epsilon_0 \in \mathcal{M}^{-1}(\epsilon')$, we have

$\mathcal{M}(\tilde{\pi}_{\varpi})(\theta' * \epsilon')=rsup\ \tilde{\pi}_{\varpi}(\theta_0 * \epsilon_0)_{(\theta_0 * \epsilon_0)\in\mathcal{M}^{-1}(\theta' * \epsilon')}$

$\succcurlyeq rmin\{rsup\ \tilde{\pi}_{\varpi}\big((\theta_0 * ƛ_0) * \epsilon_0\big)_{((\theta_0 * ƛ_0)*\epsilon_0)\in\mathcal{M}^{-1}((\theta' * ƛ')*\epsilon')}, rsup\tilde{\pi}_{\varpi}(ƛ_0)_{ƛ_0\in\mathcal{M}^{-1}(ƛ')}\}$

$= rmin\ \{\mathcal{M}(\tilde{\pi}_{\varpi})((\theta' * ƛ') * \epsilon'), \mathcal{M}(\tilde{\pi}_{\varpi})(ƛ')\}$. And

$\mathcal{M}(\alpha_{\varpi})(\theta' * \epsilon')=inf\alpha_{\varpi}(\theta_0 * \epsilon_0))_{(\theta_0 * \epsilon_0)\in\mathcal{M}^{-1}(\theta' * \epsilon')}$

$\leq max\{\ inf\alpha_{\varpi}\big((\theta_0 * ƛ_0) * \epsilon_0\big)_{((\theta_0 * ƛ_0)*\epsilon_0)\in\mathcal{M}^{-1}((\theta' * ƛ')*\epsilon')}, inf\alpha_{\varpi}(ƛ_0)_{ƛ_0\in\mathcal{M}^{-1}(ƛ')}\}$

$=max\{\mathcal{M}(\alpha_{\varpi})((\theta' * ƛ') * \epsilon'), \mathcal{M}(\alpha_{\varpi})(ƛ')\}$.

Hence, $\mathcal{M}(\varpi)$ is cubic Q-ideal of $\aleph'$.

5. Conclusion

This study explored cubic ideals in Q-algebra with the application of the idea of cubic ideals, including (BCK-ideals and Q-ideals) of Q-algebras, and discussed some of their important properties. A few relationships between a cubic sub-Q-algebra and a sub-Q-algebra using the level set of Q-algebras were presented. After that, we have proven that the image and inverse image of a cubic Q-ideal are also cubic Q-ideals by the isomorphism mapping. It is possible that in the future, we will work on the neutrosophic cubic Q-ideal and Filteristic Soft ideals of Q-algebras. Additionally, we can study the notion of a cubic intuitionistic Q-ideal.

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Conflict of Interest

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