

The Mean Time of Strip Domain Stay in Metastable State

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Abstract

The phenomenon of the thermoinduced jumps of the strip domains length in imperfect magnetic films with perpendicular with anisotropy is studied. For describing of the strip domain transitions into other metastable states, the stochastic equation of its head movement is used. In this equation the spatial non-uniformities of the medium are modeled by the restricted random function, and thermal fluctuations by the Gaussian white noise. The dependences on the temperature and the bias field of the mean time of strip domain stay in a metastable state were calculated by the methods of the Markovian processes theory.

Introduction

In the perfect magnetic films with perpendicular anisotropy the long strip domain at $H > H_{cr}$ (H is the bias field, H_{cr} is the critical bias field) should be continuously reduced, at $H < H_{cr}$ – extended, and at $H = H_{cr}$ may have any length (1). But in imperfect films the ends of the strip domain are fixed on the nonuniformities. This means that in some interval of the bias fields (H_1, H_2) including H_{cr} , the strip domain may be in a metastable state characterized by its varied length. Under influence of thermal fluctuation the domain may perform the transitions between these states which at $H > H_{cr}$ should be accompanied with discontinuous decrease of its length, and at $H < H_{cr}$ with discontinuous increase. Such thermoinduced jumps of the strip domain length were discovered in (2).

In this paper we solve the problem of main time the strip domain stay in metastable state. Our approach to solving this problem is based on the following stochastic equation of the strip domain head.

$$\xi' = \mu [R(H) + G_r(x)] + f(t) \dots \dots \dots [1.]$$

where $\xi = \xi(t)$ is the domain head coordinate, μ is the domain mobility $R(H)$ is the known function of H and other parameters (1), G is the peak value of local coercive field, $r(x)$ is a restricted ($|r(x)| \leq 1$) random function taking into account the film non-uniformities.

If the characteristic frequency of the thermal fluctuations taking into account by the Gaussian white noise which is defined by $\langle f(t) \rangle = 0$ and

$$\langle f(t+\tau)f(t) \rangle = \sigma^2(t)\delta(t) \approx 2\Delta\delta(\tau) \dots \dots \dots [2]$$

where Δ or σ^2 is the white noise intensity $\delta(t)$ is the Dirac delta function and $\langle \rangle$ designates the averaging on white noise realizations. In this case is the Markovian process and the problem of finding the meantime (T) of the strip domain stay in a metastable state mainly reduces to the first passage time problem in the random processes theory (3). Within the framework of such approach we found the general expression for T calculated T in two cases responding to various random functions $r(x)$ and determined the dependence of T on H and temperature (θ).

Theory

Suppose that at $R(H) > 0$ the domain head at the initial moment of time $t = \tau$ has the coordinate $\xi(\tau) = y$ and is within the interval $(-k, q)$ {at $R(H) < 0$ within the interval $(-q, k)$ }. Consider the left bound { $R(H) < 0$ - the right one} of this interval as reflection - the domain head can not cross it let $T(y, \tau)$ be the mean {on realization of $f(t)$ } time necessary to the domain head for the exit from the interval $(-k, q)$ in the case of any realization $\psi(x)$ of the random function $r(x)$ satisfying the conditions:

$\psi(x) < -g$ at $0 < x < q$, $\psi(0) \geq -g$ and $\psi(x) > -g$ { $g = |R(H)| / G$ }
 {at $R(H) < 0$ - the conditions $\psi(x) > g$ at $-q < x < 0$, $\psi(-q) \leq -g$ and $\psi(0 \leq g)$ }.

Then we shall define T as:

$$T = \int_0^{\infty} du P(u) \langle T(0,0) \rangle_{\psi|u} \dots \dots \dots [3]$$

where P(u) is the probability density of the realizations $\psi(x)$ with $q=u$, and $\langle \rangle_{\psi|u}$ designates the averaging on realization $\psi(x)$.

For calculating T(0,0) we shall use the method (3) based on receiving of the equation for the function $T(y,0)=T(y)$. For this purpose we shall first right down the reverse Fokker-Planck equation corresponding to equation [1]:

$$\frac{dp}{d\tau} = -\mu [r(H) + G\psi(y)] \frac{\partial P}{\partial y} \Delta \frac{\partial^2 P}{\partial y^2} \dots \dots \dots [4]$$

where $p = (x,t|y,\tau)$ is the probability density of the domain heat transition from the point $\xi(\tau) = y$ to the point $\xi(t) = x$. Then with help of this probability density we shall define the probability $w(y,t)$ of that the domain heat which does not came out from the interval $(-k,q)$ during the time (t):

$$w(y,t) = \int_{-k}^q dx P(x,t | y,0) \dots \dots \dots [5]$$

Further using the correlation.

$$T(y) = - \int_0^{\infty} t \frac{\partial}{\partial t} w(y,t) dt \dots \dots \dots [6]$$

And noticing that $w(y,0) = 1$ and $w(y,\infty) = 0$, eq.(4) yields:

$$\Delta T'(y) + \mu [R(H) + G\psi(y)] T = -1 \dots \dots \dots [7]$$

(The stroke indicates the differentiation on y). Solving conditions eq.[7] with the boundary conditions $T(q) = T(-k) = 0$ and taking account of the definition [3] the mean time of the strip domain stay in a metastable state may be written in the form:

$$T = \frac{1}{\Delta} \int_0^{\infty} du \int_0^u dx \int_0^{k+x} dz P(u < F(x, z) >_{\psi|u} \dots\dots\dots [8]$$

Here

$$F(x, z) = \exp[-az - b \int_{x-z}^x \psi(y) dy] \dots\dots\dots [9]$$

$a = \mu | R(H) | / \Delta$ and $b = \mu G / \Delta$

The similar calculations for $R(H) < 0$ leads to same expression [8]. Notice the formula [8] is applied if H satisfies the condition $R(H) \leq G$ otherwise there are no metasable state of the strip domain.

Random Jump Function

Using the general formula [8], we shall calculate T in the case where the random function $r(x)$ assumes the value 1 and -1 with the probability $1/2$ each. Suppose that the distribution function $p(s)$ of fixed sign interval of $r(x)$ depends only on their lengths and has the form $p(s) = \beta \exp(-\beta.s)$ ($\beta > 0$). In this case the probability of realizations of $r(x)$ which on the interval $(0, z)$ dose not change a sign in $\exp(-\beta.s)$ and the probability of the realizations which on the interval $(0, z)$ has n ($n \geq 1$) changes of a sign at the points $x_i = s_1 + \dots + s_i$ ($i = 1, 2, \dots, n$) belonging to intervals ds_i so

$$dW_z^n = x^n \exp(-\beta.z) \prod_{i=1}^n ds_i \dots\dots\dots [10]$$

In accordance with this we have $\langle F(x, z) \rangle_{\psi|u} = \exp(bz - az)$ at $z \leq x$, and

$$\langle F(x, z) \rangle_{\psi|u} = \exp(b-a)x \left[\exp(b+\beta)(x-z) \right] + \sum_{n=1}^{\infty} dW_{z-x}^n \exp \left(b \sum_{i=1}^n (-1)^i s_i \right) \dots\dots$$

at $z > x$ (the integration is carried out on variables s_i satisfying the conditions $x_n \leq z-x, s_i \geq 0$).

using the above expression and taking into account that in the given case $p(u) = \beta \exp(-\beta u)$, eq.[8] at $k = \infty$ reduces to

$$T = \frac{a + b + 2\beta}{\Delta\beta (a^2 + 2\beta a - b^2)} \dots\dots\dots [12]$$

For $b > a \Rightarrow A = \{(a^2 + \beta^2)^{1/2} - \beta\}$, and $T = \infty$ for $a \leq A$. Since at $\Delta \neq 0$ the strip domain head cannot be localized in any point of space, the last result means that at $a \leq A$ the realization $\psi(x)$ and $f(t)$ on which the domain heat moves to the reflecting boundary (the infinity) gives the main contribution into T . if $\Delta = 0$ then the domain head will be localized at origin and therefore, $T = \infty$ in the whole interval of baise fields where metastable states of the strip domains exist.

The Step Function With Random Size

Consider more real case (from the point of view of experimental conditions (2) when

$$\psi(x) = -\psi u(x) u(u-x) \dots\dots\dots [13]$$

Here $u(x)$ is unit step function, u and ψ are statistically independent random values distributed in intervals $(a/b, 1)$ & $(0, w)$ correspondingly. Let $D(\psi)$ and $p(u)$ be the distribution of these random values. Then according to eqs. [8], and [13], T at $k = 0$ may be written in the form.

$$T = \frac{1}{\Delta b} \int_0^w du p(u) u^{\int_0^{u(b-a)} dy D\left[\frac{y+ua}{ub}\right] \frac{e^y - y - 1}{y^2}} \dots\dots\dots [14]$$

If ψ and u are distributed uniformly, i.e. $D(\psi) = b / (b-a)$ and $p(u) = 1/w$, then eq. [14] gives ($E = w(b-a)$).

$$T = \frac{w^2}{\Delta b} \sum_{n=0}^{\infty} \frac{E^n}{(n+3)!(n+1)} \dots\dots\dots [15]$$

Numerical Results and Discussion

On the bases of the formula [15] we calculated numerically the dependence of T on H and θ . For the calculating of dependence T on H we used the expression:

$$R(H) = -2M[c \ln(1+1/c^2) + \ln(1+c^2) - 2\pi\ell/c] \dots\dots\dots [16]$$

Following from the results (1), and the equation for the equilibrium width of the strip domain (4).

$$\frac{-h}{4\sqrt{\pi M}} = \frac{1}{\pi} \left[2 \arctan \frac{1}{c} - c \ln \left\{ 1 + \frac{1}{c^2} \right\} \right] \dots\dots\dots [17]$$

where M is the magnetization, c and ℓ are the width of strip domain and the characteristic length of the material normalized on the film thickness h . the temperature dependence T was obtained on the bases of expressions.

$$\mu = \mu_0 / (f_1^{1/2}(r) f_2(r)), \quad M = M_0 m(r),$$

$$G = G_0 m(r) f_1^{3/2}(r), \quad \ell = \ell_0 f_1^{1/2}(r)$$

$$\Delta = dr / (f_1^{1/2}(r) f_2^2(r) c) \dots\dots\dots [18]$$

Here the zero subscripts denote the corresponding values at $\theta = 0$, $r = 0$, $1/3C$, C is the Curie temperature, d is the parameter, and $f_1(r)$, $f_2(r)$ and $m(r)$ are defined by (5)

$$f_1(r) = \frac{1-3r}{m^2(r)}, \quad f_2(r) = \frac{1-r}{m(r)},$$

$$m(r) = \coth(m(r)/r) - r/m(r) \dots\dots\dots [19]$$

Figure (1) illustrates the main peculiarities of the behavior T as a function of H . The bounds of the field interval of the strip domain metastability satisfying equation $\{R(H)\} = G$, are marked in figure (1) as H_1 and H_2 .

The slight asymmetry of the shown dependences concerning the dashed line is caused by the dependence of c on H . With temperature increase the metastability interval is constructed and displaced to the coordinates origin, and the maximum value of T (at $H = H_0$) is decreased.

Figure (2) illustrates the behavior of T as a function of θ at $H = H_1 + (H_2 - H_1)\eta$ ($0 \leq \eta \leq 1$). in the case when η (or $\eta = 1$) Eq. reduces to $T = w^2/6\Delta$, and the temperature dependence of T is determined only by the temperature dependence of Δ .

References

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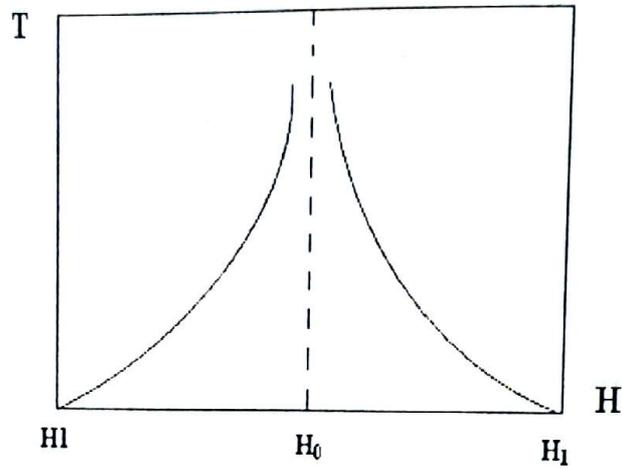


Fig.(1) The relation between T & H

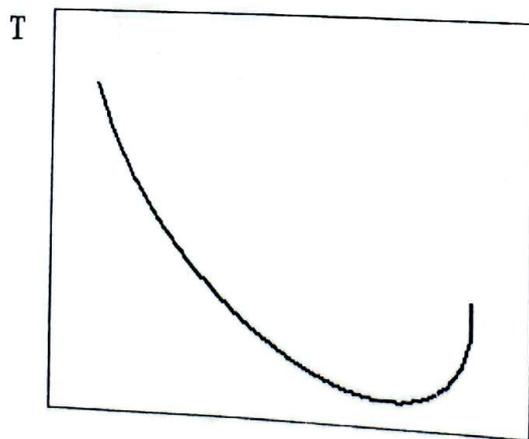


Fig.(2) The relation between T & θ

المتوسط الزمني لبقاء الحجيرات الحادة في الحالة شبه المستقرة

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الخلاصة

تمت دراسة ظاهرة القفزات الترموحتية للحجيرات الطولية الحادة في أغشية المواد المغناطيسية (الحاوية على شوائب) في أوضاع غير موحدة الخواص (anisotropy). استعملنا المعادلة العشوائية (stochastic equation) لغرض وصف المقطع الانتقالي الحاد للحجيرات في حالات شبه مستقرة أخرى عند أوج الحركة للحجيرات. في هذه المعادلة تمت صياغة عدم الانتظام الفضائي للوسط بالاعتماد على الدالة العشوائية والتغيرات الحرارية بواسطة الضوضاء الكاوسية البيضاء. وباستخدام طرق إجرائية من نظرية ماركوف تم حساب الاعتماد على درجة الحرارة واتجاه المجال المؤثر لمتوسط بقاء الحجيرات الحادة في الحالة شبه المستقرة.