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Covering Theorem for Alternating Groups

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Abstract

For the alternating group A_n, J. L. Brenner, through a series of papers showed that the class C of n- cycle or (n-1)- cycle, CCC covers A_n for n = 4k – 1, 4k and 4k + 1. In this paper, the conjugacy class of type (2)(n – 2), n > 6 and n is even was studied, this class splits into two classes of A_n denoted by C and C', CCC= $C'C'C'$ was found.

Introduction

See [1], In the symmetric group S_n (n > 4), even permutation is a product of two m – cycles if and only if [3n/4] < m ≤ n [2].
For odd m < n – 1, this is a property of a class in A_n, i.e.. the m – cycles in A_n from a single class C_m, C_m C_m covers A_n if [3n/4] < m ≤ n. For even n > 6 the class of type (2)(n – 2) splits into two classes C and C'. Covering by products of these classes is delicately dependent on this bifurcation. The above - mentioned result that $(C\cup C)(C\cup C')$ covers A_n states only that $CC \cup CC' \cup C'C'$ covers A_n in this case of the class of the type (2)(n – 2).

Results

Introduction must intervene somewhere in any attempt to refine the results of [3]. The induction commences with lemma 1.
Lemma (1): let C be the class of the (2)(6) – cycle a = (123456)(78) in A_8, let b another element in C. Then CC covers A_8 except for 1.

Proof:

$b \in C$	ab
(123456)(78)	(14)(26)
(154326)(78)	(165)
(123456)(78)	(135)(246)
(123645)(78)	(1354)(26)
(175234)(68)	(13)(24)(58)(67)
(134678)(25)	(146)(25(38)
(123458)(67)	(1347)(2486)
(126473)(58)	(162)(37548)
(154263)(78)	(16532)
(134867)(25)	(157638)(24)
(157328)(46)	(1836547)

Lemma (2): Let n > 6 be even, m = n – 2. Let C be the class of (12…….m)(n – 1 n) in A_n. The product CC includes the type (ab)(cd).

Proof:

If n = 4k +4.

(123456)(78) (124365)(78) = (14)(26)

(12345678910)(1112) (12431098765)(1112) = (14)(210)

(1234567891011121314)(11516) (1243141312111098765)(1516) = (14)(214)

(123456789101112131415161718)(1920)

(124318171615141312111098765)(1920) = (14)(218)

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(123…… m)(n – 1 n) (1243m m – 1 …… 65)(n – 1 n) = (14)(2m).

If n = 4k +6

(12345678)(910) (1243765)(910) = (14)(27)

(123456789101112)(1314) (112243111098765)(1314) = (14)(211)

(12345678910111213141516)(1718)

(1162431514131211098765)(1718) = (14)(215)

(1234567891011121314151617181920)(2122)

(1202431918171615141312111098765)(2122) = (14)(219)

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(123……m)(n – 1 n) (1m243m – 1 ….65)(n – 1 n) = (14)(2 m – 1).

Lemma(3): Let n > 6 be even, m = n – 2. Let C be the class of (12…….m)(n – 1 n) in A_n. The product CC covers the class of a 3 - cycle.

Proof:

If n = 4k +4

(123456)(78) (126543)(78) = (162)

(12345678910)(1112) (12109876543)(1112) = (1102)

(1234567891011121314)(1516) (1214131211109876543)(1516) = (1142)

(123456789101112131415161718)(1920)

(1218171615141312111098765 43)(1920) = (1182)

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(123……m)(n – 1 n) (12m m – 1 ….6543)(n – 1 n) = (1m2).

If n = 4k + 6

(12345678)(910) (18276543)(910) = (172)

(123456789101112)(1314) (112211109876543)(1314) = (1112)

(12345678910111213141516)(1718)

(116215141312111098765 43)(1718) = (1152)

(1234567891011121314151617181920)(2122)

(120219181716151413121110987654 3)(2122) = (1192)

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(123….m)(n – 1 n) (1m2m – 1……6543)(n – 1 n) = (1m – 12)

Theorem1. Let n > 6 be even, m = n – 2. Let C be the class of the permutation (12………..m)(n – 1 n) in A_n. Then CC covers A_n with the possible exception of 1.

Proof:

Lemma(1): establishes this assection for $n = 8$
Suppose the assertion to be true concerning A_{n-2}.
Let S be any permutation of A_n, let S' be a conjugate of S, such that
$S'(n\ n-2)(n-3\ n-1)$ do not move n and $n-1$,
By induction hypothesis $\Rightarrow S' = 1$ or two $n-2$-cycles b_1, b_2 in different classes in A_{n-2} exist, such that
$S' = b_1 b_2 (n\ n-2)(n-3\ n-1)$
$S'(n\ n-2)(n-3\ n-1) = b_1 b_2$
Now we put
$\xi = (n-3\ n\ n-1)$
$\beta_1 = b_1 \xi^{-1}$
$\beta_2 = \xi b_2 (n-2\ n-1\ n) \xi^{-1}$
s.t. $\beta_1 \beta_2$ are n –cycle belonging to the different classes in A_n. Then
$\beta_1 \beta_2 = S'$
To satisfy this, we must write b_1, b_2 in the following case:
$b_1 = (c_1\ c_2 \ldots\ldots c_{n-3}, n-3)(c_{n-5}\ c_{n-6})$
$b_2 = (d_1\ d_2 \ldots\ldots d_{n-3}, n-3)(d_{n-5}\ d_{n-6})$.

References

1.Brenner, J. L.and Carlitz, L. (1975)Rend Seminario Mathematics Padova, 55:81 – 90.
2.Brenner, J. L.; Cranwell, R. M. and Riddell, J. (1975). Pascific J. Mathematics, 58, 55 – 60.
3.Arad, Z. (2001) Juornal of Algebra, 103:241 – 255.

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نظرية الغطاء للزمر المتناوبة

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الخلاصة

في الزمرة المتناوبة A_n بيّن الباحث J. L. Brenner من خلال سلسلة من البحوث أن صف التكافؤ C الذي يكون من n من الدورات او من $n-1$ من الدورات،أستنتج ان CCC يغطي A_n عندما تكون $n = 4k - 1$ و $n = 4k + 1$ و $n = 4k$. في هذا البحث ، دُرس صف التكافؤ من النوع$(2)(n-2)$ ، $n > 6$ و n عدد زوجي، وينقسم صف التكافؤ هذا على قسمين ويرمز لهذين القسمين بالرمز C و C' وبينت بأنه $CCC = C'C'C'$.