



Investigation of Nuclear Symmetry Energy and its Slope for Potassium Mirror Nuclei

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Abstract

This study examines how nuclear symmetry energy and its slope change in pairs of mirror nuclei: $^{40}\text{K}-^{40}\text{Sc}$, $^{41}\text{K}-^{41}\text{Ti}$, $^{42}\text{K}-^{42}\text{V}$, $^{43}\text{K}-^{43}\text{Cr}$, and $^{44}\text{K}-^{44}\text{Mn}$, using three different Skyrme parameterizations like Msk7, Sk0, and Bsk9. This study looks at how different nuclear properties, like the difference in charge radius, the imbalance between neutrons and protons, and the thickness of the neutron skin, are affected by different symmetry energy constants. The results show that when the symmetry energy goes up, both the thickness of the neutron skin and the difference in charge radius and mirror radius also go up, while the slope parameter changes a lot with nuclear asymmetry. Comparisons among the models reveal that Bsk9 consistently predicts higher symmetry energy values compared to Msk7 and Sk0. The results highlight how important nuclear structure is in asymmetric nuclei and point out how it helps refine theoretical models related to asymmetric nuclear matter.

Keywords: Behaviour of nuclear symmetry energy, Mirror nuclei, Thickness of the neutron skin, Potassium isotopes, Skyrme model interactions, Slope parameters.

1. Introduction

Symmetry energy is a bit tricky to describe because, unlike many other nuclear properties, it changes when the density inside the nucleus goes up or down, affecting the stability and behavior of nucleons¹⁻³. The *EoS* is an essential element in the equation of state of nuclear matter, measuring the energy difference between symmetric nuclear matter (with equal numbers of protons and neutrons) and asymmetric matter. The symmetry energy is usually expressed as E_{sy} , crucial for predicting the properties of exotic nuclei and accurately modeling nuclear reactions³. Because of this, it is not simple for researchers to get a full understanding. Even small shifts in density can make a big difference in how neutrons and protons are balanced inside an atom. Scientists continue to work hard to better understand these changes. To assist with this, they often rely on a method called "Skyrme interactions," which has shown effectiveness in a variety of situations^{4,5}. Scientists have spent years trying to figure out how nuclear matter behaves. One key idea that keeps coming up is nuclear symmetry energy. It does not just affect tiny atoms; it even matters when it comes to giant objects like neutron stars¹⁻³. Symmetry energy is a little tricky To assist with this, they often rely on a method called Skyrme interactions, which has shown effectiveness in a variety of situations^{4,5}.

Understanding symmetry energy has become even more crucial recently, especially with astronomical new observations and advances in experimental nuclear physics. Following the groundbreaking discovery of gravitational waves from neutron star collisions, scientists have uncovered new avenues for studying dense nuclear matter. Improved how well one can determine neutron skin thickness, giving an important link between laboratory measurements and

how symmetry energy in extreme astrophysical ^{6,7}. Further studies have highlighted that the slope of symmetry energy, often referred to as L , plays an essential role in determining the radii and masses of mirror nuclei ⁸. These relativistic mean-field models have given more reliable predictions about nuclear properties such as binding energy and charge radius, but they still come with noticeable uncertainties, especially at higher densities ⁹⁻¹¹. Looking at pairs of mirror nuclei like $^{39}\text{K}-^{39}\text{Sc}$ and $^{41}\text{K}-^{41}\text{Ti}$ has provided more understanding of how isospin asymmetry affects nuclear structure ¹². Moreover, analyzing the differences in charge radii between mirror nuclei helps link experimental findings to key parameters, like the symmetry energy's slope ¹³. In this study, the mirror potassium isotopes were examined like $^{40}\text{K}-^{40}\text{Sc}$, $^{41}\text{K}-^{41}\text{Ti}$, $^{42}\text{K}-^{42}\text{V}$, $^{43}\text{K}-^{43}\text{Cr}$, and $^{44}\text{K}-^{44}\text{Mn}$ versions to see how changes in the symmetry energy slope (L) affect the distribution of neutron skin thickness (R_{skin}), defined as the disparity between the neutron and proton radii, the difference in proton radii of mirror pairs (ΔR_{mirr}), and the difference in charge radii (ΔR_{ch}), which helps us understand nuclear structure physics. This study aims to explore the symmetry energy (E_{sy}) by using the Hartree–Fock approach with three Skyrme parameterizations (MSk7, Sk0, and Bsk9)¹⁴. The goal is to analyze how it changes under different nuclear densities. Given that E_{sy} is vital for characterizing nuclei with a proton-neutron imbalance, comprehending its properties under varying density conditions is imperative for investigating nuclear structure, where the nuclear saturation density and these fluctuations are directly correlated with the derivative of the nuclear Equation of State (EoS). Subsequently, rate of change of this energy with respect to the slope parameter, referred to as L , will be analyzed. Collectively, these measures aim to enhance comprehension of the processes within nuclei when the neutron-to-proton ratio is unbalanced, therefore aiding the progression of theoretical nuclear models.

2. Materials and Methods.

2.1. Theoretical Framework

In nuclear physics, grasping how E_{sy} behaves is crucial for describing the Equation of State EoS in systems where neutrons and protons are not balanced. E_{sy} helps explain the energy required to convert protons into neutrons inside atomic nuclei, and it plays an important part in understanding nuclear structure and how neutron stars behave ¹⁵. At absolute zero, the EoS is generally expressed as a functional relationship between density, pressure, and energy per nucleon. In the context of symmetric nuclear matter, a prevalent representation is ^{16,17}:

$$E/A(\rho) = E_0 + \frac{K}{18} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2 \quad (1)$$

Where K is the incompressibility modulus, ρ is the density, and E_0 is the binding energy per nucleon at saturation density ρ_0 . For asymmetric nuclear matter, an additional term must be included to account for the neutron–proton imbalance. Thus, the total energy per nucleon becomes:

$$E(\rho, \gamma)/A = E(\rho)/A + E_{\text{sy}}(\rho)\gamma^2 \quad (2)$$

The $E_{\text{sy}}(\rho)$, an essential element in the EoS of nuclear matter, measures the energy disparity between symmetric nuclear matter (with equal quantities of protons and neutrons) and asymmetric matter. The symmetry energy is expressed as ³:

$$E_{\text{sy}}(\rho) \approx G_{\text{Sy}} \left(\frac{\rho}{\rho_0} \right)^\gamma \quad (3)$$

Where the parameter $\gamma = \frac{\rho_n - \rho_p}{\rho}$ is the asymmetry parameter, and the G_{Sy} are the symmetry energy coefficient, which indicates the magnitude of the symmetry energy at nuclear saturation density $\rho_0 \approx 0.16 \text{ fm}^{-3}$. The typical values for G_{Sy} lie between 30 to 34 MeV, depending on the interaction model used. The L parameter can be determined by applying ^{3, 18}:

$$L = 3\rho \left[\frac{\partial E_{\text{sy}}(\rho)}{\partial \rho} \right] \Big|_{\rho=\rho_0} \quad (4)$$

The proton or neutron skin thickness (R_{skin}) is given by ¹⁹⁻²¹:

$$R_{skin} = R_n(Z, N) - R_p(Z, N) \quad (5)$$

The $R_p(Z, N)$, and $R_n(Z, N)$ stand for the *rms* radii of the proton and neutron density distributions, respectively. Furthermore, the neutron's skin thickness value is the same as the proton's, although negative. For mirror nuclei, the disparity in root-mean-square charge radii R_{ch} , denoted as ΔR_{ch}^{19-24} , and the discrepancy in proton radii, represented as ΔR_{mirr}^{13} , is expressed as follows:

$$\Delta R_{mirr} = R_p(Z, N) - R_p(N, Z) \quad (6)$$

$$\Delta R_{ch} = R_{ch}(Z, N) - R_{ch}(N, Z) \quad (7)$$

The zero-range interactions known as the Skyrme forces are essentially made up of a zero-range three-body term and a momentum-dependent two-body term^{10, 11}.

$$V = \sum_{i < j} V_{ij}^{(2)} + \sum_{i < j < k} V_{ijk}^{(3)}, \quad (8)$$

$$\text{Were, } V_{Sk} = \sum_{i < j} V_{ij}^{(2)} \quad (9)$$

$$V_{Sk} = t_0(1 + x_0 \hat{P}_\sigma) \delta_{12} + \frac{t_1}{2}(1 + x_1 \hat{P}_\sigma)(\hat{k}'^2 \delta_{12} + \delta_{12} \hat{k}^2) + t_2(1 + x_2 \hat{P}_\sigma) \hat{k}^2 \cdot \delta_{12} \hat{k} + \frac{t_3}{6}(1 + x_3 \hat{P}_\sigma) \rho^\alpha(\vec{R}) \delta_{12} + it_4 \hat{k}'(\hat{\sigma}_1 + \hat{\sigma}_2) \times \hat{k} \delta_{12} \quad (10)$$

Where, $\vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2}$, where the operators for relative momentum operating on the eigen function are $\hat{k}$ and $\hat{k}'$, $\hat{k}'$ works to the left and $\hat{k}$ to the right. The Skyrme parameters (t_0 , t_1 , t_2 , t_3 , x , x_1 , x_2 , x_3 , and w_0) are typically ascertained by fitting them to a comprehensive and representative dataset of experimental data.

3.Results

In this study, the numerical results for the mirror nuclei studied. The physical parameters of the three Skyrme force sets (Msk7, Sk0, and Bsk9)¹⁴. in our calculations are listed in **Tables 1-4** and **Figures 1-4**.

Table 1. Parameters of the Skyrme-type force. The units of parameters are: t_0 (MeV·fm³), t_1 (MeV·fm⁵), t_2 (MeV·fm⁵), t_3 (MeV·fm^{3(α+1)}), and W_0 (MeV)¹⁴.

Parameters	Msk7	Sk0	Bsk9
t_0	-1828.230	-2103.700	-2043.913
t_1	259.400	303.400	411.599
t_2	-292.840	791.700	-194.189
t_3	13421.700	13553.000	12497.169
w_0	118.807	118.000	149.919
a	0.333	0.250	0.250
x_0	0.577	-0.210	0.515
x_1	-0.500	-2.810	-0.954
x_2	-0.500	1.460	-0.332
x_3	0.783	-0.430	0.899

Table 2. The calculated symmetry energy (E_{sy}) for ⁴⁰K-⁴⁰Sc, ⁴¹K-⁴¹Ti, ⁴²K-⁴²V, ⁴³K-⁴³Cr and ⁴⁴K-⁴⁴Mn pairs mirror nuclei using Msk7, Sk0, and Bsk9 parameterizes

The symmetry energy E_{sy} (MeV)									
Mirror Nuclei	Msk7 parameterizes			Sk0 parameterizes			Bsk9 parameterizes		
The G_{sy} coefficient	32	34	36	32	34	36	32	34	36
⁴⁰ K	34.3	35.1	34.9	36.9	37.7	37.6	39.0	39.9	39.8
⁴⁰ Sc	34.1	34.9	34.8	36.7	37.6	37.4	38.9	39.8	39.6
⁴¹ K	34.0	34.9	34.8	36.6	37.6	37.4	38.7	39.8	39.6
⁴¹ Ti	33.7	34.7	34.5	36.3	37.3	37.2	38.4	39.5	39.3
⁴² K	33.7	34.7	34.7	36.3	37.4	37.3	38.4	39.5	39.5
⁴² V	33.5	34.4	34.4	36.0	37.0	37.0	38.1	39.2	39.2
⁴³ K	33.6	34.5	34.5	36.2	37.2	37.2	38.3	39.3	39.3
⁴³ Cr	33.2	34.1	34.1	35.7	36.7	36.7	37.8	38.9	38.7
⁴⁴ K	33.4	34.3	34.3	35.9	36.9	36.9	38.0	39.0	39.0
⁴⁴ Mn	32.9	34.0	34.0	35.5	36.6	36.6	37.5	38.7	38.7

Table 3. The calculated energy slope parameter (L) for ^{40}K - ^{40}Sc , ^{41}K - ^{41}Ti , ^{42}K - ^{42}V , ^{43}K - ^{43}Cr and ^{44}K - ^{44}Mn pairs mirror nuclei using Msk7, Sk0, and Bsk9 parameters. The calculated values are compared with the experimental data²³.

The energy slope parameter L (MeV)											
Mirror Nuclei	Msk7 parameterizes			Sk0 parameterizes			Bsk9 parameterizes			Exp. ²³	
The G_{sy} coefficient	32	34	36	32	34	36	32	34	36	32	34
^{40}K - ^{40}Sc	63.1	67.9	71.9	62.5	67.2	71.3	62.6	67.3	71.3	63±11	72±13
^{41}K - ^{41}Ti	63.4	68.2	72.2	62.6	67.4	71.5	62.7	67.5	71.5		
^{42}K - ^{42}V	63.6	68.5	72.5	62.9	67.6	71.6	62.9	67.6	71.6		
^{43}K - ^{43}Cr	63.8	68.6	72.7	63.0	67.8	71.8	63.0	67.8	71.8		
^{44}K - ^{44}Mn	64.0	68.9	72.9	63.2	68.0	72.0	63.2	68.0	72.0		

Table 4. The calculated values of ΔR_{mirr} , ΔR_{ch} and ΔR_{skin} for ^{40}K - ^{40}Sc , ^{41}K - ^{41}Ti , ^{42}K - ^{42}V , ^{43}K - ^{43}Cr and ^{44}K - ^{44}Mn pairs mirror nuclei using Msk7, Sk0, and Bsk9 parameterizes.

Mirror Nuclei	Msk7 parameterizes			Sk0 parameterizes			Bsk9 parameterizes		
	ΔR_{mirr}	ΔR_{ch}	ΔR_{skin}	ΔR_{mirr}	ΔR_{ch}	ΔR_{skin}	ΔR_{mirr}	ΔR_{ch}	ΔR_{skin}
^{40}K - ^{40}Sc	-0.074	-0.076	0.125	-0.087	-0.088	0.154	-0.076	-0.077	0.132
^{41}K - ^{41}Ti	-0.108	-0.114	0.182	-0.128	-0.133	0.226	-0.111	-0.117	0.194
^{42}K - ^{42}V	-0.140	-0.151	0.234	-0.166	-0.177	0.293	-0.145	-0.155	0.252
^{43}K - ^{43}Cr	-0.171	-0.185	0.282	-0.203	-0.217	0.356	-0.177	-0.191	0.307
^{44}K - ^{44}Mn	-0.198	-0.218	0.328	-0.238	-0.256	0.416	-0.207	-0.226	0.359

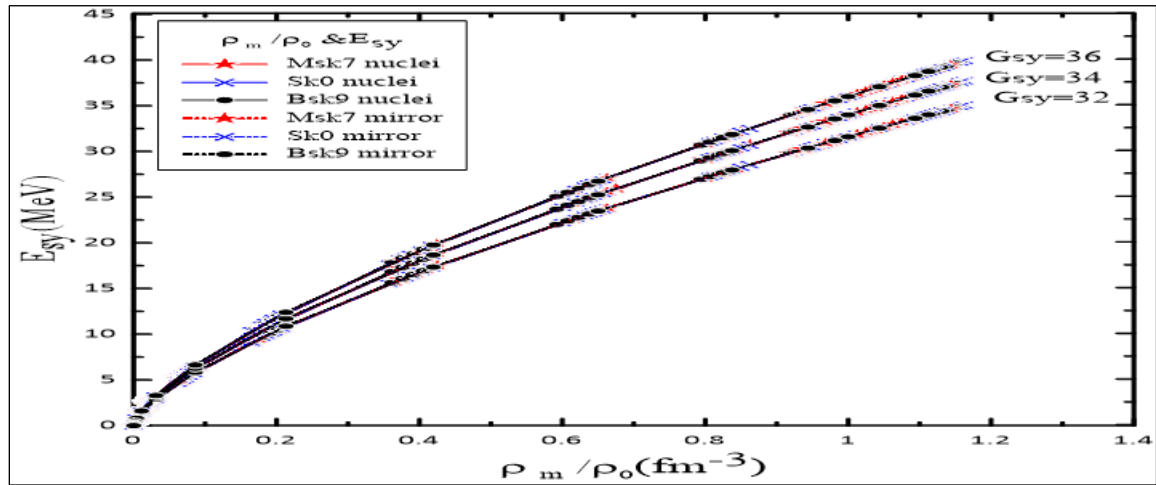


Figure 1. The relationship between nuclear matter density (ρ_m/ρ_0) and symmetry energy (E_{sy}) using the three Skyrme interactions Msk7, Sk0, Bsk9.

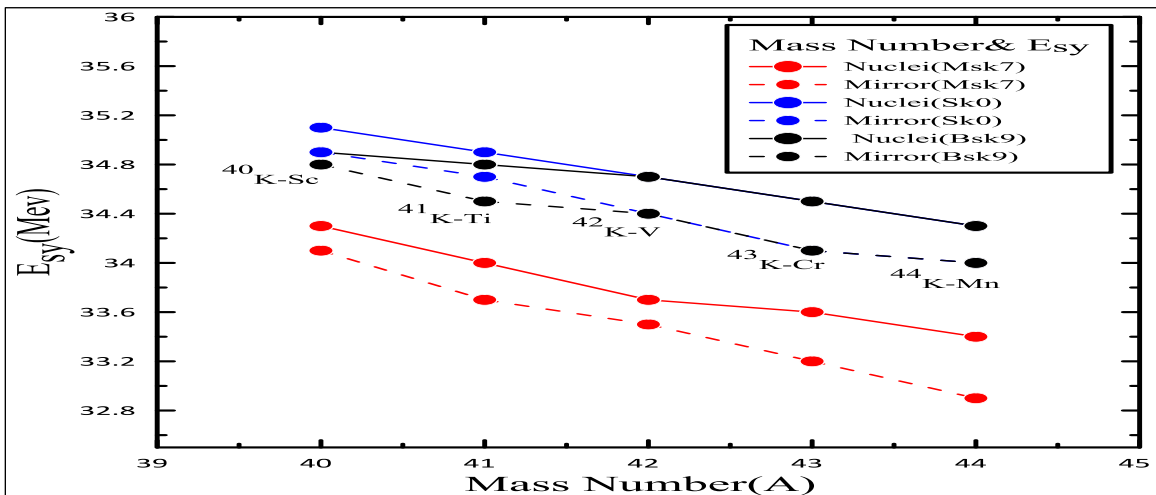


Figure 2. The relationship between mass numbers (A) and symmetry energy (E_{sy}) using the three Skyrme interactions Msk7, Sk0, Bsk9.

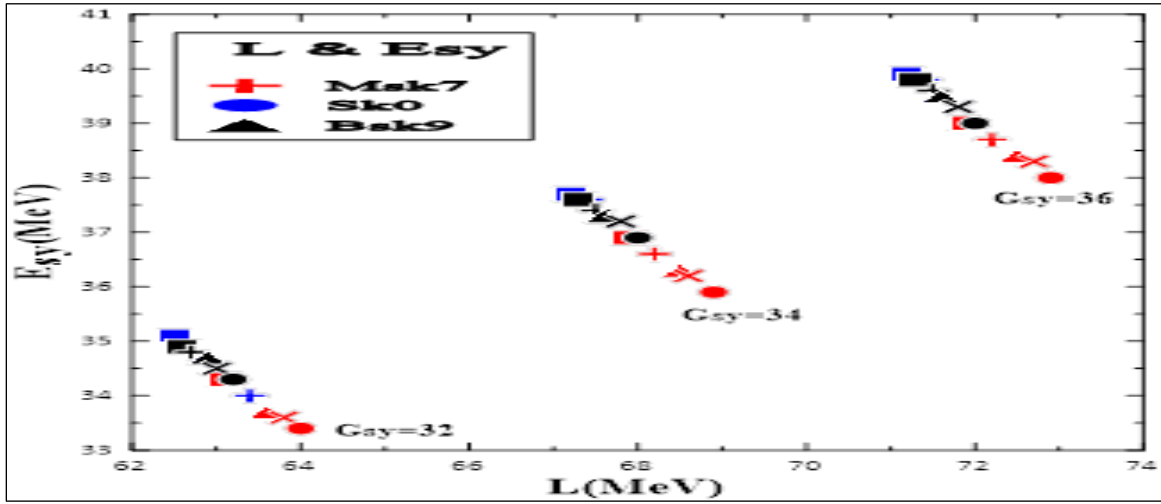


Figure 3. The relationship between the slope parameters (L) and symmetry energy (E_{sy}) using the three Skyrme interactions Msk7, Sk0, Bsk9 at different G_{ρ} Values.

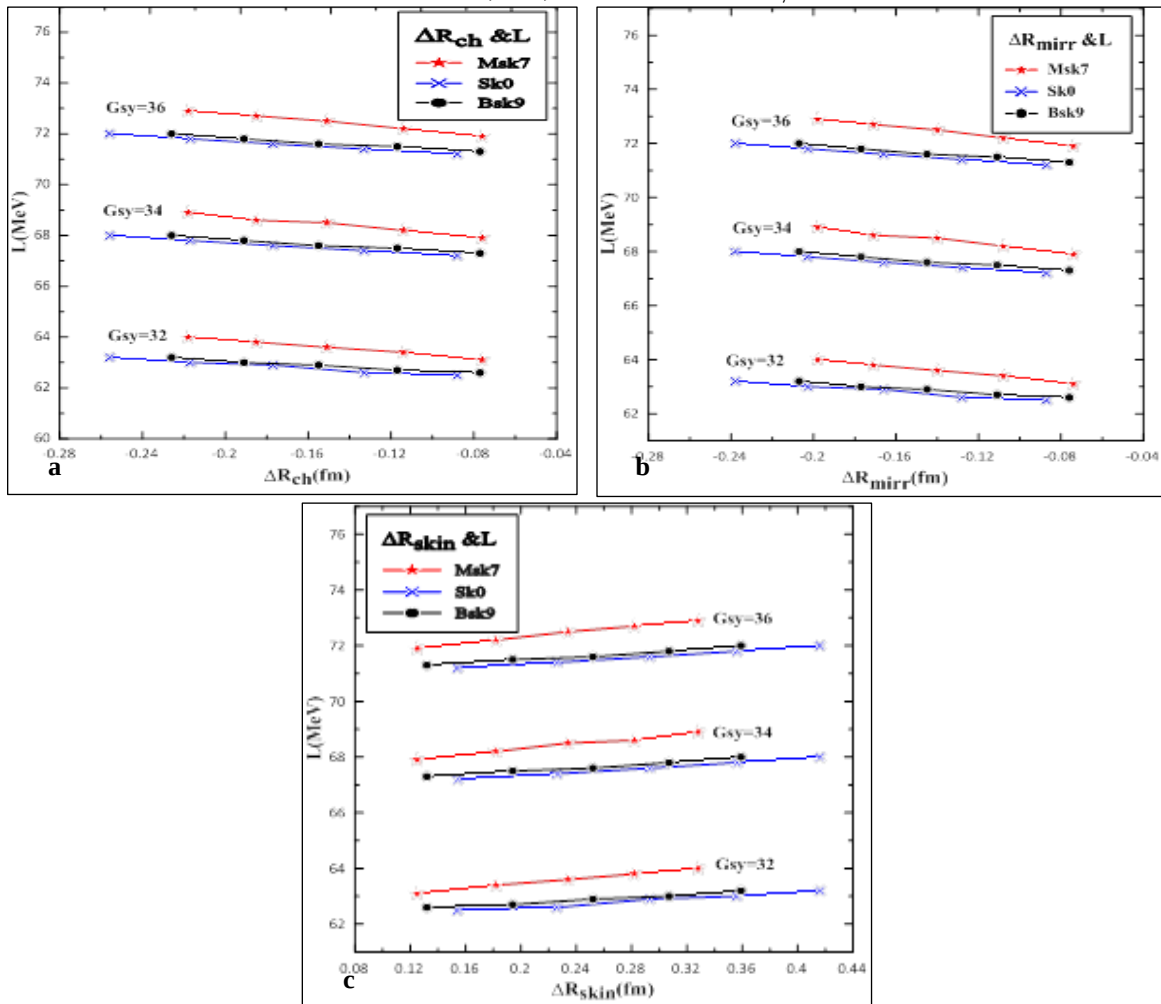


Figure 4. The relationship between the a. L and ΔR_{ch} b. L and ΔR_{mirr} c. L and ΔR_{skin} respectively, using the three Skyrme interactions at different G_{ρ} values

4. Discussion

Understanding the density dependence of nuclear symmetry energy is essential for grasping nuclear structure. This work uses the HF wave function to prototype the characteristics of the nuclear structure of mirror nuclei ^{40}K – ^{40}Sc , ^{41}K – ^{41}Ti , ^{42}K – ^{42}V , ^{43}K – ^{43}Cr , and ^{44}K – ^{44}Mn . The nuclear symmetry energy was employed to determine the parameters of the nuclear equation of

state and the slope parameter of the symmetry energy, both of which are crucial for the binding of atomic nuclei. The simulations using Skyrme interaction sets such as Msk7, Sk0, and BSk9¹⁵. The numerical calculations were carried out using the NuShellX@MSU shell-model code¹⁴, which provides reliable computational tools for determining nuclear structural properties based on Skyrme-type interactions. This method accurately depicts the interplay of Skyrme. The Skyrme parameters (t_0 , t_1 , t_2 , t_3 , x , x_1 , x_2 , x_3 , and w_0) are typically ascertained by fitting them to a comprehensive and representative dataset of experimental data. The computed symmetry energy of various pairs of mirror nuclei, from ^{40}K – ^{40}Sc to ^{44}K – ^{44}Mn . The results indicate a uniform pattern across all models: a reduction in symmetry energy correlating with the increasing mass number of the mirror pairs, despite the overall mass number being comparatively unchanged. This pattern indicates an increasing impact of isospin asymmetry in heavier isotopes. Using different symmetry energy values, using different Skyrme interaction parameters, the BSk9 parameter set gives the highest symmetry energy values, followed by Sk0, while Msk7 consistently gives the lowest values. These differences show how much the models react to the ratio of neutrons and protons. It's also clear that they have different levels of interaction when it comes to equal-spin asymmetry. The findings indicate that selecting various Skyrme parameters substantially influences the computed symmetry energy, therefore impacting the nuclear characteristics of mirror nuclei. The variation of the symmetry energy slope parameter (L) for mirror nuclei utilizing different Skyrme parameterizations, with the symmetry energy coefficients (G_{sy}). The theoretical values exhibit a progressive increase with mass number and agree with the experimental values²³. The models adhere to a consistent hierarchy in L values: Bsk9 > Sk0 > Msk7, indicating variances in their representation of nuclear matter characteristics. A modest elevation in the G_{sy} coefficient is noted in conjunction with an increase in L , signifying a direct link between the two variables²⁴. This trend indicates that an increase in neutron number amplifies the symmetry energy. The results underscore the necessity for accurate experimental data to enhance the calibration of theoretical models. and agree with the experimental values²³. These experimental values were obtained from charge-exchange and reaction cross-section measurements, as reported in²⁴. The theoretical results indicated the existence of an almost linear relationship between the difference in thickness of the neutron skin (ΔR_{skin}), and the difference in radii between the mirror nuclei (ΔR_{mirr}), along with a clear correlation with the change in the charge radius (ΔR_{ch}). The data show that both ΔR_{mirr} and ΔR_{ch} increase (a greater negative value) with the increase of ΔR_{skin} and the mass number, reflecting the influence of the neutron structure on the charge distribution in the nucleus. This behaviour has been shown to be consistent across the three Skyrme sets Bsk9, Sk0 and Msk7, which enhances the reliability of the results. Since ΔR_{ch} and ΔR_{mirr} can be measured experimentally, they can be used as indirect indicators to deduce ΔR_{skin} . This relationship contributes to improving our understanding of the distribution of neutrons and protons within unstable nuclei

The correlation between the nuclear symmetry energy E_{sy} and the scaled nuclear density $\frac{\rho_m}{\rho_0}$, where $\rho_0 \approx 0.16 \text{ fm}^{-3}$ represents the saturation density of symmetric nuclear matter. The present results were calculated for potassium nuclei with mass numbers $A=40$ to $A=44$. Three efficient Skyrme reactions were used in the theoretical calculations: Msk7, Sk0, and Bsk9. The symmetry energy increases with increasing density, which is consistent with the predictions of models of nuclear matter. This behavior is consistent with previous theoretical and experimental constraints on the density dependence of the symmetry energy^{12, 13}, which show that the symmetry energy generally increases as the nuclear matter density increases. This property demonstrates the importance of calculating the symmetry energy, especially in the cases of mirror nuclei rich in neutrons or protons. Reference lines that match the assumed values of the symmetry energy parameter G_{sy} , particularly 32, 34, and 36 MeV. The selected symmetry energy values of 32, 34, and 36 MeV fall within the empirically constrained range for the symmetry energy at nuclear saturation density^{12 - 16}. These lines represent theoretical criteria for evaluating the density-

dependent symmetry energy properties. These values are commonly used as theoretical reference points because they correspond to the predicted range of the symmetry energy at nuclear saturation density. Several studies have shown that the symmetry energy at saturation lies in the interval of 30–36 MeV, depending on the nuclear interaction model¹²⁻¹⁶. Therefore, the calculated values in this work were compared with this established range to assess their physical validity. The use of mirror nuclei is suitable for studying isospin asymmetry because mirror pairs have the same mass number but differ in proton and neutron numbers. This difference is great because it lets us separate the parts that depend on isospin, which makes mirror nuclei a really dependable way to pin down how symmetry energy changes with density^{3, 9, 13, 18}. Our research shows that using mirror nuclei systems is a smart way to improve the nuclear equation of state, especially for densities that are near or even below $\frac{\rho_m}{\rho_0}$. Basically, these results prove that mirror nuclei systems are a solid approach for figuring out the density dependence of symmetry energy, since our calculated values stay within the range of symmetry energy at saturation density that's already been proven empirically. These findings emphasize the importance of investigating symmetry energy via mirror nuclei and Skyrme interactions, providing strong limitations on the symmetry energy coefficient. Consequently, it is imperative to select Skyrme interactions judiciously to attain precise outcomes in theoretical nuclear density computations. The E_{sy} changes with the mass number (A) for different potassium isotopes and their mirror nuclei. The symmetry energy goes down as the mass number increases. It starts at $A = 40$ and becomes lower by $A = 44$. It makes sense that lighter nuclei have more even amounts of protons and neutrons, which keeps the symmetry energy higher. As the nucleus gets heavier, this balance changes, and the symmetry energy goes down as a result. There might be minor changes between each nucleus and its mirror version, but they do not really change the main pattern. The drop in symmetry energy continues clearly as A gets larger.

To investigate the relationship between the L and the E_{sy} values in potassium isotopes, three specific symmetry energy constants G_{sy} were considered namely 32, 34, and 36. The selected symmetry energy values (32, 34, and 36 MeV) lie within the widely accepted empirical range for the symmetry energy at nuclear saturation density as supported by previous theoretical and experimental studies¹²⁻¹⁶. The data show that when the slope increases, E_{sy} decreases continuously for all reactions, indicating a stable inverse relationship associated with the internal structure of the nucleus. The results indicate the Msk7 interaction yields the highest values of L and the lowest values of E_{sy} , indicating a strong sensitivity of this interaction to changes in the slope. At the same time, the Sk0 interaction shows the smallest slope values but has the highest symmetry energy. This basically means that this specific interaction is more stable when we look at its symmetry features. The BSk9 interaction, end up right in the middle of the other two, showing a more even or balanced result. Our data also makes it clear that even when L changes, E_{sy} stays inversely linked to G_{sy} , proving there's a real, direct connection between these two parts. It is concluded that the interactions predicting a larger slope for the symmetry energy tend to yield a lower symmetry energy value, reflecting the differences in how theoretical models handle the impact of nuclear asymmetry on binding energy.

offer the slope coefficients L relates to several fundamental features of a nucleus's structure. These features include the different charge radius (ΔR_{ch}), the different mirror radius (ΔR_{mirr}), and also the different neutron skin thickness (ΔR_{skin}).

that the slope coefficients L increases, the difference in charge radius between isotopes decreases, suggesting that the nuclear charge is more evenly spread out. Among the Skyrme sets, Msk7 yields steeper slopes, particularly at elevated G_{sy} values, whereas Sk0 produces the least steep slopes. These findings suggest that each Skyrme parameters exhibits a distinct response to variations in symmetry energy. Also, increasing the G_{sy} constant leads to a general rise in the slope of symmetry energy for all interactions, highlighting its importance in enhancing effects in nuclear matter. This study provides important information on how changes in charge radii appear

in different isotopes, helping to improve our understanding of nuclear structure. a similar pattern. The L values also decrease with increasing ΔR_{mirr} , and again, Msk7 gives the steepest slopes while Sk0 gives the flattest ones. The resemblance between the two pictures indicates that the mirror nuclei possess analogous structures or responses regarding the influence of mirror and charge radii on E_{sy} . a contrasting picture, showing a direct relationship between L and ΔR_{skin} . The ΔR_{skin} , which is shown with positive values expanding from left to right grows, a steady climb in L 's value is observed. Considering all three figures, the lines corresponding to G_{sy} is 36 MeV consistently display the highest L values. This clearly indicates that G_{sy} is plays a central part in setting the overall range of L 's magnitude, to sum up.

5. Conclusion

This research emphasis was placed on the energy of nuclear symmetry and its role in determining the parameters of the equation of state of the nuclear state, especially in systems rich in neutrons or protons. The results indicated that the symmetry energy increases with nuclear density, in line with theoretical predictions, and the calculated G_{sy} values also fall within the reference range (32-36 MeV), which indicates a good agreement with the experimental values. The nuclear symmetry energy basically keeps dropping as the mass number A for the mirror nuclei pairs goes from 40 to 44. This mainly happens because the whole neutron-proton asymmetry starts playing a bigger role when the isotopes get heavier. Out of the three, BSk9 gave the highest results, then Sk0, while Msk7 always ended up with the lowest values. This just shows that picking the right Skyrme coefficients is a big deal when you're trying the Skyrme coefficient group significantly affects the theoretical results. The nuclear symmetry energy basically keeps dropping as the mass number A gets bigger for the mirror nuclei pairs going from ^{40}K – ^{40}Sc to ^{44}K – ^{44}Mn . This mainly happens because the whole neutron-proton asymmetry starts playing a much bigger role in those heavier isotopes. The results also showed clear differences in the values of the symmetry energy calculated using different parameters of the Skyrme reaction, where BSk9 showed the highest values, followed by Sk0, while Msk7 gave the lowest values, and thus it was concluded that the choice of the Skyrme coefficient group significantly affects the theoretical results and must be done carefully to obtain accurate models of the nuclear structure. Also, the L increases when E_{sy} decreases continuously for all reactions, indicating a stable inverse relationship associated with the internal structure of the nucleus. The result of this study indicates a linear link between the ΔR_{skin} and the ΔR_{mirr} , with a distinct correlation to the variation in ΔR_{ch} . The findings indicate that both ΔR_{mirr} and ΔR_{ch} rise as ΔR_{skin} and mass increase. The L inversely correlates with ΔR_{ch} and ΔR_{mirr} but increases with ΔR_{skin} , revealing distinct nuclear structure sensitivities across Skyrme interactions (Msk7> Sk0). Higher G_{sy} values systematically enhance L , emphasizing its key role in symmetry energy effects. These results refine our understanding of charge distribution and neutron skin in nuclei.

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Conflict of Interest

The authors declare that they have no conflict of interest that could influence this work, and they all agree to publish this paper under ethical academic.

Ethical Clearance

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