Modified Iterative Solution of Nonlinear Uniformly Continuous Mappings Equation in Arbitrary Real Banach Space

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Abstract

In this paper, we study the convergence theorems of the Modified Ishikawa iterative sequence with mixed errors for the uniformly continuous mappings and solving nonlinear uniformly continuous mappings equation in arbitrary real Banach space.

Introduction and preliminaries

Throughout this paper, we assume that X be a Banach space with norm $\| \quad \|$, the dual space X^* and J denote the normalized duality from X into 2^{X^*} give by:

$$J(x) = \{f \in X^* : \langle x, f \rangle = \|x\| \|f\|, \ \|f\| = \|x\|, \ \ \forall x \in X\} \quad \text{... (1.1)[1]}$$

Where $\langle ., . \rangle$ denotes the generalized duality pairing.

A mapping T with domain D(T) and range R(T) in X is called accretive if the inequality holds:

$$\|x - y\| \le \|x - y + s(Tx - Ty)\| \quad \text{... (1.2)[2]}$$

For every $x, y \in D(T)$ and for all $s > 0$.

A mapping T is called a strongly pseudocontraction if there exists $t > 0$ such that $x, y \in D(T)$ and $r > 0$, the following inequality holds:

$$\|x - y\| \le \|(1 + r)(x - y) + rt(Tx - Ty\| \quad \text{... (1.3)}$$

If t=1 in inequality (1.3), then T is called pseudocontractive.

Also, as a consequence of Kato [3], if follows from the inequality (1.3) that T is strongly pseudocontractive if and only if the following inequality holds:

$$\langle (I - T)x - (I - T)y, \ j(x - y) \rangle \ge k\|x - y\|^2 \quad \text{... (1.4)}$$

For all $x, y \in D(T)$ and for some $j(x - y) \in J(x - y)$, $where\ k = \frac{(t-1)}{t} \in (0, 1)$. Consequently, it follows easily, again from Kato [3] and (1.3), that T is strongly pseudocontractive if and only if the inequality holds:

$$\|x - y\| \le \|x - y + s[(I - T - kI)x - (I - T - kI)y]\| \quad \text{... (1.5)}$$

For every $x, y \in D(T)$ and for all $s > 0$.

Closely related to the class of pseudocontractive mapping is the class of accretive mapping.

It is clear that T is strongly accretive mapping if and only if I-T is strongly pseudocontractive mapping.

Let us recall the following iterative scheme du to Mann [4], Ishikawa [5], Xu[6] and Cho [7], respectively.

Definition 1.1 Let C be a convex subset of X and $T: C \to C$ be a mapping, then:

i . For any $x_1 \in C$, the sequence $\{x_n\}$ is defined by:

$$y_n = (1 - \beta_n)x_n + \beta_n T x_n, \qquad x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T y_n, \quad n \ge 1$$

Is called **Ishikawa iteration sequence**, where $\{\alpha_n\}\ and\ \{\beta_n\}$ are two real sequences in [0, 1] satisfying some conditions.

ii. . If $\beta_n = 0$ for all $n \ge 1$ in (1.6), then the sequence $\{x_n\}$ is defined by:

$$x_1 \in C, \quad x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T y_n, \quad n \ge 1 \quad \text{... (1.7)}$$

Is called **Mann iteration sequence**.

iii. For any $x_1 \in C$, the sequence $\{x_n\}$ is defined by:

$$y_n = \grave{a}_n x_n + \grave{b}_n T x_n + \grave{c}_n v_n, \qquad x_{n+1} = a_n x_n + b_n T y_n + c_n u_n, \quad n \ge 1 \quad \text{... (1.8)}$$

Is called **Ishikawa iteration sequence with random errors**. Here $\{u_n\}, \{v_n\}$ are two bounded sequences in C; $\{\grave{a}_n\}$, $\{\grave{b}_n\}$, $\{\grave{c}_n\}$, $\{a_n\}$, $\{b_n\}$, $\{c_n\}$are real sequences in [0, 1] satisfying

$\grave{a}_n + \grave{b}_n + \grave{c}_n = a_n + b_n + c_n$ =1 for all $n \ge 1$

v. If $\grave{b}_n = \grave{c}_n = 0$ for all $n \ge 1$ in (1.8), then the sequence $\{x_n\}$ defined by:

$$x_0 \in C, \quad x_{n+1} = a_n x_n + b_n T x_n + c_n u_n, \quad n \ge 1 \quad \text{... (1.9)}$$

Is called **Mann iteration sequence with random errors**.

iv. . For any $x_1 \in C$, the sequence $\{x_n\}$ is defined by:

$$y_n = \grave{a}_n x_n + \grave{b}_n T^n x_n + \grave{c}_n v_n, \qquad x_{n+1} = a_n x_n + b_n T^n y_n + c_n u_n, \quad n \ge 1 \quad \text{... (1.10)}$$

Is called **modified Ishikawa iteration sequence with random errors**. Here $\{u_n\}, \{v_n\}$ are two bounded sequences in C; $\{\grave{a}_n\}$, $\{\grave{b}_n\}$, $\{\grave{c}_n\}$, $\{a_n\}$, $\{b_n\}$, $\{c_n\}$are real sequences in [0, 1] satisfying

$\grave{a}_n + \grave{b}_n + \grave{c}_n = \ a_n + b_n + c_n = 1$ for all $n \geq 1$.

iiv. . If $\grave{b}_n = \grave{c}_n = 0$ for all $n \geq 0$ in (1.10), then the sequence $\{x_n\}$ is defined by:

$x_1 \in C, \quad x_{n+1} = a_n x_n + b_n T^n x_n + c_n u_n, \quad n \geq 1$ ... (1.11)

Is called **modified Mann iteration sequence with random errors.**

It is clear that the Mann, Ishikawa iterative sequences and Mann, Ishikawa iterative sequence with random errors are special cases of Modified iterative sequences with errors.

Definition 1.2 [8, 9] A mapping T: C→C called:

i. Lipschitz if there exists a constant L>0 such that

$$\|T(x) - T(y)\| \leq L\|x - y\| \quad \ldots(1.12)$$

for all $x, y \in C$.

ii. Uniformly L-Lipschitzian if there exists a constant L>0 such that

$$\|T^n(x) - T^n(y)\| \leq L\|x - y\| \quad \ldots(1.13)$$

for all $x, y \in C$, $n \geq 1$

Definition 1.3 [10] Let C be a convex subset of vector space and f real valued function defined on C, then f is called generalized convex if for any three points x, y, z ∈ C and $a, b, c \in [0, 1]$ such that

$$f(ax + by + cz) = af(x) + b\,f(y) + c\,f(z).$$

Example 1.4 Let $f: [0, \infty) \rightarrow [0, \infty)$ such that $f(t) = t^2$ $for\ all\ t \in [0, \infty)$. Show that f is generalized convex.

Solution: for any three points x, y, z ∈ [0, ∞) and $a, b, c \in [0, 1]$ such that

$$f(ax + by + cz) = (ax + by + cz)^2 \leq ax^2 + b\,y^2 + c\,z^2$$

Thus f is a generalized convex.

Several researchers proved that the Mann iterative scheme, Ishikawa iterative scheme , the Mann iterative scheme with random errors and Ishikawa iterative scheme with random errors can be used to approximate solutions of the equations Tx=f where T is strongly pseudocontractive mapping or ø- strongly pseudocontractive mapping.

Very recently, Arifiq [10], proved a related result that deals with the Mann iterative approximation of the fixed point for the class of strongly pseudocontractive mapping in arbitrary real Banach space. At the same time, he puts forth an open problem:

It is not Known whether or not the modified Ishikawa iteration method converges uniformly continuous mapping. This open problem has been studied extensively in case of Mann iterative scheme with random errors and Ishikawa iterative scheme with random errors by many of the researchers (see, [1-3], [6], [10-16]).

The objective of this paper is to introduce the modified Ishikawa iterative method a class of sequence which much more general than the important class of Mann iterative scheme with random errors and Ishikawa iterative scheme with random errors, and to study problem of approximation fixed point by modified Ishikawa iterative processes with random errors for uniformly continuous mappings and this mapping satisfies some conditions.

We will prove that the answer of Airfiq' s open problem is affirmative if X is an arbitrary real Banach space and $T : X \rightarrow X$ is an uniformly continuous mapping and satisfying some conditions. The results presented in this paper improve, generalize and unify results of [1-2], [11-16].

The following two lemmas play crucial roles in the proofs of our main results:

Lemma 1.5 [2] Let $J : X \rightarrow 2^{X^*}$ be the normalized duality mapping. Then for the $x, y \in X$, we have

$$\|x + y\|^2 \leq \|x\|^2 + 2\langle y, j(x + y)\rangle, \quad \forall j(x + y) \in J(x + y).$$

Lemma 1.6 [2] If there exists a positive integer N such that for all $n \geq N, n \in N$,

$$\alpha_{n+1} \leq (1-b_n)\alpha_n + c_n \quad n \geq 1$$

Then $\lim_{n\to\infty} \alpha_n = 0, where \ \ b_n \in [0,1] \ for\ each\, n \in N, \ \sum_{n=1}^{\infty} b_n = \infty \ \ and$
$c_n = 0(b_n)$,

Main result

Theorem 2.1 Let X be an arbitrary real Banach space , C be a convex subset of X, T:C $\rightarrow C$ be an uniformly continuous mapping with bounded range and T satisfies the condition

$$\|x-y\| \leq \|x-y+r[(I-T^n-kI)x-(I-T^n-kI)y]\| \qquad \dots (2.1)$$

where I is the identity mapping on C, for all $x, y \in C, \ k \in (0,1), \ r > 0$.

If q is a fixed point of T and for arbitrary x$\in C$, then modified Ishikawa iterative scheme with random errors defined by (1.10) which satisfies the conditions:

i. $\sum_{n=1}^{\infty} b_n = \infty$

ii. $c_n = 0(b_n)$,

iii. $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \acute{b}_n = \lim_{n\to\infty} \acute{c}_n = 0$.

is converges strongly to unique fixed point q of T.

Proof: Since T bounded range, we set

$$M_1 = \|x_1 - q\| + sup_{n\geq 1}\|T^n y_n - q\| + sup_{n\geq 1}\|u_n - q\|$$

Obviously $M_1 < \infty$.

It is clear that $\|x_1 - q\| \leq M_1$. Let $\|x_n - q\| \leq M_1. Next\ we\ will\ prove\ that$
$\|x_{n+1} - q\| \leq M_1$.

Consider

$$\begin{aligned}\|x_{n+1} - q\| &= \|a_n x_n + b_n T^n y_n + c_n u_n - q\| \\ &= \|a_n x_n + b_n T^n y_n + c_n u_n - (a_n + b_n + c_n)q\| \\ &= \|a_n(x_n - q) + b_n(T^n y_n - q) + c_n(u_n - q)\| \\ &\leq a_n\|x_n - q\| + b_n\|T^n y_n - q\| + c_n\|u_n - q\| \\ &\leq (1-b_n)\|x_n - q\| + b_n\|T^n y_n - q\| + c_n\|u_n - q\| \\ &\leq (1-b_n)M_1 + b_n\|T^n y_n - q\| + c_n\|u_n - q\| \\ &\leq (1-b_n)(\|x_1 - q\| + sup_{n\geq 1}\|T^n y_n - q\| + sup_{n\geq 1}\|u_n - q\|) + \\ &\qquad b_n\|T^n y_n - q\| \\ &\quad + c_n\|u_n - q\| \\ &\leq \|x_1 - q\| + sup_{n\geq 1}\|T^n y_n - q\| + sup_{n\geq 1}\|u_n - q\| - b_n(\|x_1 - q\| \\ &\quad + sup_{n\geq 1}\|T^n y_n - q\| + sup_{n\geq 1}\|u_n - q\|) + b_n\|T^n y_n - q\| + c_n\|u_n - q\| \\ &\leq M_1.\end{aligned}$$

So from the above discussion, we can conclude that the sequence $\{x_n\}_{n=1}^{\infty}$ is bounded.

Let $M_2 = sup_{n\geq 1}\|y_n - q\|$. Denote $M = M_1 + M_2$. Obviously $M < \infty$.

Consider $\|x_{n+1} - q\|^2 = \|a_n x_n + b_n T^n y_n + c_n u_n - q\|^2$

$$\begin{aligned}&= \|a_n x_n + b_n T^n y_n + c_n u_n - (a_n + b_n + c_n)q\|^2 \\ &= \|a_n(x_n - q) + b_n(T^n y_n - q) + c_n(u_n - q)\|^2 \\ &\leq \left(a_n\|x_n - q\| + b_n\|T^n y_n - q\| + c_n\|u_n - q\| \ \right)^2 \\ &\leq \left((1-b_n)\|x_n - q\| + b_n\|T^n y_n - q\| + c_n\|u_n - q\| \ \right)^2\end{aligned}$$

From def.(1.3)

$$\begin{aligned}&\leq (1-b_n)\|x_n - q\|^2 + b_n\|T^n y_n - q\|^2 + c_n\|u_n - q\|^2 \\ &\leq (1-b_n)\|x_n - q\|^2 + b_n M^2 + c_n M^2 \qquad \dots (2.2)\end{aligned}$$

Now from lemma (1.5) for all $n \geq 1$, we obtain

$$\begin{aligned}\|x_{n+1} - q\|^2 &= \|a_n x_n + b_n T^n y_n + c_n u_n - q\|^2 \\ &= \|a_n(x_n - q) + b_n(T^n y_n - q) + c_n(u_n - q)\|^2 \\ &\leq (1-b_n)^2\|x_n - q\|^2 + b_n\|T^n y_n - q\|^2 + c_n\|u_n - q\|^2 \\ &\leq (1-b_n)^2\|x_n - q\|^2 + 2\langle b_n(T^n y_n - q) + c_n(u_n - q), j(x_{n+1} + q)\rangle\end{aligned}$$

$$\leq (1-b_n)^2\|x_n-q\|^2+2\langle b_n(T^ny_n-q),j(x_{n+1}-q)\rangle$$
$$+2\langle c_n(u_n-q),j(x_{n+1}-q)\rangle$$
$$\leq (1-b_n)^2\|x_n-q\|^2+2b_n\langle T^ny_n-q,j(x_{n+1}-q)\rangle \quad +2c_n\langle u_n-q,j(x_{n+1}-q)\rangle$$
$$\leq (1-b_n)^2\|x_n-q\|^2+2b_n\langle T^nx_{n+1}-q,j(x_{n+1}-q)\rangle$$
$$+2b_n\langle T^ny_n-T^nx_{n+1},j(x_{n+1}-q)\rangle \;+2M^2c_n$$
$$\leq (1-b_n)^2\|x_n-q\|^2+2b_n\langle T^nx_{n+1}-q,j(x_{n+1}-q)\rangle$$
$$+2b_n\|T^ny_n-T^nx_{n+1}\|\|x_{n+1}-q\| \;+2M^2c_n$$
$$\leq (1-b_n)^2\|x_n-q\|^2+2b_n(1-k)\|x_{n+1}-q\|^2 \quad +2b_n\|T^ny_n-T^nx_{n+1}\|\|x_{n+1}-q\| +2M^2c_n$$
$$\leq (1-b_n)^2\|x_n-q\|^2+2b_n(1-k)\|x_{n+1}-q\|^2 \quad +2b_nM\|T^ny_n-T^nx_{n+1}\| \;+2M^2c_n$$
$$\leq (1-b_n)^2\|x_n-q\|^2+2b_n(1-k)\|x_{n+1}-q\|^2 \quad +2b_nd_n \;+2M^2c_n \qquad \ldots(2.3)$$

Where $d_n=M\|T^ny_n-T^nx_{n+1}\|$ ... (2.4)

From (1.10) we have

$$\|y_n-x_{n+1}\|=\|(\grave{a}_n-a_n)x_n+\grave{b}_nT^nx_n+\grave{c}_n\grave{v}_n \quad +b_nT^ny_n+c_nu_n)\| \qquad \ldots(2.5)$$

By conditions (ii-iii) and (2.5) then;

$\lim_{n\to\infty}\|y_n-x_{n+1}\|=0 \Rightarrow \lim_{n\to\infty}\|Ty_n-Tx_{n+1}\|=0 \Rightarrow \lim_{n\to\infty}\|T^ny_n-T^nx_{n+1}\|=0$

$$\lim_{n\to\infty} d_n=0. \qquad \ldots(2.6)$$

Substi

$$\|x_{n+1}-q\|^2 \leq (1-b_n)^2\|x_n-q\|^2+2b_n(1-k)\|x_{n+1}-q\|^2 \quad +2b_nd_n \;+2M^2c_n$$
$$\leq \left(1-b_n)^2\|x_n-q\|^2+2b_n((1-b_n)\;\; \|x_n-q\|^2 \quad +M^2b_n+M^2c_n\right)$$
$$+2b_nd_n \;+2M^2c_n$$
$$\leq (1-b_n)^2\;\|x_n-q\|^2+2b_n(1-k)(1-b_n)\;\;\|x_n-q\|^2$$
$$+2b_n(1-k)M^2b_n+2b_n(1-k)M^2c_n+2b_nd_n \;+2M^2c_n$$
$$=\;\|x_n-q\|^2[(1-b_n)^2\;+2b_n(1-k)(1-b_n)\;\;]\quad +2b_n[M^2(1-k)(b_n+\;\;c_n)$$
$$+d_n]\;+2M^2c_n \qquad \ldots(2.7)$$

By $\lim_{n\to\infty} b_n=0,$ *there exists a natural number* $n_o\in N$ *such that for all* $n\geq n_o$

We have $b_n\leq\frac{1}{2}$. *From* (2.7), *we get*

$$\|x_{n+1}-q\|^2\leq\left(1-ka_n\right)\;\;\|x_n-q\|^2 \;\;+2b_n[M^2(1-k)(b_n+\;\;c_n\right)+d_n\;\;+2M^2t_n \qquad (2.8)$$

where $t_n=\frac{b_n}{c_n}$

Now with the help of (i-ii), (2.6) and lemma 1.6

$$\lim_{n\to\infty}\|x_n-q\|=0.$$

Then $\{x_n\}$ converges strongly to fixed point $q\in F(T)$.

If p also is a fixed point of T; we will show that q is uniqueness

$$\|q-p\|^2=\langle q-p,j(q-p)\rangle=\langle Tq-Tp,j(q-p)\rangle\leq(1-k)\|q-p\|^2$$

Since $k\in(0,1)$, it follows that $\|q-p\|^2\leq 0$, which implies the uniqueness. ■

Theorem 2.2 Let X be an arbitrary Banach space , C be a convex subset of X, $T{:}C\to C$ be L-Lipschitzian mapping with bounded range and T satisfies the condition $\|x-y\|\leq\|x-y+r[(I-T^n-kI)x-(I-T^n-kI)y]\|$

where I is the identity mapping on C, for all $x,y\in C$, $k\in(0,1)$, $r>0$.

If q is a fixed point of T, then the modified Ishikawa iterative scheme with random errors defined by (1.10) which satisfies the conditions:

i $\sum_{n=1}^{\infty}b_n=\infty$

$ii.\ c_n = 0(b_n),$
$iii.\ \lim_{n\to\infty\partial} b_n = \lim_{n\to\infty} \acute{b}_n = \lim_{n\to\infty} \acute{c}_n = 0.$
Then the sequence $\{x_n\}$is converges strongly to unique fixed point q of T.

Theorem 2.3 Let X be an arbitrary Banach space , T: X $\rightarrow X$ be an uniformly continuous mapping and T satisfies the condition

$\langle T^n x - T^n y, j(x-y)\rangle \geq k\|x-y\|^2,\ n \geq 1$ for all $x, y \in X,\ k \in (0,1).$

For given $f \in X^*$; let x^*denote the unique solution of the equation $T^n x = f$. Define the mapping H: X $\rightarrow X$ such that $H^n x = f + x - T^n x$, and suppose that the range of H is bounded $Let\ \{x_n\}_{n=1}^{\infty}$ the modified Ishikawa iterative scheme with random errors defined by :

$x_1 \in X,\ \ y_n = \grave{a}_n x_n + \grave{b}_n H^n x_n + \grave{c}_n v_n,\quad x_{n+1} = a_n x_n + b_n H^n y_n + c_n u_n,\quad n \geq 1$

Here $\{u_n\}, \{v_n\}$ are two bounded sequences in X; $\{\grave{a}_n\}$, $\{\grave{b}_n\}$,$\{\grave{c}_n\}$, $\{a_n\}$, $\{b_n\}$, $\{c_n\}$are real sequences in [0, 1] satisfying

$\grave{a}_n + \grave{b}_n + \grave{c}_n = a_n + b_n + c_n$ =1 for all $n \geq 1$ .which satisfying the conditions:
i $\sum_{n=1}^{\infty} b_n = \infty$
$ii.\ c_n = 0(b_n),$
$iii.\ \lim_{n\to\infty\partial} b_n = \lim_{n\to\infty} \acute{b}_n = \lim_{n\to\infty} \acute{c}_n = 0.$
Then the sequence $\{x_n\}$ is converges strongly to unique solution of $T^n x = f$.

Theorem 2.4 Let X be arbitrary Banach space, T: X $\rightarrow X$ be L-Lipschitizan mapping and T satisfies the condition

$\langle T^n x - T^n y, j(x-y)\rangle \geq k\|x-y\|^2,\ n \geq 1$ for all $x, y \in X,\ k \in (0,1).$

For given $f \in X$; let x^*denotes the unique solution of the equation $T^n x = f$. Define the mapping H: X $\rightarrow X$ such that $H^n x = f + x - T^n x$, and suppose that the range of H is bounded For any $x_1 \in X,\ let\ \{x_n\}_{n=1}^{\infty}$ the modified Ishikawa iterative scheme with random errors defined by

$y_n = \grave{a}_n x_n + \grave{b}_n H^n x_n + \grave{c}_n v_n,\qquad x_{n+1} = a_n x_n + b_n H^n y_n + c_n u_n,\quad n \geq 1$

Here $\{u_n\}, \{v_n\}$ are two bounded sequences in X; $\{\grave{a}_n\}$, $\{\grave{b}_n\}$,$\{\grave{c}_n\}$, $\{a_n\}$, $\{b_n\}$, $\{c_n\}$are real sequences in [0, 1] satisfying

$\grave{a}_n + \grave{b}_n + \grave{c}_n = a_n + b_n + c_n$ =1 for all $n \geq 1$ .which satisfying the conditions:
i $\sum_{n=1}^{\infty} b_n = \infty$
$ii.\ c_n = 0(b_n),$
$iii.\ \lim_{n\to\infty\partial} b_n = \lim_{n\to\infty} \acute{b}_n = \lim_{n\to\infty} \acute{c}_n = 0.$
Then the sequence $\{x_n\}$ is converges strongly to unique solution of $T^n x = f$.

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الحل التكراري المطور لمعادلة غير خطية على دوال مستمرة بانتظام في فضاء بناخ الحقيقي الأختياري

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ا
لخلاصة
في هذا البحث نقوم بدراسة نظريات التقارب لمتتابعة ايشكاوا المطورة الممزوجة بالخطأ الى دوال مستمره بانتظام وحل معادلات غير خطية على دوال مستمرة بانتظام في فضاء بناخ الحقيقي الأختياري.