On A Bitopological (1,2)*- Proper Functions

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Abstract

In this paper, we introduce a new type of functions in bitopological spaces, namely, (1,2)*-proper functions. Also, we study the basic properties and characterizations of these functions . One of the most important of equivalent definitions to the (1,2)*-proper functions is given by using (1,2)*-cluster points of filters . Moreover we define and study (1,2)*-perfect functions and (1,2)*-compact functions in bitopological spaces and we study the relation between (1,2)*-proper functions and each of (1,2)*-closed functions , (1,2)*-perfect functions and (1,2)*-compact functions and we give an example when the converse may not be true .

Key words: (1,2)*-proper functions, (1,2)*-perfect functions , (1,2)*-compact functions , (1,2)*-cluster points, (1,2)*- T_2 -spaces , (1,2)*-compactly closed sets and (1,2)*-K-spaces .

Introduction

The concept of a bitopological space (X,τ_1,τ_2) was first introduced by Kelly [1] , where X is a nonempty set and τ_1 , τ_2 are topologies on X . Lellis Thivagar et. al. [2] introduced the concepts of (1,2)*-compact spaces and studied their properties . Ravi et. al. [3] introduced the concepts of (1,2)*-closed functions . The purpose of this paper is to introduce a new class of functions, namely , (1,2)*-proper functions . We give the definition by depending on the definition of (1,2)*-closed functions . Also, we study the characterizations and basic properties of (1,2)*-proper functions . We can prove that a (1,2)*-continuous function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is (1,2)*-proper if and only if whenever ξ is a filter on X and $y\in Y$ is a (1,2)*-cluster point of $f(\xi)$, then there is a (1,2)*-cluster point x of ξ such that $f(x)=y$. Moreover we study (1,2)*-perfect functions and (1,2)*-compact functions in bitopological spaces and study the relation between (1,2)*-proper functions and each of (1,2)*-closed functions , (1,2)*-perfect functions and (1,2)*-compact functions and we give an example when the converse may not be true .

Throughout this paper (X,τ_1,τ_2), (Y,σ_1,σ_2) and (Z,η_1,η_2) (or simply X , Y and Z) represent non-empty bitopological spaces on which no separation axioms are assumed, unless otherwise mentioned . If $A\subseteq X$, then $(A,\tau_{1_A},\tau_{2_A})$ is called a bitopological subspace of (X,τ_1,τ_2) .

1.Preliminaries

First we recall the following definitions:

Definition(1.1)[4]: A subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$-open if $A=U_1\bigcup U_2$ where $U_1\in\tau_1$ and $U_2\in\tau_2$. The complement of a $\tau_1\tau_2$-open set is called $\tau_1\tau_2$-closed .

Notice that $\tau_1\tau_2$- open sets need not necessarily form a topology **[4]** .

Definition(1.2)[4]: Let A be a subset of a bitopological space (X,τ_1,τ_2) .Then:

i) The $\tau_1\tau_2$- closure of A , denoted by $\tau_1\tau_2 cl(A)$, is defined by:-

$\tau_1\tau_2 cl(A)=\bigcap\{F : A\subseteq F \,\&\, F \text{ is } \tau_1\tau_2-\text{closed}\}$.

ii) The $\tau_1\tau_2$- interior of A , denoted by $\tau_1\tau_2\,\text{int}(A)$, is defined by:-

$\tau_1\tau_2\,\text{int}(A)=\bigcup\{U : U\subseteq A \,\&\, U \text{ is } \tau_1\tau_2-\text{open}\}$.

Definition(1.3)[2]: A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ from a bitopological space (X,τ_1,τ_2) into a bitopological space (Y,σ_1,σ_2) is called (1,2)*-continuous if $f^{-1}(V)$ is $\tau_1\tau_2$-closed set in X for every $\sigma_1\sigma_2$-closed set V in Y .

Definition(1.4)[5]: Let $(X_\alpha,\tau_\alpha,\tau'_\alpha)_{\alpha\in\wedge}$ be a family of bitopological spaces . On the product set $X=\underset{\alpha\in\wedge}{\pi}X_\alpha$ we define a bitopological structure (τ,τ') by taking τ as the product topology generated by the τ_α s and τ' as the product topology generated by the τ'_α s .

Definition(1.5)[6]: A filter ξ on a set X is a non-empty collection of non-empty subsets of X which has the following properties:-

i) Every finite intersection of sets of ξ belongs to ξ .

ii) Every subset of X which contains a set of ξ belongs to ξ .

Definition(1.6)[6]: A non-empty collection ξ_0 of non-empty subsets of a set X is called a filter base for some filter on X if and only if for each $F_0^1, F_0^2 \in \xi_0$, there exists $F_0^3 \in \xi_0$ such that $F_0^3 \subseteq F_0^1 \cap F_0^2$.

Definition(1.7):A subset A of a bitopological space (X, τ_1, τ_2) is called a (1,2)*-neighborhood of a point x in X if there exists a $\tau_1\tau_2$-open set U in X such that $x \in U \subseteq A$. The family of all (1,2)*-neighborhoods of a point $x \in X$ is denoted by $N^*(x)$.

Definition(1.8): A filter ξ on a bitopological space (X, τ_1, τ_2) has $x \in X$ as an (1,2)*-cluster point (written $\xi \overset{(1,2)*}{\propto} x$) iff each $F \in \xi$ meets each $N \in N^*(x)$.

Remark(1.9): A filter ξ on a bitopological space (X, τ_1, τ_2) has $x \in X$ as an (1,2)*-cluster point iff $x \in \bigcap\{\tau_1\tau_2 cl(F) : F \in \xi\}$.

Proof: To prove that $\xi \overset{(1,2)*}{\propto} x \Leftrightarrow x \in \bigcap\{\tau_1\tau_2 cl(F) : F \in \xi\}$.

$$\therefore \xi \overset{(1,2)*}{\propto} x \Leftrightarrow \forall N \in N^*(x) \ \&\ \forall F \in \xi, N \cap F \neq \phi$$
$$\Leftrightarrow \forall N \in N^*(x), F \cap N \neq \phi, \forall F \in \xi$$
$$\Leftrightarrow x \in \tau_1\tau_2 cl(F), \forall F \in \xi$$
$$\Leftrightarrow x \in \bigcap\{\tau_1\tau_2 cl(F) : F \in \xi\}.$$

Definition(1.10): A filter base ξ_0 on a bitopological space (X, τ_1, τ_2) has $x \in X$ as an (1,2)*-cluster point (written $\xi_0 \overset{(1,2)*}{\propto} x$) iff each $F \in \xi_0$ meets each $N \in N^*(x)$.

Definition(1.11)[5]: ***A bitopological space*** (X, τ_1, τ_2) ***is called a (1,2)*-*** T_2***-space (or quasi-Hausdorff space if for any two distinct points x and y of*** X***, there are two*** $\tau_1\tau_2$***-open sets U and V such that*** $x \in U$, $y \in V$ ***and*** $U \cap V = \phi$**.**

Definition(1.12)[2]: A bitopological space (X, τ_1, τ_2) is said to be a (1,2)*-compact space if and only if every $\tau_1\tau_2$-open cover of X has a finite subcover .

Theorem(1.13): A bitopological space (X, τ_1, τ_2) is (1,2)*-compact if and only if given any family $\{F_\alpha\}_{\alpha \in \wedge}$ of $\tau_1\tau_2$-closed subsets of X such that the intersection of any finite number of the F_α is non-empty, then $\bigcap_{\alpha \in \wedge} F_\alpha \neq \phi$.

Proof: It is Obvious .

Theorem(1.14): A bitopological space (X,τ_1,τ_2) is (1,2)*-compact if and only if every filter
on X has an (1,2)*-cluster point .

Proof: $\Rightarrow$ Suppose that (X,τ_1,τ_2) is a (1,2)*-compact space and ξ be a filter on X .
Take $\Omega=\{\tau_1\tau_2cl(F):F\in\xi\}$ **,** then by (1.5) Ω has the finite intersection property (f.i.p.) .
But X is (1,2)*-compact , then by (1.13) , $\bigcap\Omega\neq\phi$. This implies that any point of this intersection is an (1,2)*-cluster point of ξ .

$\Leftarrow$ Let Ω be a collection of $\tau_1\tau_2$-closed subsets of X such that Ω has the f.i.p .
Then there exists a filter ξ on X such that $\Omega\subseteq\xi$.
$\Rightarrow\bigcap\{\tau_1\tau_2cl(F):F\in\xi\}\subseteq\bigcap\{\tau_1\tau_2cl(A):A\in\Omega\}$. Since ξ has an (1,2)*-cluster point , then there exists $x\in\bigcap\{\tau_1\tau_2cl(F):F\in\xi\}\Rightarrow\bigcap\{\tau_1\tau_2cl(F):F\in\xi\}\neq\phi$.
Since $\bigcap\{\tau_1\tau_2cl(F):F\in\xi\}\subseteq\bigcap\{\tau_1\tau_2cl(A):A\in\Omega\}$, then $\bigcap\{\tau_1\tau_2cl(A):A\in\Omega\}\neq\phi$. But Ω is
a collection of $\tau_1\tau_2$-closed subsets of X , then $\bigcap\{A:A\in\Omega\}=\bigcap\{\tau_1\tau_2cl(A):A\in\Omega\}$.
Hence $\bigcap\{A:A\in\Omega\}\neq\phi$ and by (1.13) , X is a (1,2)*-compact space .

2. (1,2)*-CLOSED FUNCTIONS

Definition(2.1)[3]: A function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ from a bitopological space (X,τ_1,τ_2)
into a bitopological space (Y,σ_1,σ_2) is called (1,2)*-closed (resp. (1,2)*-open) if $f(F)$ is $\sigma_1\sigma_2$-
closed (resp. $\sigma_1\sigma_2$-open) in Y for every $\tau_1\tau_2$-closed (resp. $\tau_1\tau_2$-open) set F in X .

Examples(2.2):
i) Let $f:(\Re,\mu,\mu)\rightarrow(\Re,\mu,\mu)$ be a function which is defined by : $f(x)=0$, $\forall\ x\in\Re$.
Then f is an (1,2)*-closed function .

ii) An inclusion function $i:(F,\tau_F,\tau'_F)\rightarrow(X,\tau,\tau')$ is (1,2)*-closed iff F is a $\tau\tau'$-closed set in X .

Theorem(2.3): A function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is (1,2)*-closed if and only if for each subset B of Y and each $\tau_1\tau_2$-open set U in X containing $f^{-1}(B)$, there exists a $\sigma_1\sigma_2$-open set V in Y containing B such that $f^{-1}(V)\subseteq U$.

Proof: $\Rightarrow$ Suppose that B is an arbitrary subset in Y and U is an arbitrary $\tau_1\tau_2$-open set in X containing $f^{-1}(B)$. Put $V=Y-f(X-U)$. Then by (2.1) , V is a $\sigma_1\sigma_2$-open set in Y . Since $f^{-1}(B)\subseteq U\Rightarrow X-U\subseteq f^{-1}(Y-B)\Rightarrow f(X-U)\subseteq Y-B\Rightarrow B\subseteq Y-f(X-U)\Rightarrow B\subseteq V$ and $f^{-1}(V)\subseteq U$.

Conversely , Let F be any $\tau_1\tau_2$-closed set in X . Put $B=Y-f(F)$, then we have $f^{-1}(B)\subseteq X-F$. Since $X-F$ is $\tau_1\tau_2$-open , then by hypothesis there exists a $\sigma_1\sigma_2$-open set

V in Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq X - F$. Therefore, we obtain $f(F) = Y - V$ and hence $f(F)$ is $\sigma_1\sigma_2$-closed in Y . This shows that f is a (1,2)*-closed function .

Theorem(2.4): Let (X,τ_1,τ_2) , (Y,σ_1,σ_2) and (Z,η_1,η_2) be three bitopological spaces ,
and $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, $g:(Y,\sigma_1,\sigma_2) \to (Z,\eta_1,\eta_2)$ be two functions . Then:-
i) If f and g are (1,2)*-closed , then $g \circ f$ is (1,2)*-closed .
ii) If $g \circ f$ is (1,2)*-closed and f is (1,2)*-continuous and onto, then g is (1,2)*-closed .
iii) If $g \circ f$ is (1,2)*-closed and g is (1,2)*-continuous and one-to-one ,then f is (1,2)*-closed.

Proof:
i) To prove that $g \circ f:(X,\tau_1,\tau_2) \to (Z,\eta_1,\eta_2)$ is a (1,2)*-closed function . Let F be any $\tau_1\tau_2$-closed subset of X . Since f is a (1,2)*-closed function, then $f(F)$ is a $\sigma_1\sigma_2$-closed set in Y . Since g is a (1,2)*-closed function , then $g(f(F))$ is a $\eta_1\eta_2$-closed set in Z , hence $(g \circ f)(F)$ is a $\eta_1\eta_2$-closed set in Z . Thus $g \circ f:(X,\tau_1,\tau_2) \to (Z,\eta_1,\eta_2)$ is a (1,2)*-closed function .

ii) To prove that $g:(Y,\sigma_1,\sigma_2) \to (Z,\eta_1,\eta_2)$ is a (1,2)*-closed function . Let F be any $\sigma_1\sigma_2$-closed subset of Y . Since f is (1,2)*-continuous, then $f^{-1}(F)$ is a $\tau_1\tau_2$-closed set in X . Since $g \circ f$ is (1,2)*-closed, then $(g \circ f)(f^{-1}(F)) = g(f \circ f^{-1}(F))$ is a $\eta_1\eta_2$-closed set in Z . Since f is onto ,then $g(F)$ is a $\eta_1\eta_2$-closed set in Z . Thus $g:(Y,\sigma_1,\sigma_2) \to (Z,\eta_1,\eta_2)$ is a (1,2)*-closed function .

iii) To prove that $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is a (1,2)*-closed function . Let F be any $\tau_1\tau_2$-closed subset of X . Since $g \circ f$ is (1,2)*-closed, then $(g \circ f)(F)$ is a $\eta_1\eta_2$-closed set in Z . Since g is (1,2)*-continuous, then $g^{-1}(g \circ f(F)) = (g^{-1} \circ g)(f(F))$ is a $\sigma_1\sigma_2$-closed set in Y . Since g is one-to-one, then $f(F)$ is a $\sigma_1\sigma_2$-closed set in Y . Thus $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is a (1,2)*-closed function .

Theorem(2.5): Let $f:(X,\tau,\tau') \to (Y,\sigma,\sigma')$ be a (1,2)*-closed function . Then for each subset T of Y , the function $f_T:(f^{-1}(T),\tau_{f^{-1}(T)},\tau'_{f^{-1}(T)}) \to (T,\sigma_T,\sigma'_T)$ which agrees with f on $f^{-1}(T)$ is also (1,2)*-closed function .

Proof: Let F be a $\tau_{f^{-1}(T)}\tau'_{f^{-1}(T)}$-closed subset of $f^{-1}(T)$. Then there is a $\tau\tau'$-closed subset F_1 of X such that $F = F_1 \cap f^{-1}(T)$. Since $f_T(F) = f(F_1) \cap T$ and f is (1,2)*-closed function , then $f(F_1)$ is $\sigma\sigma'$-closed in Y . Hence $f(F_1) \cap T$ is a $\sigma_T\sigma'_T$-closed set in T . Thus f_T is a (1,2)*-closed function .

Theorem(2.6): If $f:(X,\tau,\tau') \to (Y,\sigma_1,\sigma_2)$ is a (1,2)*-closed function ,then the restriction of f
to a $\tau\tau'$-closed subset F of X is a (1,2)*-closed function of F into Y .
Proof: Since F is a $\tau\tau'$-closed set in X , then the inclusion function $i:(F,\tau_F,\tau'_F) \to (X,\tau,\tau')$

is a (1,2)*-closed function . Since $f:(X,\tau,\tau')\rightarrow(Y,\sigma_1,\sigma_2)$ is a (1,2)*-closed function , then by (2.4) $f\circ i:(F,\tau_F,\tau'_F)\rightarrow(Y,\sigma_1,\sigma_2)$ is a (1,2)*-closed function . But $f\circ i=f/F$, thus the restriction function $f/F:(F,\tau_F,\tau'_F)\rightarrow(Y,\sigma_1,\sigma_2)$ is a (1,2)*-closed function .

Remark(2.7): If $f_1:(X_1,\tau_1,\tau_2)\rightarrow(Y_1,\sigma_1,\sigma_2)$ and $f_2:(X_2,\tau'_1,\tau'_2)\rightarrow(Y_2,\sigma'_1,\sigma'_2)$ are two (1,2)*-closed functions . Then $f_1\times f_2:(X_1\times X_2,\tau_1\times\tau'_1,\tau_2\times\tau'_2)\rightarrow(Y_1\times Y_2,\sigma_1\times\sigma'_1,\sigma_2\times\sigma'_2)$ is not necessarily a (1,2)*-closed function .

Example: Let $f_1:(\Re,\mu,\mu)\rightarrow(\Re,\mu,\mu)$ be a function which is defined by : $f_1(x)=0$, $\forall\ x\in\Re$. And Let $f_2:(\Re,\mu,\mu)\rightarrow(\Re,\mu,\mu)$ be a function which is defined by : $f_2(x)=x$, $\forall\ x\in\Re$. Where f_2 is the identity function on $\Re$. Clearly f_1 and f_2 are (1,2)*-closed functions , but $f_1\times f_2:(\Re\times\Re,\mu',\mu')\rightarrow(\Re\times\Re,\mu',\mu')$ such that $(f_1\times f_2)(x,y)=(0,y)$, $\forall\ (x,y)\in\Re\times\Re$ (where μ' is the product topology on $\Re\times\Re$) is not a (1,2)*-closed function , since the set $A=\{(x,y)\in\Re\times\Re : x\,y=1\}$ is $\mu'\mu'$-closed in $\Re\times\Re$, but $(f_1\times f_2)(A)=\{0\}\times\Re\setminus\{0\}\cong\Re\setminus\{0\}$ is not $\mu'\mu'$-closed in $\Re\times\Re$.

Theorem(2.8): Let $f_1:(X_1,\tau_1,\tau_2)\rightarrow(Y_1,\sigma_1,\sigma_2)$ and $f_2:(X_2,\tau'_1,\tau'_2)\rightarrow(Y_2,\sigma'_1,\sigma'_2)$ be functions . If $f_1\times f_2:(X_1\times X_2,\tau_1\times\tau'_1,\tau_2\times\tau'_2)\rightarrow(Y_1\times Y_2,\sigma_1\times\sigma'_1,\sigma_2\times\sigma'_2)$ is (1,2)*-closed , then f_1 and f_2 are also (1,2)*-closed functions .

Proof: Suppose that $f_1\times f_2:(X_1\times X_2,\rho_1,\rho_2)\rightarrow(Y_1\times Y_2,\rho_3,\rho_4)$ is a (1,2)*-closed function where $\rho_i, i=1,2,3,4$ be the product topology on $X_1\times X_2$ and $Y_1\times Y_2$ respectively . To prove that $f_1:(X_1,\tau_1,\tau_2)\rightarrow(Y_1,\sigma_1,\sigma_2)$ is (1,2)*-closed . Let F be a $\tau_1\tau_2$-closed subset of X_1, to prove that $f_1(F)$ is $\sigma_1\sigma_2$-closed in Y_1. Suppose that $G=f_1(F)$ $\Rightarrow$ $F\times X_2$ is $\rho_1\rho_2$-closed in $X_1\times X_2$. Since $f_1\times f_2$ is (1,2)*-closed $\Rightarrow$ $(f_1\times f_2)(F\times X_2)=f_1(F)\times f_2(X_2)=G\times f_2(X_2)$ is $\rho_3\rho_4$-closed in $Y_1\times Y_2$. i.e. $\rho_3\rho_4 cl(G\times f_2(X_2)=G\times f_2(X_2)$ $\Rightarrow$ $\sigma_1\sigma_2 cl(G)=G$ $\Rightarrow$ $G=f_1(F)$ is $\sigma_1\sigma_2$-closed in Y_1 $\Rightarrow$ $f_1:(X_1,\tau_1,\tau_2)\rightarrow(Y_1,\sigma_1,\sigma_2)$ is a (1,2)*-closed function . By the same way we can prove that f_2 is a (1,2)*-closed function . Thus f_1 and f_2 are (1,2)*-closed functions .

Definition(2.9)[7]: A function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ from a bitopological space (X,τ_1,τ_2)
into a bitopological space (Y,σ_1,σ_2) is called (1,2)*-homeomorphism if :-

i) f is (1,2)*-continuous .
ii) f is one-to-one and onto .
iii) f is (1,2)*-closed (or (1,2)*-open) .

3. (1,2)*-PROPER FUNCTIONS

In this section we introduce a new type of functions in bitopological spaces which we call (1,2)*-proper functions . Besides we give examples and theorems .

Definition(3.1): A function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ from a bitopological space (X,τ_1,τ_2) into a bitopological space (Y,σ_1,σ_2) is called (1,2)*-proper if :-

i) f is (1,2)*-continuous .

ii) $f\times I_Z:(X\times Z,\tau_1\times\tau,\tau_2\times\tau')\to(Y\times Z,\sigma_1\times\tau,\sigma_2\times\tau')$ is (1,2)*-closed for every bitopological space (Z,τ,τ') .

Examples(3.2):

i) Let $f:(\Re,\mu,\mu)\to(\Re,\mu,\mu)$ be a function which is defined by : $f(x)=0$, $\forall\ x\in\Re$. Notice that f is a (1,2)*-closed function , but f is not a (1,2)*-proper function , since for the usual bitopological space $(\Re,\mu,\mu)$ the function $f\times I_{\Re}:(\Re\times\Re,\mu\times\mu,\mu\times\mu)\to(\Re\times\Re,\mu\times\mu,\mu\times\mu)$ which is defined by $(f\times I_{\Re})(x,y)=(0,y),\forall\ (x,y)\in\Re\times\Re$ is not a (1,2)*-closed function .

ii) An inclusion function $i:(F,\tau_F,\tau'_F)\to(X,\tau,\tau')$ is (1,2)*-proper iff F is a $\tau\tau'$-closed set in X .

Theorem(3.3): Every (1,2)*-proper function is a (1,2)*-closed function .

Proof: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a (1,2)*-proper function , then the function $f\times I_Z:(X\times Z,\tau_1\times\tau,\tau_2\times\tau')\to(Y\times Z,\sigma_1\times\tau,\sigma_2\times\tau')$ is (1,2)*-closed for every bitopological space (Z,τ,τ') . Let $Z=\{t\}$, then $X\times Z=X\times\{t\}\cong X$ & $Y\times Z=Y\times\{t\}\cong Y$ and we can replace $f\times I_Z$ by f . Thus $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is a (1,2)*-closed function .

Remark(3.4): The converse of (3.3) may not be true in general . Consider the following example:

Example: In (3.2) (i) , $f:(\Re,\mu,\mu)\to(\Re,\mu,\mu)$ is a (1,2)*-closed function, but it is not a (1,2)*-proper function .

Theorem(3.5): Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a (1,2)*-continuous and one-to-one function.

Then the following statements are equivalent:-

i) f is (1,2)*-proper .

ii) f is (1,2)*-closed .

iii) f is a (1,2)*-homeomorphism of X onto a $\sigma_1\sigma_2$-closed subset of Y **.**

Proof: By theorem (3.3), $(i\to ii)$.

$(ii\to iii)$. Assume that $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is a (1,2)*-closed function . Since X is a $\tau_1\tau_2$-closed set in X , then $f(X)$ is a $\sigma_1\sigma_2$-closed set in Y . Since f is (1,2)*-continuous and one-to-one, then f is a (1,2)*-homeomorphism of X onto a $\sigma_1\sigma_2$-closed subset $f(X)$ of Y **.**

$(iii\to i)$. To prove that $f\times I_Z:(X\times Z,\rho_1,\rho_2)\to(Y\times Z,\rho_3,\rho_4)$ is (1,2)*-closed for every bitopological space (Z,τ,τ'), where $\rho_i, i=1,2,3,4$ be the product topology on $X\times Z$ and $Y\times Z$ respectively . Since f is a (1,2)*-homeomorphism of X onto a $\sigma_1\sigma_2$-closed subset F of Y , then $f\times I_Z$ is a (1,2)*-homeomorphism of $X\times Z$ onto a $\rho_3\rho_4$-closed subset $F\times Z$ of

$Y \times Z$ and therefore $f \times I_Z$ is (1,2)*-closed . Thus $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is a (1,2)*-proper function .

Corollary(3.6): Every (1,2)*-homeomorphism is a (1,2)*-proper function .

Remark(3.7): The converse of (3.6) may not be true in general . Consider the following example:-

Example: Let $f:([0,1],\mu',\mu') \to (\Re,\mu,\mu)$ be a function which is defined by :
$f(x) = x$, $\forall\ x \in [0,1]$.
(where μ' is the relative usual topology on [0,1]) . Clearly that f is a (1,2)*-proper function, but it
is not (1,2)*-homeomorphism .

Theorem(3.8): Let $f:(X,\tau,\tau') \to (Y,\sigma,\sigma')$ be a (1,2)*-proper function . Then for each subset T of Y , the function $f_T:(f^{-1}(T),\tau_{f^{-1}(T)},\tau'_{f^{-1}(T)}) \to (T,\sigma_T,\sigma'_T)$ which agrees with f on $f^{-1}(T)$ is also (1,2)*-proper .

Proof: To prove that $f_T:(f^{-1}(T),\tau_{f^{-1}(T)},\tau'_{f^{-1}(T)}) \to (T,\sigma_T,\sigma'_T)$ is (1,2)*-proper . Since f is (1,2)*-continuous , then so is f_T. Since f is (1,2)*-proper , then for every bitopological space (Z,τ,τ') the function $f \times I_Z:(X \times Z,\rho_1,\rho_2) \to (Y \times Z,\rho_3,\rho_4)$ is (1,2)*-closed , where ρ_i , $i = 1,2,3,4$ be the product topology on $X \times Z$ and $Y \times Z$ respectively . Since $f_T \times I_Z = (f \times I_Z)_{T \times Z}$, then by (2.5) $f_T \times I_Z$ is (1,2)*-closed . Thus $f_T:(f^{-1}(T),\tau_{f^{-1}(T)},\tau'_{f^{-1}(T)}) \to (T,\sigma_T,\sigma'_T)$ is a (1,2)*-proper function .

Definition(3.9): If the function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is (1,2)*-proper and (X,τ_1,τ_2) is a
(1,2)*-T_2-space, then f is called a (1,2)*-perfect function .

Corollary(3.10): Every (1,2)*-perfect function is a (1,2)*-proper function .

Remark(3.11): The converse of (3.10) may not be true in general . Consider the following example:-

Example: Let $f:(\Re,\tau_{cof.},\tau_{cof.}) \to (\Re,\tau_{cof.},\tau_{cof.})$ be the identity function , where $\tau_{cof.}$ be the cofinite
topology on $\Re$. Then f is a (1,2)*-homeomorphism and by (3.6) , f is (1,2)*-proper . Since
$(\Re,\tau_{cof.},\tau_{cof.})$ is not a (1,2)*-T_2-space, then f is not a (1,2)*-perfect function .

Theorem(3.12): Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2) \to (Z,\eta_1,\eta_2)$ be two (1,2)*-
continuous functions . Then:-
i) If f and g are (1,2)*-proper, then $g \circ f$ is (1,2)*-proper .
ii) If $g \circ f$ is (1,2)*-proper and f is onto, then g is (1,2)*-proper .
iii) If $g \circ f$ is (1,2)*-proper and g is one-to-one , then f is (1,2)*-proper .

Proof:

i) It is clear that $g\circ f:(X,\tau_1,\tau_2)\to(Z,\eta_1,\eta_2)$ is (1,2)*-continuous . Let (Z_1,τ,τ') be any bitopological space **.** We have : $(g\circ f)\times I_{Z_1}=(g\times I_{Z_1})\circ(f\times I_{Z_1})$. Since f and g are (1,2)*-proper, then $f\times I_{Z_1}$ and $g\times I_{Z_1}$ are (1,2)*-closed . Hence by (2.4) , no.(i) $(g\circ f)\times I_{Z_1}$ is (1,2)*-closed . Thus $g\circ f$ is a (1,2)*-proper function .

ii) To prove that $g\times I_{Z_1}:(Y\times Z_1,\sigma_1\times\tau,\sigma_2\times\tau')\to(Z\times Z_1,\eta_1\times\tau,\eta_2\times\tau')$ is (1,2)*-closed for every bitopological space Z_1. Since $g\circ f$ is (1,2)*-proper, then

$(g\circ f)\times I_{Z_1}=(g\times I_{Z_1})\circ(f\times I_{Z_1})$

is (1,2)*-closed . Since f is (1,2)*-continuous and onto, then so is $f\times I_{Z_1}$, hence by (2.4), no. (ii) $g\times I_{Z_1}$ is (1,2)*-closed . Thus $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ is a (1,2)*-proper function .

iii) To prove that $f\times I_{Z_1}:(X\times Z_1,\tau_1\times\tau,\tau_2\times\tau')\to(Y\times Z_1,\sigma_1\times\tau,\sigma_2\times\tau')$ is (1,2)*-closed for every bitopological space Z_1 . Since $g\circ f$ is (1,2)*-proper, then

$(g\circ f)\times I_{Z_1}=(g\times I_{Z_1})\circ(f\times I_{Z_1})$

is (1,2)*-closed . Since g is one-to-one and (1,2)*-continuous, then so is $g\times I_{Z_1}$, hence by (2.4) , no. (iii) $f\times I_{Z_1}$ is (1,2)*-closed . Thus $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is a (1,2)*-proper function .

Corollary(3.13): If $f:(X,\tau,\tau')\to(Y,\sigma_1,\sigma_2)$ is a (1,2)*-proper function, then the restriction of f to a $\tau\tau'$-closed subset F of X is a (1,2)*-proper function of F into Y .

Proof: Since F is a $\tau\tau'$-closed set in X, then the inclusion function $i:(F,\tau_F,\tau'_F)\to(X,\tau,\tau')$ is a (1,2)*-proper function . Since $f:(X,\tau,\tau')\to(Y,\sigma_1,\sigma_2)$ is a (1,2)*-proper function , then by (3.12) , no.(i) $f\circ i:(F,\tau_F,\tau'_F)\to(Y,\sigma_1,\sigma_2)$ is a (1,2)*-proper function . But $f\circ i=f/F$, thus the restriction function $f/F:(F,\tau_F,\tau'_F)\to(Y,\sigma_1,\sigma_2)$ is a (1,2)*-proper function .

Corollary(3.14): If $f:(X,\tau,\tau')\to(Y,\sigma_1,\sigma_2)$ is a (1,2)*-perfect function, then the restriction of f to a $\tau\tau'$-closed subset F of X is a (1,2)*-perfect function of F into Y .

Proof: It is Obvious .

Corollary(3.15): The composition of two (1,2)*-perfect functions is a (1,2)*-perfect function.

Corollary(3.16): Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ be functions . If f is a (1,2)*-perfect function and g is a (1,2)*-proper function . Then $g\circ f$ is a (1,2)*-perfect function .

Theorem(3.17): If $f_1:(X_1,\tau_1,\tau_2)\to(Y_1,\sigma_1,\sigma_2)$ and $f_2:(X_2,\tau'_1,\tau'_2)\to(Y_2,\sigma'_1,\sigma'_2)$ are two (1,2)*-proper functions . Then $f_1\times f_2:(X_1\times X_2,\tau_1\times\tau'_1,\tau_2\times\tau'_2)\to(Y_1\times Y_2,\sigma_1\times\sigma'_1,\sigma_2\times\sigma'_2)$ is also (1,2)*-proper function .

Proof: Let (Z,τ,τ') be any bitopological space .We can write $f_1\times f_2\times I_Z$ by the composition of $I_{Y_1}\times f_2\times I_Z$ and $f_1\times I_{X_2}\times I_Z$. Since f_1 and f_2 are (1,2)*-proper functions, then

$f_1 \times I_{X_2} \times I_Z$ and $I_{Y_1} \times f_2 \times I_Z$ are (1,2)*-closed functions , hence by (2.4), no. (i) $(I_{Y_1} \times f_2 \times I_Z) \circ (f_1 \times I_{X_2} \times I_Z)$ is (1,2)*-closed . But $f_1 \times f_2 \times I_Z = (I_{Y_1} \times f_2 \times I_Z) \circ (f_1 \times I_{X_2} \times I_Z)$ $\Rightarrow$ $f_1 \times f_2 \times I_Z$ is a (1,2)*-closed function . Thus $f_1 \times f_2 : (X_1 \times X_2, \tau_1 \times \tau'_1, \tau_2 \times \tau'_2) \to (Y_1 \times Y_2, \sigma_1 \times \sigma'_1, \sigma_2 \times \sigma'_2)$ is a (1,2)*-proper function .

Theorem(3.18): Let $f_1 : (X_1, \tau_1, \tau_2) \to (Y_1, \sigma_1, \sigma_2)$ and $f_2 : (X_2, \tau'_1, \tau'_2) \to (Y_2, \sigma'_1, \sigma'_2)$ be two (1,2)*-continuous functions such that $f_1 \times f_2$ is a (1,2)*-proper function . Then , f_1 and f_2 are (1,2)*-proper .

Proof: Let (Z, τ, τ') be any bitopological space . Since $f_1 \times f_2$ is (1,2)*-proper , then $f_1 \times f_2 \times I_Z : (X_1 \times X_2 \times Z, \rho_1, \rho_2) \to (Y_1 \times Y_2 \times Z, \rho_3, \rho_4)$ is (1,2)*-closed , where $\rho_i , i = 1,2,3,4$ be the product topology on $X_1 \times X_2 \times Z$ and $Y_1 \times Y_2 \times Z$ respectively . To prove that $f_2 \times I_Z : (X_2 \times Z, \eta_1, \eta_2) \to (Y_2 \times Z, \eta_3, \eta_4)$ is (1,2)*-closed , where $\eta_i , i = 1,2,3,4$ be the product topology on $X_2 \times Z$ and $Y_2 \times Z$ respectively . Let F be any $\eta_1\eta_2$-closed set in $X_2 \times Z$ and $G = (f_2 \times I_Z)(F)$. To prove that G is $\eta_3\eta_4$-closed in $Y_2 \times Z$. Since $X_1 \neq \phi$, then $X_1 \times F$ is $\rho_1\rho_2$-closed in $X_1 \times X_2 \times Z$. Since $f_1 \times f_2 \times I_Z$ is (1,2)*-closed, then $(f_1 \times f_2 \times I_Z)(X_1 \times F) = f_1(X_1) \times G$ is $\rho_3\rho_4$-closed in $Y_1 \times Y_2 \times Z$ $\Rightarrow$ $\rho_3\rho_4 cl(f_1(X_1) \times G) = f_1(X_1) \times G$ $\Rightarrow$ $\eta_3\eta_4 cl(G) = G$ $\Rightarrow$ $G = (f_2 \times I_Z)(F)$ is $\eta_3\eta_4$-closed in $Y_2 \times Z$. Therefore $f_2 \times I_Z$ is (1,2)*-closed . Thus f_2 is a (1,2)*-proper function . By the same way we can prove that f_1 is (1,2)*-proper .

Lemma(3.19): Let (X, τ_1, τ_2) be any bitopological space such that the constant function $f : (X, \tau_1, \tau_2) \to P = \{w\}$ is (1,2)*-proper .Then X is a (1,2)*-compact space, where w is any point which does not belong to X .

Proof: Let ξ be a filter on X and let $X' = X \bigcup \{w\}$. Then $\xi' = \{M \bigcup \{w\} : M \in \xi\}$ is a filter on X'. This filter with ϕ form a topology on X' say τ. Hence (X', τ, τ) is a bitopological space
associated with ξ . Let $\Delta \subseteq X \times X'$ such that $\Delta = \{(x, x) : x \in X\}$ and let $\rho_1\rho_2 cl(\Delta) = F$ be the
$\rho_1\rho_2$-closure of Δ in $(X \times X', \rho_1, \rho_2)$, where $\rho_i , i = 1,2$ be the product topology on $X \times X'$.
Since $f : (X, \tau_1, \tau_2) \to P$ is (1,2)*-proper , then $f \times I_{X'} : X \times X' \to P \times X'$ is (1,2)*-closed . But
$P \times X' \cong X'$ so $pr_2 : X \times X' \to X'$ is (1,2)*-closed . Hence $pr_2(F)$ is $\tau\tau$-closed in X' . Since
$(x, x) \in \Delta$ for each $x \in X$ $\Rightarrow$ $x = pr_2(x, x) \in pr_2(\Delta)$ for each $x \in X$ $\Rightarrow$ $X \subseteq pr_2(F)$ $\Rightarrow$
$\tau\tau cl(X) \subseteq \tau\tau cl(pr_2(F)) = pr_2(F)$. Since $w \in \tau\tau cl(X)$ $\Rightarrow$ $w \in pr_2(F)$ $\Rightarrow$ $\exists\, x \in X$ such that
$(x, w) \in \rho_1\rho_2 cl(\Delta) = F$ .By the definition of the bitopology of $X \times X'$, this means that for each

(1,2)*-neighborhood V of x in X and each $M \in \xi$, we have $(V \times M) \bigcap \Delta \neq \phi \Rightarrow V \bigcap M \neq \phi$.

Hence x is a (1,2)*-cluster point of the filter ξ . Thus X is a (1,2)*-compact space .

Theorem(3.20): Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a (1,2)*-continuous function . Then the following statements are equivalent:-

i) f is (1,2)*-proper .

ii) f is (1,2)*-closed and $f^{-1}(y)$ is (1,2)*-compact for each $y \in Y$.

iii) If ξ is a filter on X and if $y \in Y$ is a (1,2)*-cluster point of $f(\xi)$,then there is a (1,2)*-cluster point x of ξ such that $f(x) = y$.

Proof: $(i \to ii)$.

If f is (1,2)*-proper ,then by (3.3) f is (1,2)*-closed . To prove that $f^{-1}(y)$ is (1,2)*-compact for each $y \in Y$. Since f is (1,2)*-proper , then by (3.8) $f_{\{y\}}: f^{-1}(y) \to \{y\}$ is (1,2)*-proper for each $y \in Y$. By lemma (3.19) , we get $f^{-1}(y)$ is (1,2)*-compact for each $y \in Y$.

$(ii \to i)$.

To prove that $h = f \times I_Z : (X \times Z, \rho_1, \rho_2) \to (Y \times Z, \rho_3, \rho_4)$ is (1,2)*-closed for every bitopological space (Z, τ, τ'), where $\rho_i, i = 1,2,3,4$ be the product topology on $X \times Z$ and $Y \times Z$ respectively . Let C be any $\rho_1\rho_2$-closed in $X \times Z$. To prove that $h(C) = D$ is $\rho_3\rho_4$-closed in $Y \times Z$. Let $(y,s) \in D^c \Rightarrow h^{-1}(y,s) \in h^{-1}(D^c) \Rightarrow (f \times I_Z)^{-1}(y,s) \in h^{-1}(D^c) \Rightarrow$ $(f^{-1} \times I_Z^{-1})(y,s) \in h^{-1}(D^c) \Rightarrow f^{-1}(y) \times \{s\} \subseteq C^c$, where C^c is $\rho_1\rho_2$-open in $X \times Z$ $\Rightarrow$ $\exists$ $\tau_1\tau_2$-open set U in X and $\tau\tau'$-open set V in Z such that $f^{-1}(y) \times \{s\} \subseteq U \times V \subseteq C^c$ $\Rightarrow$ $f^{-1}(y) \subseteq U$ and $\{s\} \subseteq V$. Since f and I_Z are (1,2)*-closed , then by (2.3) $\exists$ $\sigma_1\sigma_2$-open set U' in Y and $\tau\tau'$-open set V' in Z such that $\{y\} \subseteq U'$, $\{s\} \subseteq V'$, $f^{-1}(U') \subseteq U$ and $I_Z^{-1}(V') \subseteq V$ $\Rightarrow$ $(y,s) \in U' \times V' \subseteq D^c \Rightarrow D^c$ is $\rho_3\rho_4$-open $\Rightarrow$ D is $\rho_3\rho_4$-closed in $Y \times Z$. Hence $f \times I_Z : (X \times Z, \rho_1, \rho_2) \to (Y \times Z, \rho_3, \rho_4)$ is (1,2)*-closed . Thus $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is (1,2)*-proper .

$(ii \to iii)$.

Let ξ be a filter on X and $y \in Y$ be a (1,2)*-cluster point of $f(\xi)$

$\Rightarrow y \in \bigcap\{\sigma_1\sigma_2 cl(f(F)) : F \in \xi\}$. Since f is (1,2)*-closed and (1,2)*-continuous , then by [1,8]
$\sigma_1\sigma_2 cl(f(F)) = f(\tau_1\tau_2 cl(F))$ for every $F \in \xi \Rightarrow y \in f(\tau_1\tau_2 cl(F))$ for every $F \in \xi \Rightarrow$
$f^{-1}(y) \bigcap \tau_1\tau_2 cl(F) \neq \phi$ for every $F \in \xi$. Let $\xi_0 = \{f^{-1}(y) \bigcap \tau_1\tau_2 cl(F) : F \in \xi\} \Rightarrow \xi_0$ is a filter
base on $f^{-1}(y)$ whose elements are $\tau_{1f^{-1}(y)}\tau_{2f^{-1}(y)}$-closed subsets of $(f^{-1}(y), \tau_{1f^{-1}(y)}, \tau_{2f^{-1}(y)})$.
Since $f^{-1}(y)$ is (1,2)*-compact , then by (1.14) there exist $x \in f^{-1}(y)$ such that
$x \in \bigcap\{f^{-1}(y) \bigcap \tau_1\tau_2 cl(F) : F \in \xi\} \Rightarrow \exists\ x \in f^{-1}(y)$ such that $x \in \tau_1\tau_2 cl(F)$ for every $F \in \xi$
$\Rightarrow \xi \overset{(1,2)^*}{\propto} x$ and $f(x) = y$.

$(iii \rightarrow ii)$.

Let A be a non-empty $\tau_1\tau_2$-closed subset of X and let ξ be the filter of subsets of X which
contains $A \Rightarrow A$ is the set of (1,2)*-cluster points of ξ . Let B be the set of (1,2)*-cluster
points of $f(\xi)$ on Y $\Rightarrow$ B is $\sigma_1\sigma_2$-closed set in Y and $f(A) \subseteq B$. To prove that $B \subseteq f(A)$,
let $y \in B \Rightarrow f(\xi) \overset{(1,2)^*}{\propto} y$ by (iii), $\exists\ x \in X$ such that $\xi \overset{(1,2)^*}{\propto} x$ and $f(x) = y$. But A is the set of
all (1,2)*-cluster points of ξ , then $x \in A$ and $f(x) = y \Rightarrow y \in f(A) \Rightarrow B \subseteq f(A) \Rightarrow$
$B = f(A)$
$\Rightarrow$ f is (1,2)*-closed . Now , to prove that $f^{-1}(y)$ is (1,2)*-compact for each $y \in Y$. Let
$y \in Y$, then either $f^{-1}(y) = \phi$ or $f^{-1}(y) \neq \phi$. If $f^{-1}(y) = \phi \Rightarrow f^{-1}(y)$ is (1,2)*-compact . If
$f^{-1}(y) \neq \phi$, then let ξ be a filter on $f^{-1}(y) \Rightarrow f(\xi)$ be a filter generated by f on $\{y\}$, but $\{y\}$
is (1,2)*-compact and $y \in \{y\}$, then $f(\xi) \overset{(1,2)^*}{\propto} y$ in $\{y\} \subseteq Y$. This implies that $f(\xi) \overset{(1,2)^*}{\propto} y$ in
Y . By (iii) $\exists\ x \in X$ such that $\xi \overset{(1,2)^*}{\propto} x$ and $f(x) = y \Rightarrow x \in f^{-1}(y)$ and $f^{-1}(y)$ is (1,2)*-compact for each $y \in Y$.

Corollary(3.21): A bitopological space (X, τ_1, τ_2) is (1,2)*-compact if and only if the constant
function $f : (X, \tau_1, \tau_2) \rightarrow P = \{w\}$ is (1,2)*-proper .

Proof: It is Obvious .

Theorem(3.22): If (X, τ_1, τ_2) is any (1,2)*-compact space and (Y, σ_1, σ_2) is any bitopological space , then the projection $pr_2 : (X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2) \to (Y, \sigma_1, \sigma_2)$ is a (1,2)*-proper function .

Proof: Since (X, τ_1, τ_2) is a (1,2)*-compact space, then by (3.21) $f : (X, \tau_1, \tau_2) \to P$ is (1,2)*-Proper . Since $I_Y : (Y, \sigma_1, \sigma_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*-proper , then by (3.17) $f \times I_Y : X \times Y \to P \times Y \cong Y$ is (1,2)*-proper . But $pr_2 = f \times I_Y$. Thus $pr_2 : (X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2) \to (Y, \sigma_1, \sigma_2)$ is a (1,2)*-proper function .

Definition(3.23): A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ from a bitopological space (X, τ_1, τ_2) into a bitopological space (Y, σ_1, σ_2) is called (1,2)*-compact if the inverse image of every (1,2)*-compact set in Y is a (1,2)*-compact set in X .

Theorem(3.24): Every (1,2)*-proper function is a (1,2)*-compact function .

Proof: Let $f : (X, \tau, \tau') \to (Y, \sigma, \sigma')$ be a (1,2)*-proper function and K be a (1,2)*-compact subset of Y, then by (3.8) , $f_K : (f^{-1}(K), \tau_{f^{-1}(k)}, \tau'_{f^{-1}(k)}) \to (K, \sigma_K, \sigma'_K)$ is (1,2)*-proper . Since $K \to P$ is (1,2)*-proper (by (3.21)) it follows from (3.12) , no.(i) that the composition $f^{-1}(K) \xrightarrow{f_K} K \to P$ is (1,2)*-proper . Hence by (3.21), $f^{-1}(K)$ is (1,2)*-compact set in X .

Remark(3.25): The converse of (3.24) may not be true in general .Consider the following example:

Example: Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$ and $\sigma_2 = \{\phi, Y, \{b\}\}$. The sets in $\{\phi, X, \{a\}, \{b, c\}\}$ are $\tau_1\tau_2$-closed sets in X and the sets in $\{\phi, Y, \{b, c\}, \{a, c\}, \{c\}\}$ are $\sigma_1\sigma_2$ - closed sets in Y . Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function which is defined by : $f(a) = a$, $f(b) = b$ and $f(c) = c$ $\Rightarrow$ f is a (1,2)*-compact function, but it is not (1,2)*-proper function ,since f is not (1,2)*-closed function .

Theorem(3.26): Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be two (1,2)*-continuous functions . Then:-

i) If f and g are (1,2)*-compact, then $g \circ f$ is (1,2)*-compact .

ii) If $g \circ f$ is (1,2)*-compact and f is onto, then g is (1,2)*-compact .

iii) If $g \circ f$ is (1,2)*-compact and g is one-to-one , then f is (1,2)*-compact .

Proof: The proof is similar of theorem (3.12) .

Definition(3.27): A subset F of be a bitopological space (X,τ_1,τ_2) is said to be (1,2)*-compactly closed if $F\cap K$ is (1,2)*-compact for each (1,2)*-compact set K in X .

Remark(3.28): Every $\tau_1\tau_2$-closed subset of a bitopological space (X,τ_1,τ_2) is (1,2)*-compactly closed . But the converse is not true in general . Consider the following example:-

Example: Let $X=\{a,b,c\}$, $\tau_1=\{\phi,X,\{a\}\}$ and $\tau_2=\{\phi,X\}$. The sets in $\{X,\phi,\{b,c\}\}$ are $\tau_1\tau_2$-closed .
Thus $\{a\}$ is (1,2)*-compactly closed in X , but it is not $\tau_1\tau_2$-closed .

Definition(3.29): A (1,2)*-T_2-space (X,τ_1,τ_2) is called a (1,2)*-K-space if every (1,2)*-compactly closed subset of X is $\tau_1\tau_2$-closed .

Theorem(3.30): Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a (1,2)*-continuous function such that Y is a (1,2)*-K-space . Then f is a (1,2)*-proper function if and only if f is a (1,2)*-compact function .

Proof: $\Rightarrow$ By (3.24) every (1,2)*-proper function is a (1,2)*-compact function .
Conversely , since f is a (1,2)*-compact function and {y} is a (1,2)*-compact set in Y , then by (3.23) , $f^{-1}(y)$ is (1,2)*-compact in X for each $y\in Y$. Now , to prove that f is (1,2)*-closed . Let F be any $\tau_1\tau_2$-closed set in X , to prove that $f(F)$ is a $\sigma_1\sigma_2$-closed set in Y . Suppose that K is a (1,2)*-compact set in Y , then $f^{-1}(K)$ is a (1,2)*-compact set in X . Since $F\cap f^{-1}(K)$ is a (1,2)*-compact set in X and f is (1,2)*-continuous, then by [5] $f(F\cap f^{-1}(K))$ is a (1,2)*-compact set in Y .
Since $f(F\cap f^{-1}(K))=f(F)\cap K$, then by (3.27) $f(F)$ is a (1,2)*-compactly closed set in Y . But Y is a (1,2)*-K-space , then by (3.29) $f(F)$ is a $\sigma_1\sigma_2$-closed set in Y . Therefore by (3.20) f is a (1,2)*-proper function .

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حول الدوال السديدة -*(1,2) في الفضاءات التبولوجية الثنائية

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الخلاصة

في هذا البحث قدمنا نوعا جديدا من الدوال في الفضاءات التبولوجية الثنائية أسميناها بالدوال السديدة -*(1,2) ((1,2)*-proper functions) . كذلك درسنا الخواص الأساسية والمكافئات للدوال السديدة -*(1,2) . احد أهم لتعارف المكافئة لهذه الدوال أعطي باستخدام النقاط العنقودية-*(1,2) للمرشحات ((1,2)*-cluster points of filters) .

فضلا عن ذلك عرفنا و درسنا الدوال التامة-*(1,2) ((1,2)*-perfect functions) و الدوال المتراصة-*(1,2) ((1,2)*-compact functions) في الفضاءات التبولوجية الثنائية. كذلك درسنا العلاقة بين الدوال السديدة-*(1,2) وكل من الدوال المغلقة-*(1,2) ((1,2)*-closed functions) والدوال التامة-*(1,2) والدوال المتراصة-*(1,2) على التوالي مع أعطاء مثال للاتجاه غير الصحيح.

الكلمات المفتاحية: الدوال السديدة -*(1,2)، الدوال التامة-*(1,2)، الدوال المتراصة-*(1,2)، النقاط العنقودية-*(1,2) ، فضاءات- T_2 -*(1,2) ، المجموعات المغلقة رصا-*(1,2) ، فضاءات-K -*(1,2) .