Solution of High Order Ordinary Boundary Value Problems Using Semi-Analytic Technique

Luma N. M. Tawfiq
Samaher M. Yassien
Dept. of Mathematics/College of Education for Pure Science (Ibn AL-Haitham)
University of Baghdad

Received in: 19 October 2011 , Accepted in: 16 November 2011

Abstract

The aim of this paper is to present a method for solving high order ordinary differential equations with two point's boundary condition, we propose semi-analytic technique using two-point oscillatory interpolation to construct polynomial solution. The original problem is concerned using two-point oscillatory interpolation with the fit equal numbers of derivatives at the end points of an interval [0 , 1] .
Also, many examples are presented to demonstrate the applicability, accuracy and efficiency of the method by comparing with conventional methods.

Key wards : ODE , BVP's , Oscillator Interpolation .

Introduction

In the study of nonlinear phenomena in physics, engineering and other sciences, many mathematical models lead to two-point BVP's associated with non-linear high order ordinary differential equations . In recent decades, many works have been devoted to the analysis of these problems and many different techniques have been used or developed in order to deal with two main questions: existence and uniqueness of solutions [1],[2] and computation of solutions.

In this paper, we use two-point oscillatory interpolation, essentially this is a generalization of interpolation using Taylor polynomials. The idea is to approximate a function y by a polynomial P in which values of y and any number of its derivatives at given points are fitted by the corresponding function values and derivatives of P .

We are particularly concerned with fitting function values and derivatives at the two end points of a finite interval, say [0,1] where a useful and succinct way of writing oscillatory interplant P2n+1 of degree 2n + 1 was given for example by Phillips [3] as :

$$\mathbf{P2n+1(x)} = \sum_{j=0}^{n} \{ y^{(j)}(0)\, q^{j}(x) + (-1)^{j}\, y^{(j)}(1)\, q^{j}(1-x) \} \quad , \quad (1)$$

$$q^{j}(x) = (x^{j} / j!)(1-x)^{n+1} \sum_{s=0}^{n-j} \binom{n+s}{s} x^{s} = Q^{j}(x) / j! \quad , \quad (2)$$

so that (1) with (2) satisfies :

$$y^{(j)}(0) = P_{2n+1}^{(j)}(0) \; , \quad y^{(j)}(1) = P_{2n+1}^{(j)}(1) \; , \quad j = 0, 1, 2,\dots, n .$$

implying that P2n+1 agrees with the appropriately truncated Taylor series for y about x = 0 and x = 1. We observe that (1) can be written directly in terms of the Taylor coefficients a_i and b_i about x = 0 and x = 1 respectively, as :

$$\mathbf{P2n+1(x)} = \sum_{j=0}^{n} \{ a_j\, Q^{j}(x) + (-1)^{j}\, b_j\, Q^{j}(1-x) \} \quad , \quad (3)$$

Suggested Solution of Two-Point High Order BVP's for ODE

A general form of nth - order ordinary BVP's is :-

$$y^{(n)}(x) = f(x, y, y^{(1)}, y^{(2)}, y^{(3)}, y^{(4)}, \dots , y^{(n-1)}) \; , \quad 0 \le x \le 1 \quad , \quad (4)$$

subject to the boundary conditions :

$$y^{(i)}(0)= A_i \; , \quad y^{(j)}(1)= B_j \quad , i= 0,1,\dots, k-1 \; , j= 0,1 , \dots , n- k-1 \; , \quad (5a)$$

Or

$$y^{(2i)}(0)= A_i \; , \quad y^{(2i)}(1)= B_i \; , i= 0,1,\dots,(n-2) / 2 \; , \text{if } n \text{ is even} \quad , \quad (5b)$$

The simple idea of semi - analytic method is using a two - point polynomial interpolation to replace y in problems (4)and (5) by a P2n+1 which enables any unknown derivatives of y to be computed, the first step therefore is to construct the P2n+1,to do this we need evaluate Taylor coefficients of y about x = 0 :

$$y^{(j)}(0) = P_{2n+1}^{(j)}(0) \; , \quad y^{(j)}(1) = P_{2n+1}^{(j)}(1) \; , \quad j = 0, 1, 2,\dots, n .$$

Implying that P2n+1 agrees with the appropriately truncated Taylor series for y about x = 0 and x = 1. We observe that (1) can be written directly in terms of the Taylor coefficients a_i and b_i about x = 0 and x = 1 respectively, as :

$$\mathbf{P2n+1(x)} = \sum_{j=0}^{n} \{ a_j\, Q^j(x) + (-1)^j\, b_j\, Q^j(1-x) \} \quad , \quad (3)$$

Suggested Solution of Two-Point High OrderBVP's for ODE

A general form of nth - order ordinary BVP's is :-

$$\mathbf{y(n)(x) = f(x, y, y(1), y(2), y(3), y(4), \dots , y(n-1)) , \quad 0 \le x \le 1 \quad , \quad (4)}$$

subject to the boundary conditions :

$$\mathbf{y(i)(0)= Ai \ , \ y(j)(1)= Bj \ , i= 0,1,\dots, k-1 , j= 0,1 , \dots , n- k-1 , \quad (5a)}$$

Or

$$\mathbf{y(2i)(0)= Ai \ , \ y(2i)(1)= Bi \ , i= 0,1,\dots,(n-2) / 2 , \text{ if n is even} \quad , \quad (5b)}$$

The simple idea of semi - analytic method is using a two - point polynomial interpolation to replace y in problems (4)and (5) by a P2n+1 which enables any unknown derivatives of y to be computed, the first step therefore is to construct the P2n+1,to do this we need evaluate Taylor coefficients of y about x = 0 :

$$\mathbf{y} = \sum_{i=0}^{\infty} a_i \, \mathbf{xi} \qquad \ni a_i = \mathbf{y(i)(0) / i!} \quad , \quad (6a)$$

Then insert the series form (6a) into (4) and equate the coefficients of powers of x to obtain a_n . Also, evaluate Taylor coefficients of y about x =1 : y $= \sum_{i=0}^{\infty} b_i (x - 1)$i

$\ni b_i =$ y(i) (1) / i! , (6b)

Then insert the series form (6b) into (4) and equate coefficients of powers of (x-1) , to obtain b_n ,then derive equation (4) with respect to x and iterate the above process to obtain a_{n+1} and b_{n+1} ,now iterate the above process many times to obtain a_{n+2}, b_{n+2},then a_{n+3}, b_{n+3} and so on, that is ,we can get a_i and b_i , for all i ≥ n .

Now, to evaluate a_i , b_i, for i < n, we get half number of these unknown coefficients from given boundary condition ,then use all these s̊ and s̊ to construct P2n+1 of the form :

$$\mathbf{P2n+1\ (x)} = \sum_{i=0}^{n} \{ a_i\, Q_i(x) + (-1)_{\mathbf{i}} \quad , \quad \mathbf{(7a)}$$

Where

$$Q^j(x) / j! = (x^j / j!)(1-x)^{n+1} \sum_{s=0}^{n-j} \binom{n+s}{s} xs \quad , \quad (7b)$$

we see that (7a) have n unknown coefficients .

Now, to evaluate the remainder coefficients integrate equation (4) on [0 , x] n - times to obtain :

$$\mathbf{y(n-1)(x)–(n-1)}\,!a_{n-1} = \int_0^x \mathbf{f(s,y ,y', y '',\dots,y(n-1)) \, ds} \quad ,(81\,)$$

$$\mathbf{y(n-2)(x)} - (n-2)!\, a_{n-2} - (n-1)!\, a_{n-1} x = \int_0^x \mathbf{(1-s)f(s,y,y',y'',\dots,y(n-1))ds\ , (82\,)}$$

y(x)- a_0-$a_1 x$-…-(n-2) !$a_{n-2} x$n-2/(n-2) !-(n-1) ! $a_{n-1} x$n-1/(n-1) !=$\int_0^x$ f(s,y,y',y'',...,y(n-1)ds,(8n) wheres̊ defined in (6a).

useP2n+1as a replacement of y,y',y",…,y(n-1) in (8) and putting $x = 1$ in all above integration,then wehave system of n equations with n unknown coefficients which can be solved using the MATLABpackage, version 7.9, to get the unknown coefficients, thus insert it into(7),thus (7) represent the solution of (4) .

Now, we introducemanyexamples of higher order TPBVP's for ODE to illustrate suggested method , a semi - analytic method will be tested by discussing three non - linear BVP's of 9th–order, 10th – order, and 12th–order respectively. Accuracy and efficiency of the suggested method is established through comparison with homotopy perturbation method (HPM)[4] .

Example 1

Consider the following linear ninth-order BVP's :

$$y^{(9)}(x) = -9e^{x} + y(x) \quad , \quad 0 < x < 1 ,$$

subject to the BC :

$$y^{(i)}(0) = (1-i) , i= 0,1, ... , 4 ,$$
$$y^{(i)}(1) = -ie \quad , \quad i= 0,1, 2, 3.$$

The exact solution for this problem is:$y(x) = (1-x)e^{x}$.

Now, we solve this equation using semi-analytic method from equations (2) and (3) we have :

$$P_{27} = 0.002485613x^{27} - 0.033505860x^{26} + 0.208738965x^{25} - 0.795864547x^{24} + 2.072175246x^{23} - 3.89122197x^{22} + 5.422696100x^{21} - 5.679126548x^{20} + 4.470612805x^{19} - 2.613188832x^{18} + 1.102724758x^{17} - 0.318212417x^{16} + 0.056301206x^{15} - 0.004614519x^{14} - 0.000000002x^{13} - 0.000000023x^{12} - 0.000000251x^{11} - 0.00000248x^{10} - 0.000022046x^{9} - 0.000173611x^{8} - 0.001190476x^{7} - 0.006944444x^{6} - 0.033333333x^{5} - 0.125x^{4} - 0.333333333x^{3} - 0.5x^{2} + 1.$$

For more details ,table (1) gives the results for different nodes in the domain, for n = 13, i.e. P27 anderrors obtained by comparing it with the exact solution. Higher accuracy can be obtained by evaluating higher n. Table (2) gives a comparison between the P27 and Homotopy perturbation method (HPM) given in[4] to illustrate the accuracy of suggested method. Also, figure (1) gives the accuracy of the suggested method .

We close our analysis by discussing a 12th-order BVP's.

Example 2

Consider the following nonlinear tenth-order BVP's :

$$y^{(10)}(x) = e^{-x} y^{2}(x) , \quad 0 < x < 1 ,$$

subject to the BC: $y^{(2i)}(0) = 1, i= 0,1,2,3,4$ and $y^{(2i)}(1) = e, i= 0, 1,2,3,4$

The exact solution for this problem is $y(x) = e^{x}$.

Now, we solve this equation using semi-analytic method from equations (2) and (3) we have :

$$P_{29} = -0.011117887x^{29} + 0.161312269x^{28} - 1.088112264x^{27} + 4.523212156x^{26} - 12.946818468x^{25} + 26.996204463x^{24} - 42.295420191x^{23} + 50.587892556x^{22} - 46.425913997x^{21} + 32.539784329x^{20} - 17.150630564x^{19} + 6.593586235x^{18} - 1.748504966x^{17} + 0.286398369x^{16} - 0.021872038x^{15} + 1\times10^{-11} x^{14} + 2\times10^{-11} x^{13} + 2\times10^{-10} x^{12} + 0.000000025x^{11} + 0.000000276x^{10} + 0.000002756x^{9} + 0.000024802 x^{8} + 0.000198413x^{7} + 0.001388889x^{6} + 0.008333333x^{5} + 0.041666667x^{4} + 0.166666667x^{3} + 0.5x^{2} + x + 1$$

For more details ,table(3) gives the results for different nodes in the domain, for n = 14, i.e. P29 anderrors obtained by comparing it with the exact solution. Higher accuracy can be obtained by evaluating higher n. Table (4) give a comparison between the P29 and Homotopy perturbation method (HPM) given in[4] to illustrate the accuracy of suggested method. Also, figure (2) gives the accuracy of the suggested method .

We close our analysis by discussing a 12th-order BVP's.

Example 3

Consider the nonlinear 12th-order nonlinear BVP's.

$$y^{(12)}(x) = 2 e^x y^2(x) + y^3(x) \quad , \quad 0 < x < 1 ,$$

subject to the BC: $y^{(2i)}(0) = 1$, $y^{(2i)}(1) = e^{-1}$, i= 0,1,2,3,4,5,

with exact solution is : $y = e^{-x}$.

Now, we solve this equation using semi-analytic method from equations (2) and (3) we have :

$$\begin{aligned}P_{29} = &-0.077122663x^{29} + 1.117564781x^{28} - 7.528039942x^{27} + 31.247141987x^{26} - 89.295134116x^{25} + 185.869480290x^{24} - 290.649995930x^{23} + 346.909553666x^{22} - 317.637729257x^{21} + 222.066553374x^{20} - 116.713964807x^{19} + 44.7295364639x^{18} - 11.81947533530183x^{17} + 1.928212142x^{16} - 0.146580649x^{15} + 0.000000000012x^{14} - 0.0000000002x^{13} + 0.000000002x^{12} - 0.000000025x^{11} + 0.000000276x^{10} - 0.000002756x^{9} + 0.000024802x^{8} - 0.000198413x^{7} + 0.001388889x^{6} - 0.008333333x^{5} + 0.041666667x^{4} - 0.166666667x^{3} + 0.5x^{2} - 1.0x + 1.0\end{aligned}$$

For more details ,table(5) gives the results for different nodes in the domain, for n = 14, i.e. P29 anderrors obtained by comparing it with the exact solution. Higher accuracy can be obtained by evaluating higher n. Table (6) gives a comparison between the P29 and homotopy perturbation method (HPM) given in[4] to illustrate the accuracy of suggested method. Also, fig (3) gives the accuracy of the suggested method .

Conditioning of BVP's

In particular ,BVP's for which a small change to the ODE or boundary conditions results in a small change to the solution must be considered, a BVP's that has this property is said to be well-conditioned.[5] Otherwise, the BVP's is said to be ill-conditioned. To be useful in applications, a BVP's should be well posed. This means that given the input to the problem there exists a unique solution, which depends continuously on the input. Consider the following nth-order BVP's

$$y^{(n)}(x) = f(x, y(x), y'(x), \ldots , y^{(n-1)}(x)) \ , x\in[0, 1] \quad , \qquad (9a)$$

With

$$BC: y^{(i)}(0) = A_i , y^{(j)}(1) = B_j , \ i= 0,1,\ldots,k-1 , \ j= 0,1,\ldots, n-k-1, \qquad (9b)$$

For a well-posed problem we now make the following assumptions:

1. Equation (9) has an approximate solution $P \in C^n[0, 1]$, with this solution and $\rho > 0$, we associate the spheres :

$$S_\rho(P(x)) := \{ y \in IR^n: | P(x) - y(x) | \le \rho \}$$

2. $f(x, P(x), P'(x), \ldots , P^{(n-1)}(x))$ is continuously differentiable with respect to P, and $\partial f / \partial P$is continuous .

This property is important due to the error associated with approximate solutions to BVP's, depending on the semi-analytic technique, approximate solution $\check{y}$ to the linear nth-order BVP's (9) may exactly satisfy the perturbed ODE :

$$\check{y}^{(n)} = u(x) \check{y}^{(n-1)} +\ldots+ d(x)\check{y}'+ q(x) \check{y} + r(x) \ ; \ 0 < x < 1 \quad ; \qquad (10a)$$

where r : R → Rm , and the linear BC :

$$B0\check{y}(0) + B1\, \check{y}(1) = \beta + \quad ; \qquad (10b)$$

where β + = σ , σ ∈Rm and {,β, σ} are constants. If y̌ is a reasonably good approximate solution to (9), then ‖r(x)‖ and ‖σ‖ are small. However, this may not imply that y̌ is close to the exact solution y. A measure of conditioning for linear BVP's that relates both ‖r(x)‖ and ‖σ‖ to the error in the approximate solution can be determined. The following discussion can be extended to nonlinear BVP's by considering the variational problem on small sub domains of the nonlinear BVP's [6].

Letting : e(x) = |y̌ (x) - y(x)| ;then subtracting the original BVP's (9) from the perturbed BVP's (10) results in :

$$\mathbf{e(n)(x) = \check{y}(n)(x)\text{-}0\ \ y(n)(x)} \qquad . \qquad \mathbf{(11a)}$$

$$\mathbf{e(n)(x)= u(x)\, e(n\text{-}1)(x) + \ldots + d(x)\, e'(x) + q(x)\, e(x) + r(x);\ \ 0 < x < 1\ ;} \qquad \mathbf{(11b)}$$

with BC : B0 e(0) + B1 e(1) = σ ; (11c) However, the form of the solution can be further simplified by letting : Θ(x) = Y(x) Q-1 ; where Y is the fundamental solution and Q is defined in (7b) . Then the general solution can be written as :

$$\mathbf{e(x) = \Theta(x)\, \sigma + \int_0^1 G(x, t)\, r(t)\, dt} \qquad . \qquad \mathbf{(12)}$$

where G(x, t) is Green's function [7], taking norms of both sides of (12) and using the Cauchy - Schwartz inequality [7] results in :

$$\mathbf{\| e(x) \|_\infty \le k1 \| \ \sigma\ \|_\infty + k2 \| r(x) \|_\infty} \qquad . \qquad \mathbf{(13)}$$

$$\textbf{where}\ \ \mathbf{k1 = \| Y(x)Q\text{-}1 \|_\infty} \quad ; \quad \textbf{and}\ \ \mathbf{k2} = \sup_{0\le x\le 1} \int_0^1 \mathbf{\| G(x, t) \|_\infty dt} \ ,$$

In (13), the L∞ norm, sometimes called a maximum norm, is used due to the common use of this norm in numerical BVP's software. For any vector v ∈ RN, the L∞ norm is defined as :

$\| \ v \ \|_\infty = \max_{1\le i\le N} |\ vi\ |$:The measure of conditioning is called the conditioning constant k, and it is given by

$$\mathbf{k = max(k1, k2);} \qquad \mathbf{(14)}$$

When the conditioning constant is of moderate size,then the BVP's is said to be well-conditioned.

Referring again to (13), the constant k thus provides an upper bound for the norm of the error associated with the perturbed solution,

$$\mathbf{\| e(x) \|_\infty \le k\ [\, \| \sigma \|_\infty + \| r(x) \|_\infty]} \qquad . \qquad \mathbf{(15)}$$

It is important to note that the conditioning constant only depends on the original BVP's and not the perturbed BVP's. As a result, the conditioning constant provides a good measure of conditioning that is independent of any numerical technique that may cause such perturbations. The well-conditioned nature of a BVP's and the local uniqueness of its desired solution are assumed in order to solve numerically the problem .

References

1. Agarwal R. P., (1986) , " BVP's for High ODE, World Scientific Singapore .

2. Wang Y., Zhao Y.B.,and Wei G.W., (2003) ," A note on the numerical solution of high-order differential Equations" , Journal of Computational and Applied Mathematics 159: 387–398 .

3. Phillips G .M. , (1973) , " Explicit forms for certain Hermite approximations ", BIT 13, 177-180.

4. Nadjafi J. S. and Zahmatkesh SH. , (2010)," HPM For Solving Higher Order BVP's ", Applied Mathematical and Computational Sciences,1, (2):199-224.

5. Howell K.B., (2009) , " ODE" , Spring, USA .

6. Shampine L .F., (2002) , " Singular BVP's for ODEs " , Southern Methodist University J., 1:1-23.

7.Boisvert J. J. , January(2011) , A Problem-Solving Environment for the Numerical Solution of BVP's ,MSC Thesis, in the Department of Computer Science, University of Saskatchewan ,Saskatoon , Canada

Table(1): The result of the method for P27 of example 1

x	Exact Solution y(x)	Osculatory Interpolation P27	Errors |y(x) - P27|
0	1.000000000000000	1.000000000000000	0.000000000000000
0.1	0.994653826268083	0.994653826268080	2.55351E-15
0.2	0.977122206528136	0.977122206528065	7.10543E-14
0.3	0.944901165303202	0.944901165299981	3.22098E-12
0.4	0.895094818584762	0.895094818553623	3.11386E-11
0.5	0.824360635350064	0.824360635249051	1.01013E-10
0.6	0.728847520156204	0.728847520003405	1.52799E-10
0.7	0.604125812241143	0.604125812102035	1.39108E-10
0.8	0.445108185698494	0.445108185603449	9.50442E-11
0.9	0.245960311115695	0.245960311068211	4.74832E-11
1	0.000000000000000	0.000000000000000	0.000000000000000
S.S.E =6.533167667712323E-020			

Table(2):a comparison between P27 and HPM method for Example 1.

x	Exact Solution y(x)	HPM y1(x)	Errors |y(x) -y1(x)|	Errors |y(x) - P27|
0	1.000000000000000	1.0000000000	0.000000	0.000000000000000
0.1	0.994653826268083	0.9946538264	3.6E-9	2.55351E-15
0.2	0.977122206528136	0.9771222066	3.4E-9	7.10543E-14
0.3	0.944901165303202	0.9449011654	4.6E-9	3.22098E-12
0.4	0.895094818584762	0.8950948186	1.4E-9	3.11386E-11
0.5	0.824360635350064	0.8243606355	4.5E-9	1.01013E-10
0.6	0.728847520156204	0.7288475206	-6.E-6	1.52799E-10
0.7	0.604125812241143	0.6041258131	-3.1E-9	1.39108E-10
0.8	0.445108185698494	0.4451081876	-2.4E-9	9.50442E-11
0.9	0.245960311115695	0.2459603145	-4.5E-9	4.74832E-11
1	0.000000000000000	0.0000000000	0.000000	0.000000000000000

Table (3): The result of the method for P29 of example 2

x	Exact solution y(x)	Osculatory interpolation P29	Errors |y(x) –P29|
0	1.000000000000000	1.000000000000000	0.000000000000000
0.1	1.105170918075648	1.105170918075648	0.000000000000000
0.2	1.221402758160170	1.221402758160129	4.04121E-14
0.3	1.349858807576003	1.349858807572658	3.34510E-12
0.4	1.491824697641270	1.491824697601597	3.96738E-11
0.5	1.648721270700128	1.648721270550529	1.49599E-10
0.6	1.822118800390509	1.822118800118783	2.71776E-10
0.7	2.013752707470477	2.013752707133349	3.37128E-10
0.8	2.225540928492468	2.225540928114606	3.77862E-10
0.9	2.459603111156950	2.459603110725136	4.31814E-10
1	2.718281828459046	2.718281828459046	0.000000000000000
S.S.E = 8.396449182677434E-019			

Table (4):a comparison between P29 and HPM method for Example 2

x	Exact solution y(x)	HPM y1(x)	Errors |y(x) - y1(x)|	Errors |y(x) –P29|
0	1.000000000000000	1.000000000	0.000000000	0.000000000000000
0.1	1.105170918075648	1.10517233	-1.41E-6	0.000000000000000
0.2	1.221402758160170	1.221405446	-2.69E-6	4.04121E-14
0.3	1.349858807576003	1.349862509	-3.70E-6	3.34510E-12
0.4	1.491824697641270	1.49182905	-4.35E-6	3.96738E-11
0.5	1.648721270700128	1.648725849	-4.58E-6	1.49599E-10
0.6	1.822118800390509	1.822123158	-4.36E-6	2.71776E-10
0.7	2.013752707470477	2.013756415	-3.71E-6	3.37128E-10
0.8	2.225540928492468	2.225543623	-2.69E-6	3.77862E-10
0.9	2.459603111156950	2.459604528	-1.42E-6	4.31814E-10
1	2.718281828459046	2.7182830	2.00E-9	0.000000000000000

Table (5): The result of the method for P29 of example 3

x	Exact solution y(x)	Oscillatory interpolationP29	Errors |y(x) –P29 |
0	1.000000000000000	1.000000000000000	0.000000000000000
0.1	0.904837418035960	0.904837418035958	1.22125E-15
0.2	0.818730753077982	0.818730753077706	2.75446E-13
0.3	0.740818220681718	0.740818220660028	2.16899E-11
0.4	0.670320046035639	0.670320045785891	2.49749E-10
0.5	0.606530659712633	0.606530658818836	8.93798E-10
0.6	0.548811636094027	0.548811634621416	1.47261E-9
0.7	0.496585303791409	0.496585302234367	1.55704E-9
0.8	0.449328964117222	0.449328962716291	1.40093E-9
0.9	0.406569659740599	0.406569658858897	8.81702E-10
1	0.367879441171442	0.367879441171442	0.000000000000000
S.S.E =1.738398767933298E-017			

Table (6):A comparison between P29 and HPM method for Example3

X	Exact solution y(x)	HPM y1(x)	Errors |y(x) - y1(x)|	Errors |y(x) –P29 |
0	1.000000000000000	1.000000000	0.00000	0.000000000000000
0.1	0.904837418035960	0.904837579	-1.61E-7	1.22125E-15
0.2	0.818730753077982	0.818731060	-3.07E-7	2.75446E-13
0.3	0.740818220681718	0.740818643	-4.22E-7	2.16899E-11
0.4	0.670320046035639	0.670320543	-4.97E-7	2.49749E-10
0.5	0.606530659712633	0.606531182	-5.22E-7	8.93798E-10
0.6	0.548811636094027	0.548812133	-4.97E-7	1.47261E-9
0.7	0.496585303791409	0.496585726	-4.22E-7	1.55704E-9
0.8	0.449328964117222	0.44932971	-3.07E-7	1.40093E-9
0.9	0.406569659740599	0.406569821	-1.61E-7	8.81702E-10
1	0.367879441171442	0.367879441	2.00E-10	0.000000000000000

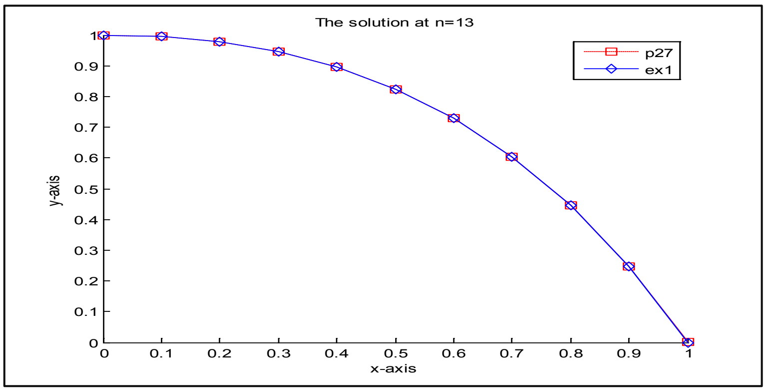

Fig.(1): Comparison between the exact and semi-analytic solution P27 of example1

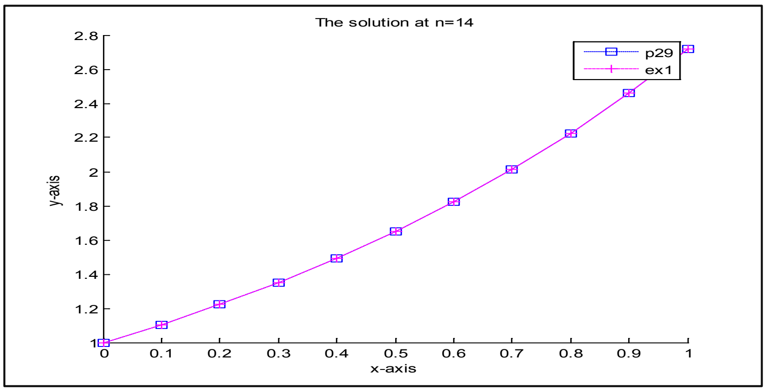

Fig.(2): Comparison between the exact and semi-analytic solution P29 of example 2

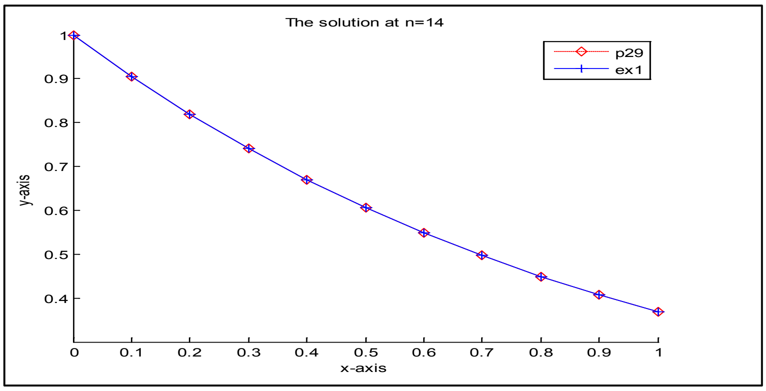

Fig.(3):Comparison between the exact and semi-analytic solution P29 of example3

حل مسائل القيم الحدودية الاعتيادية ذات الرتب العالية باستخدام التقنية شبه التحليلية

لمى ناجي محمد توفيق
سماهر مرز ياسين
قسم علوم الرياضيات / كلية التربية للعلوم الصرفة (ابن الهيثم) / جامعة بغداد

استلم البحث في: 19 تشرين الاول 2011 ، قبل البحث في: 16 تشرين الثاني 2011

الخلاصة

الهدف من هذا البحث عرض طريقة لحل معادلات تفاضلية اعتيادية من الرتبة العالية ذي الشروط الحدودية عند نقطتين اذ أننا نقترح التقنية شبه التحليلية باستعمال الاندراج التماسي ذي النقطتين للحصول على الحل بوصفها متعدده حدود،أن أصل المسالة يتعلق باستعمال الاندراج التماسي ذي النقطتين الذي يتفق مع الدالة ومشتقاتها عند نقطتي نهاية المدة [0,1]و ناقشنا بعض الأمثلة ايضاً لتوضيح الدقة، و الكفاية وسهولة الأداء للطريقة المقترحة من خلال المقارنة مع الطرائق التقليدية الأخرى.

الكلمات المفتاحية : معادلات تفاضلية اعتيادية ،مسائل القيم الحدودية ،الاندراج التماسي