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Discuss of Error Analysis of Gauss-Jordan Elimination For Linear Algebraic Systems

S.Ab. M. Abbas
Department Of Mathematics - College Of Science University Of Baghdad

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Abstract

The paper establishes explicit representations of the errors and residuals of approximate solutions of triangular linear systems by Jordan elimination and of general linear algebraic systems by Gauss-Jordan elimination as functions of the data perturbations and the rounding errors in arithmetic floating-point operations. From these representations strict optimal componentwise error and residual bounds are derived. Further, stability estimates for the solutions are discussed. The error bounds for the solutions of triangular linear systems are compared to the optimal error bounds for the solutions by back substitution and by Gaussian elimination with back substitution, respectively. The results confirm in a very detailed form that the errors of the solutions by Jordan elimination and by Gauss-Jordan elimination cannot be essentially greater than the possible maximal errors of the solutions by back substitution and by Gaussian elimination, respectively. Finally, the theoretical results are illustrated by two numerical examples.

Key words: Jordan elimination, data perturbations, error bounds, Gaussian elimination.

Introduction

The Gauss-Jordan algorithm is ideally suited for vector computers [1]. This justifies the study of the numerical stability of the algorithm under data perturbations and rounding errors of floating-point arithmetic. It uses the same direct method of forward analysis as our rounding error analysis of Gaussian elimination in [2]. Both the solution of general linear systems by the Gauss-Jordan algorithm and of upper triangular linear system by Jordan elimination are analyzed[3]. The main results of the paper are optimal componentwise error and residual estimates, bounds for the stability of solutions and residuals, and upper bound for the errors of the solutions of Jordan elimination and Gauss-Jordan elimination in terms of the optimal error bounds for back substitution and Gaussian elimination respectively. The results will prove that the error of the Gauss-Jordan solution cannot be much greater than the possible maximal error of the solution by Gaussian elimination with back substitution. However, the residual bounds of the Gauss-Jordan solution can be big if the solution vector has components with big relative errors.

The first step of the error analysis consists in the derivation of explicit analytical representations of the errors and residuals of approximate solutions of linear algebraic systems as functions of the data errors and the rounding errors of the arithmetic floating-operations. Under standard assumptions on the data errors of the problem and the rounding errors of the floating-point arithmetic these error and residual representations readily yield strict componentwise and, save for terms of higher order in the accuracy constant η. optimal error and residual estimates for the solutions of upper triangular linear systems in the following theorem {The residual of the computed approximate solution vector $\bar{x}$ of the

triangular linear system is bounded componentwise, optimally with respect to the error distributions by $|U\bar{x} - Z| \le \eta T = \eta_D T^D + \eta_R T^R$, [2]

Where U upper triangular linear system, η accuracy constant, $0 < \eta_R < 4/(3n)$, η_D data accuracy, η_R rounding accuracy in application $\eta_R << \eta_D$, matrix Z defined in the Jordan algorithm see [1] for the solution of regular triangular linear systems;

$u_{11}x_1 + u_{12}x_2 + \ldots + u_{1n}x_n = z_1$

$U_x = Z: u_{22}x_2 + \ldots + u_{2n}x_n = z_2$

$u_{nn}x_n = z_n$

A general linear systems in the following theorem {the residuals of the approximate solutions $\bar{x}$ of general linear systems by Gauss-Jordan elimination satisfy the componentwise optimal estimates $|A\bar{x} - y| \le m$, [4]. Where Y is column vector,η accuracy constant

A basic tool for the formulation of the error and residual bounds are the associated data, rounding, and total condition numbers $\sigma_i^D, \sigma_i^R, \sigma_i$ of the components of the computed solutions $\bar{x}_i$ and T_j^D, T_j^R, T_j of the components of the associated residuals $(A\bar{x} - y)j$. In addition, using these condition numbers, the stability constants $w_i = \sigma_i^R / \sigma_i^D$ of the solutions and $\psi_j = T_j^R / T_j^D$ of the residuals are formed which measure the ratio of the contribution to the total error bound due to the rounding errors in floating-point operations on the one hand and the data perturbations on the other hand. The size of the possible residuals can be assessed by means of the residual stability constants ψ_j, for Jordan solutions $\bar{x}_i \neq 0, i = 1, 2, \ldots, n$, of upper triangular linear systems the upper bound $\psi_j = \frac{T_j^R}{T_j^D} \le \frac{\rho_m^R}{1 - \rho_m^R \eta_R}$, j= 1, 2, …, n,whewe ρ_m^R is the maximal relative rounding condition number of the solution vector, η^R rounding aceurauy [4].

If $\rho_m^R \eta_R < 1$. An analagus estimate holds for general linear system.

The magnitudes of the possible maximal errors are measured componentwise, using the total condition numbers $\sigma_{i,j}$ and $\sigma_{i,G}$, i= 1, 2, …, n, by $\sigma_i \eta = \sigma_i^D \eta_D + \sigma_i^R \eta_R, \sigma_i^R = \sigma_i^0 + \sigma_i^1$

The term $\sigma_i^D \eta_D$ is the bound for the contribution of the data perturbations to the total error of the solution, the term $\sigma_i^0 \eta_R, \sigma_i^1 \eta_R$ bound the contributions by eliminations in the lower and in the upper triangle of the coefficient matrix, respectively establishes the estimate.

$\sigma_{i,j}^D \eta_D + \sigma_{i,j}^0 \eta_R \le q\left(\sigma_{i,G}^D \eta_D + \sigma_{j,G}^0 \eta_R\right)$

$+ q^2 p_m, G \eta \sigma_{i,j}^1 \eta_R$, [3]

For solutions $\bar{x}_{i,j} \neq 0, \bar{x}_{i,G} \neq 0$ i = 1, 2, …, n, For sufficient small accuracy constant $\eta = \max(\eta_D, \eta_R)$ the constant q is close to 1.

1. The Gauss-Jordan Method

The following error analysis deals with the Gauss- Jordan algorithm for solving linear systems

$$Ax = y : \sum_{k=1}^{n} a_{ik} x_k = y, i = 1,2,...n$$

…(1)

We shall assume that A=(a_{iK})is nonsingular and that rows and columns of A have been ordered such that A possesses a triangular factorization. In [5] we have shown how error and residual estimates are used when pivoting is taken into account.

Let $A_1 = (A, y) = (a_{iK}^{1})$ be the n×n+1 coefficient matrix of the linear system (1). The Gauss-Jordan algorithm successively eliminates by means of the pivotal equation t the unknown x_t where the vector $x= (x_t, x_{t+1}, \ldots, x_n)$ of (1) is obtained simply by $x_i = w_i / u_{ii},\ i = 1,...,n$ from all other equations i= 1, 2, …., n, i≠t, for t=1,…, n. The coefficients $A_t = (a_{ik})^t$ of the reduced linear systems thus obtained are specified by the equations

$$a_{ik}^{t+1} = a_{ik}^{t} - m_{it} u_{tk}, i = 1,...,n, k = 1,...,n+1$$

…(2)

Using the coefficients u_{tk} of the pivot equation t and the row multipliers m_{it}, defined by

$$u_{tk} = a_{tk}^{t}, m_{it} = a_{it}^{t} / u_{tt}, i = 1,2,...,n, i \neq t;$$
$$k = 1,...n+1; u_{tt} \neq 0; m_{tt} = 1;$$

…..(3)

For t=1, …, n. In this way,

$$a_{ik}^{t+1} = 0, i \neq k, i = 1,...,n, k = 1,...,t, t = 1,...,n$$

…..(4)

That is, in the first t columns of the matrix A_{t+1} all off- diagonal entries are zero. Hence the coefficient matrix A_{n+1} has the form $A_{n+1}= (D, w)$ with an n-by-n diagonal matrix D and the vector *w* where

$$D = diag(u_{11},...,u_{nn}); W = (w_i),$$
$$w_i = a_{i,n+1}^{n+1}, i = 1,...,n$$

…..(5)

From the associated reduced linear system

$$Dx = w : u_{ii} x_i = w_i, i = 1,...,n,$$

…..(6)

The solution vector $x=(x_1,\ldots, x_n)$ of (1) is obtained simply by $x_i = w_i / u_{ii}$, i=1,…,n, [6]

The residual of the computed approximate solution vector $\bar{x}$ of the triangular linear system is bounded componentwise, optimally with respect to the error distribution by

$$|U\bar{X} - Z| \leq \eta^T = \eta_D T^D + \eta_R T^R$$

If $\bar{x} \neq 0$ for i = 1,. . . ,n and $\rho_m \eta < 1$, the absolute and relative error of the approximate solution $\bar{X}$ satisfy componentwise, in first order optimal error estimates holds.

$$|\bar{x}_i - x_i| \leq \frac{\sigma_i \eta}{1-\rho_m \eta},$$

$$\left|\frac{\bar{x}_i - x_i}{\bar{x}_i}\right| \leq \frac{\rho_i \eta}{1-\rho_m \eta}, \text{i} = 1,\ldots,\text{n}, \quad [5]$$

…..(7)

Lemma: The vector of data condition numbers is bounded from below by:

$$\sigma^D \geq 2|\bar{x}| - \sigma^R \eta_R ;[2]$$

The rounding condition numbers can be written in the form $\sigma_n^R \geq |\bar{x}_n|$.

Jordan elimination in comparison with back substitution

1. The behaviour of the errors of approximated solutions of triangular linear systems by Jordan elimination (J) will now be compared with that of solutions by Gaussian with back substitution (G) in linear algebra, Jordan elimination brings a matrix to reduced row echelon form, whereas gaussian elimination takes it only as for as row echelon form. Every matrix has a reduced row echelon form, and this algorithm is guaranteed to produced it see [6]. It will be shown that the vectors of data and rounding condition number σ_j^D, σ_j^R of the Jordan solutions can be bounded from above by the corresponding condition numbers σ_G^D, σ_G^R of the solutions by back substitution. This result means that the errors of the computed Jordan solutions $\bar{x}_j$ cannot be essentially bigger than the possible maximal errors of the corresponding computed solutions $\bar{x}_G$ by back substitution.
2. when triangular linear systems are solved by back substitution the associated solution and residual stability constants $w_{i,G}$, $\psi_{j,G}$ of the solutions $\bar{x}_{i,G}$ are bounded below and above by

$$\left.\begin{aligned} \frac{1}{2+\mu_R} \leq w_{i,G} \leq \frac{n+2-i}{2-(n-i)\eta_R}, & \quad \text{for } i \neq n \\ \frac{1}{2+\mu_R} \leq \omega_{j,G} \leq \frac{n+2-j}{2-(n-j)\eta_R}, & \quad \text{for } j \neq n \end{aligned}\right\} \text{for i,j=1,…,n}$$

…..(8)

These estimates are obtained from the error and residual bounds in [2]

3. Solving a triangular linear system both by Jordan elimination and by back substitution gives two approximations $\bar{x}_j$, $\bar{x}_G$ for the searched solution vector x see [7].

These two approximations satisfy the error estimates of residual of the computed approximate solution vector $\bar{X}$ of the triangular linear system is bounded componentwise, optimally with respect to the error distributions by $|u\bar{x} - z| \leq \eta^T = \eta_D T^D + \eta^R T^R$ in first order optimal estimates, that is,

$$\left|\frac{\bar{x}_{i,j} - x_i}{\bar{x}_{i,j}}\right| \leq \frac{\rho_i, j\eta}{1-\rho_m, j\eta},$$

$$\left|\frac{\bar{x}_{i,G} - x_i}{\bar{x}_{i,G}}\right| \leq \frac{\rho_i, G\eta}{1-\rho_m, G\eta},$$

….(9)

for i =1, …,n, where

1. $\rho_m G\eta \neq 1$
2. $\rho_m J\eta \neq 1$
3. ρ_i, is defined $\rho_i = \frac{\sigma_i}{|\bar{x}_i|}$, i = 1, . . ., n.
4. $\bar{x}_{i,j} \neq 0$
5. $\bar{x}_{i,G} \neq 0$

For comparing the error behavior of the two algorithms .We need the following result

Lemma[8]: Let the two approximate solutions $\bar{x}_j$, $\bar{x}_G$ have non-vanishing components and let the maximal relative total condition numbers of these solution be bounded by $(\rho_{m,j}+\rho_{m,G})\eta<\frac{1}{2}$

Then the componentwise estimates $|\bar{x}_j| \le q|\bar{x}_G|, |\bar{x}_G| \le q|\bar{x}_j|$

are valid using the constant

$$q = 1/\left(1-\frac{\rho_m j\eta}{1-\rho_m j\eta}-\frac{\rho_m G\eta}{1-\rho_m G\eta}\right) > 1,$$

$$\text{and } 1-\rho_m j\eta \neq 0, 1-\rho_m G\eta \neq 0$$

Lemma: Let the two approximate solutions $\bar{x}_j, \bar{x}_G$ have nonvanishing components and let the maximal relative total condition numbers of these solutions be bounded by $(\rho_{m,j} + \rho_{MG})$ $\eta<\frac{1}{2}$ then the component wise estimates $|\bar{X}_j| \le q|\bar{X}_G|, |\bar{X}_G| \le q|\bar{x}_j|$ are valid

Theorem: (under the assumption of above Lemma the estimates

$$\sigma_{i,j}^D \le \frac{1}{1-q^2\omega_{i,j}\rho_{m,G}^D\eta_R} q\sigma_i^D, G$$

provided denominador $\neq 0$

$$\sigma_{i,j}^R \le \frac{3}{2}.$$

$$\frac{n+1-i}{1-\left\{\frac{3}{4}\left(n+\frac{5}{3}-i\right)+\frac{3}{2}(n+1-i)q^2\rho_{m,G}^R\right\}\eta_R}.q\sigma_{i,G}^R \text{ Hold for i} = 1, \ldots, \text{n}$$

when. η_R is so small that the denominators are positive,[8]

Solution of general linear system by Gauss- Jordan elimination

The solution of general linear systems $Ax = y$ by Gauss- Jordan elimination will be analyzed. In the context of rounding error analysis the algorithm is considered as being

composed of Gaussian forward elimination and a subsequent solution of the upper triangular linear system $Ux = z$ by Jordan elimination see [3].

The computed trapezoidal factors $\overline{L}, \overline{U}_1$ of A_1= (A, y) or, under data perturbations, $\overline{A}_1 = (\overline{A}, \overline{y})$ satisfy, according to [2, 1. (13)], the relation $\overline{L}\overline{U}_1 = \overline{A_1} + F_G$ …..(10)

With an error matrix F_G. Using trapezoidal factorization $A_1=LU_1$ of the n-by-(n+1) coefficient matrix, one readily derives from (10) the equation

$\Delta L U_1 + \overline{L}\Delta U_1 = \Delta A_1 + F_G$ ….(11)

Since $U_1 \hat{x} = Ux - z = 0$ …(12)

$\Delta L U_1 + \overline{L}\Delta U_1 = \Delta A_1 + F_G$ Implies the representation $\Delta U_1 \hat{x} = \overline{L}^{-1}(\Delta A_1 + F_G)\hat{x}$ …(13)

This result establishes the dependence of the errors ΔU_1 of the n-by-(n+1) coefficient matrix $\overline{U}_1$ of the upper triangular linear system, which has to be solved by Jordan elimination, upon the data errors and the rounding errors in forward elimination, where F_G denotes the errors matrix of forward elimination in and F_J the errors matrix of the solution by Jordan elimination.

Comparsion of Gauss-Jordan elimination with Gaussian elimination

The behavior of the error of the solution of general linear system by Gaussian elimination on the one hand and Gauss-Jordan elimination on the other hand will be compared with each other. In both cases we assume the same perturbed data $\overline{A}, \overline{y}$. Then for both methods also computed triangular factors $\overline{L}, \overline{u}$ of $\overline{A}$ and computed coefficients $\overline{U}, \overline{Z}$ of the upper triangular factor system are the same. The vector of data and rounding condition numbers of Gaussian elimination are specified by (see [8, 64]).

The solution vector $\overline{x_j}, \overline{x_G}$ of the two method satisfy error estimates of the form

$$\left|\overline{x_i} - x_i\right| \le \frac{\sigma_i \eta}{1 - \rho m^{\eta}}, \left|\frac{\overline{x_i} - x_i}{\overline{x_i}}\right| \le \frac{\rho_i \eta}{1 - \rho m^{\eta}}, i = 1, \ldots, n.$$

The following equation shows that the total condition numbers

$$\sigma_{i,j} = \frac{\eta_D}{\eta}\sigma_{i,j}^{D} + \frac{\eta_R}{\eta}\sigma_{i,j}^{R}, i = 1, \ldots, n, \eta = \max(\eta_D, \eta_R),$$

Of the Gauss – jordan algorithm can be bounded from above by for corresponding total condition number $\sigma_{i,G}$ of Gaussian elimination for sufficiently small accuracy constant η, the condition number $\sigma_{i,j}$ is, in essence, less than or equal to $\frac{3}{2}(n+1-i)$ − times $\sigma_{i,j}$ because the constant q and the denominator are close to 1.The condition number σ_i constitute , save for terms of higher order in η, optimal bounds for the absolute errors $\Delta x_i = \overline{x_i} - x_i$. Hence, the result say that for sufficiently small η the absolute errors $\Delta x_{i,j}$ of the solutions by Gauss-

Jordan elimination cannot be, in absolute value, essentially grater than $\frac{3}{2}(n+1-i)-times$ the possible maximal errors $\Delta x_{i,G} = \bar{x}_{i,G} - x_i, 1, ,\ldots, n$ of the solution by Gaussian elimination.

Numerical example

The results of the paper are now illustrated by simple numerical example. The first example is the upper triangular linear system

$0.826354\ x_1+0.432175\ x_2+0.613256\ x_3+0.614227\ x_4 = 0.722872$

$0.000547x_2+0.814712\ x_3+0.816328\ x_4 =0.15424$

$0.915316\ x3+0.814275\ x4 =0.109844$ ….(14)

$0.982176\ x_4 =0.602286$

Of Peters-Wilkinson [2].Table 1 contains the condition numbers, stability constants and the solution by Jordan elimination which were computed in high accuracy and then rounded to 6 decimal digits. Table 2 shows the corresponding results of the solution by back substitution.

Since the data condition numbers ρ_i^D, τ_i^D in essence, depend on the problem only but not on the algorithm these condition numbers coincide in both algorithms in the first 6 digits. The residual condition number $\tau_{i,j}^R = 3343$ of the Jordan solution and the corresponding residual stability constant $\psi_{1,j} = 1699$ (see table 1) are much bigger in this example than those of the solution by back substitution where $\psi_{j,G} \leq 1.3$ for j=1,…, 4. (see table 2)

In contrast, the relative rounding condition numbers of the solutions and thus the solution stability constants w_i do not differ much for the two algorithms.

2. The second example is a linear system with the 5×5 Hilbert matrix A of Wilkinson [10, III, 34] and the right-hand side 1.

$$A = \frac{50*10!}{10^8}\left(\frac{1}{i+k}\right)_{i,k=1,\ldots,5}; y^t, = (1,1,1,1,1)$$

….(15)

The first 6 digit of the solution, the condition numbers and stability constants for Gauss-Jordan elimination and Gaussian elimination, all computed in high accuracy, are found in table 3 and table 4.

The matrix A is ill-conditioned so that the relative data and rounding condition numbers ρ_i^D, ρ_i^R are big in value. The relative error of the computed solution may effect up to 6 decimal digits.

Nevertheless, the maximal residual stability constant $\psi_{i,j} = 15.5$ of Gauss-Jordan elimination is only about 5.3- times bigger than the maximal residual stability constant $\psi_{4,G} = 2.9$ of Gaussian elimination.

The relative rounding condition numbers ρ_i^R of the two algorithms are practically equal. Therefore also the stability constants wi of the two algorithms coincide in the leading decimal digits.

Conclusion

Error and residual bounds can be computed numerically together with the solutions of the linear systems. The calculated condition numbers and stability constants of the solutions by Gauss-Jordan elimination as well as by Gaussian elimination are determined. The examples show in accordance with the theoretical results that the numerical solutions by the two methods are of comparable accuracy in spite of the ill-conditioning of the two problems.

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Table(1): condition numbers, stability constants and solution of the Jordan algorithm for triangular linear system (15).

i	$\overline{\chi}_i$	ρ_i^D	ρ_i^R	w_i	τ_i^D	τ_i^R	Ψ_i
1	0.413155	4639	7207	1.55	1.97	3343	1699
2	0.614928	5955	3313	0.56	1.00	3.20	3.19
3	-0.425517	5.13	4.56	0.89	0.99	2.28	2.28
4	0.613216	2.00	1.00	0.50	1.21	0.60	0.50

Table(2): condition numbers, stability constants and solution by back substitution of the triangular linear system (15).

i	$\overline{\chi}_i$	ρ_i^D	ρ_i^R	w_i	τ_i^D	τ_i^R	Ψ_i
1	0.413155	4639	5531	1.19	1.97	2.54	1.29
2	0.614928	5955	7100	1.19	1.00	1.19	1.19
3	-0.425517	5.13	4.56	0.89	0.99	1.28	1.28
4	0.613216	2.00	1.00	0.50	1.21	0.60	0.50

Table(3): condition numbers, stability constants and solution of the "Gauss –Jordan" algorithm for the linear system Ax=y.

i	$\overline{\chi}_i$	ρ_i^D	ρ_i^R	w_i	τ_i^D	τ_i^R	Ψ_i
1	1.65344	1.07	1.36	1.27	1.29	2.00	15.5
2	-2.31481	8.69	1.11	1.28	5.05	1.61	15.3
3	9.25926	7.33	9.38	1.28	8.90	1.32	14.8
4	-1.38889	6.34	8.13	1.28	7.72	1.12	14.5
5	6.94444	5.59	7.18	1.28	6.82	9.69	14.2

Table(4): condition numbers, stability constants and solution of "Gaussian elimination" of the linear system Ax=y.

i	$\overline{\chi}_i$	ρ_i^D	ρ_i^R	w_i	τ_i^D	τ_i^R	Ψ_i
1	1.65344	1.07	1.36	1.27	1.29	1.93	1.50
2	-2.31481	8.69	1.11	1.28	1.05	2.63	2.50
3	9.25926	7.33	9.38	1.28	8.90	2.47	2.77
4	-1.38889	6.34	8.13	1.28	7.72	2.21	2.86
5	6.94444	5.59	7.18	1.29	6.82	1.99	2.92

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مناقشة تحليل الأخطاء لكاوس جوردن للحذف لمنظومة المعادلات الخطية

شوقي عبد المطلب عباس
قسم الرياضيات ، كلية العلوم ، جامعة بغداد
استلم البحث في:22 كانون الثاني 2012 قبل البحث في: 21 ايار 2012

الخلاصة

يتعلق هذا البحث بالتمثيلات الصريحة للأخطاء والبواقي للحلول التقريبية للأنظمة الخطية المثلثية بطريقة جوردن للحذف والطريقة العامة. أن حل المعادلات الجبرية الخطية بطريقه كاوس – جوردن للحذف توصف على أنها دالة للبيانات القلقة وعمليات أخطاء التدوير لحساب الفارزة السائبة ومن هذه التمثيلات تم اشتقاق الجزء الأساسي المتصل بالأخطاء وتحديد البواقي.

إن تخمين الاستقرار للحلول قد تم شرحه في هذا البحث. كما حددت قيود الأخطاء للحلول ومقارنتها مع أفضل الحدود للأخطاء التي احتسبت بالتعويض المتراجع وطريقة الحذف لكاوس على التوالي. وتؤكد النتائج وبشكل مفصل ان الحلول بطريقتي جوردن للحذف وطريقة كاوس – جوردن للحذف هي ليست أساساً أعظم من الأخطاء الكبيرة المحتملة بطريقه التعويض المتراجع والحذف لكاوس على التوالي. ولقد وضحت النتائج النظرية بمثالين عددين.

الكلمات المفتاحية: طريقة جوردن للحذف، البيانات المضطربة، حدود الأخطاء، طريقة كاوس – جوردن للحذف.