



On Generalized Regular Continuous Functions In Topological Spaces

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Abstract

In this paper we introduce a new type of functions called the generalized regular continuous functions. These functions are weaker than regular continuous functions and stronger than regular generalized continuous functions. Also, we study some characterizations and basic properties of generalized regular continuous functions. Moreover we study another types of generalized regular continuous functions and study the relation among them.

Key words: generalized regular continuous functions, regular continuous functions and regular generalized continuous functions.

Introduction

The concept of regular continuous functions was first introduced by Arya, S.P. and Gupta, R. [1]. Later Palaniappan, N. and Rao, K.C. [2] studied the concept of regular generalized continuous functions. Also, the concept of generalized regular closed sets in topological spaces was introduced by Bhattacharya, S. [3]. The purpose of this paper is to introduce a new class of functions, namely, generalized regular continuous functions. This class is placed properly between the class of regular continuous functions and the class of regular generalized continuous functions. Also, we study some characterizations and basic properties of generalized regular continuous functions. Moreover we study the perfectly generalized regular continuous functions, contra generalized regular continuous functions, generalized regular irresolute functions, contra generalized regular irresolute functions and we study the relation among them.

Throughout this paper (X, τ) , (Y, τ') and (Z, τ'') (or simply X , Y and Z) represent non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When

A is a subset of X , $\text{cl}(A)$, A° and A^c denote the closure, the interior and the complement of a set A respectively.

Preliminaries

First we recall the following definitions:

(1.1) Definition: A subset A of a topological space X is said to be :

- i) A generalized closed (briefly g-closed) set [4] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- ii) A regular closed (briefly r-closed) set [5] if $\text{cl}(\text{int}(A)) = A$.



- iii) A regular generalized closed (briefly rg-closed) set [2] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- iv) A generalized regular closed (briefly gr-closed) set [3] if $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X , where $\text{rcl}(A) = \bigcap \{F : A \subseteq F, F \text{ is a regular closed subset of } X\}$.
- v) A b-closed set [6] if $\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq A$.
- vi) A generalized b-closed (briefly gb-closed) set [7] if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X , where $\text{bcl}(A) = \bigcap \{F : A \subseteq F, F \text{ is a b-closed subset of } X\}$.
- The complement of a g-closed (resp. r-closed, rg-closed, gr-closed, b-closed, gb-closed) set is called a g-open (resp. r-open, rg-open, gr-open, b-open, gb-open) set.

Remarks :

1) closed sets and gr-closed sets are in general independent. Consider the following examples:-

Examples:

- i) Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. Then $A = \{b\}$ is a gr-closed set, but not closed.
- ii) Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}, \{b, c\}, \{b\}\}$. Then $A = \{c\}$ is a closed set, but not gr-closed.
- 2) Every gr-closed set is a g-closed set, but the converse in general is not true. In (1) no.(ii), $A = \{c\}$ is a g-closed set, but not gr-closed.
- 3) Every gr-closed set is a rg-closed set, but the converse in general is not true. In (1) no.(ii), $A = \{c\}$ is a rg-closed set, but not gr-closed.
- 4) Every gr-closed set is a gb-closed set, but the converse in general is not true. In (1) no.(ii), $A = \{c\}$ is a gb-closed set, but not gr-closed.
- 5) Every r-closed set is a gr-closed (resp. g-closed, gb-closed, rg-closed) set, but the converse in general is not true. In (1) no.(i), $A = \{b\}$ is a gr-closed (resp. g-closed, gb-closed, rg-closed) set, but not r-closed.

Definition: The intersection of all gr-closed subsets of X containing a set A is called the generalized regular-closure of A and is denoted by $\text{grcl}(A)$.
if A is a gr-closed set, then $\text{grcl}(A) = A$. The converse is not true, since the intersection of gr-closed sets need not be gr-closed. [3].

Theorem: Let A be a subset of a topological space X . Then $x \in \text{grcl}(A)$ if and only if for any gr-open set U containing x , $A \cap U \neq \phi$.

Proof: \Rightarrow

Let $x \in \text{grcl}(A)$ and suppose that, there is a gr-open set U in X s.t $x \in U$ and $A \cap U = \phi$
 $\Rightarrow A \subset U^c$ which is gr-closed in $X \Rightarrow \text{grcl}(A) \subseteq \text{grcl}(U^c) = U^c$.

$\therefore x \in U \Rightarrow x \notin U^c \Rightarrow x \notin \text{grcl}(A)$, this is a contradiction.

Conversely,

Suppose that, for any gr-open set U containing x , $A \cap U \neq \phi$. To prove that $x \in \text{grcl}(A)$.
Suppose that $x \notin \text{grcl}(A)$, then there is a gr-closed set F in X such that $x \notin F$ and $A \subseteq F$.

$\therefore x \notin F \Rightarrow x \in F^c$ which is gr-open in X .

$\therefore A \subseteq F \Rightarrow A \cap F^c = \phi$, this is a contradiction. Thus $x \in \text{grcl}(A)$.

Definition: A function $f : X \rightarrow Y$ from a topological space X into a topological space Y is called :

- 1) A generalized continuous (briefly g-continuous) [8] if $f^{-1}(V)$ is g-closed set in X for every closed set V in Y .



- 2) A regular generalized continuous (briefly rg-continuous) [2] if $f^{-1}(V)$ is rg-closed set in X for every closed set V in Y .
- 3) A regular continuous (briefly r-continuous)[1] if $f^{-1}(V)$ is r-closed set in X for every closed set V in Y .
- 4) A generalized b-continuous (briefly gb-continuous)[9] if $f^{-1}(V)$ is gb-closed set in X for every closed set V in Y .
- 5) A generalized irresolute (briefly g-irresolute)[8] if $f^{-1}(V)$ is g-closed set in X for every g-closed set V in Y .
- 6) A regular generalized irresolute (briefly rg-irresolute) [2] if $f^{-1}(V)$ is rg-closed set in X for every rg-closed set V in Y .

Generalized Regular Continuous Functions

In this section we introduce the concept of generalized regular continuous functions in topological spaces and study the characterizations and basic properties of generalized regular continuous functions. Also, we study another types of generalized regular continuous functions and we study the relation among them.

Definition: A function $f : X \rightarrow Y$ from a topological space X into a topological space Y is called a generalized regular continuous (briefly gr-continuous) if $f^{-1}(V)$ is gr-closed set in X for every closed set V in Y .

Theorem . A function $f : X \rightarrow Y$ from a topological space X into a topological space Y is gr-continuous iff $f^{-1}(V)$ is gr-open set in X for every open set V in Y .

Proof: It is Obvious .

Theorem: Let $f : X \rightarrow Y$ be a function from a topological space X into a topological space Y . If $f : X \rightarrow Y$ is gr-continuous, then $f(\text{grcl}(A)) \subseteq \text{cl}(f(A))$ for every subset A of X .

Proof: Since $f(A) \subseteq \text{cl}(f(A)) \Rightarrow A \subseteq f^{-1}(\text{cl}(f(A)))$. Since $\text{cl}(f(A))$ is a closed set in Y and f is gr-continuous, then by (2.1) $f^{-1}(\text{cl}(f(A)))$ is a gr-closed set in X containing A .

Hence $\text{grcl}(A) \subseteq f^{-1}(\text{cl}(f(A)))$. Therefore $f(\text{grcl}(A)) \subseteq \text{cl}(f(A))$.

Theorem: Let $f : X \rightarrow Y$ be a function from a topological space X into a topological space Y . Then the following statements are equivalent:-

i) For each point x in X and each open set V in Y with $f(x) \in V$, there is a gr-open set U in X such that $x \in U$ and $f(U) \subseteq V$.

ii) For each subset A of X , $f(\text{grcl}(A)) \subseteq \text{cl}(f(A))$.

iii) For each subset B of Y , $\text{grcl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$.

Proof: (i) \rightarrow (ii).

Suppose that (i) holds and let $y \in f(\text{grcl}(A))$ and let V be any open neighborhood of y .

Since $y \in f(\text{grcl}(A)) \Rightarrow \exists x \in \text{grcl}(A)$ s.t $f(x) = y$.

Since $f(x) \in V$, then by (i) \exists a gr-open set U in X s.t $x \in U$ and $f(U) \subseteq V$.

Since $x \in \text{grcl}(A)$, then by (1.4) $U \cap A \neq \emptyset$ and hence $f(A) \cap V \neq \emptyset$.

Therefore we have $y = f(x) \in \text{cl}(f(A))$. Hence $f(\text{grcl}(A)) \subseteq \text{cl}(f(A))$

(ii) \rightarrow (i)



If (ii) holds and let $x \in X$ and V be any open set in Y containing $f(x)$.

Let $A = f^{-1}(V^c) \Rightarrow x \notin A$. Since $f(\text{grcl}(A)) \subseteq \text{cl}(f(A)) \subseteq V^c \Rightarrow \text{grcl}(A) \subseteq f^{-1}(V^c) = A$.

Since $x \notin A \Rightarrow x \notin \text{grcl}(A)$ and by (1.4) there exists a gr-open set U containing x such that $U \cap A = \emptyset$ and hence $f(U) \subseteq f(A^c) \subseteq V$.

(ii) \rightarrow (iii).

Suppose that (ii) holds and let B be any subset of Y . Replacing A by $f^{-1}(B)$ we get from (ii) $f(\text{grcl}(f^{-1}(B))) \subseteq \text{cl}(f(f^{-1}(B))) \subseteq \text{cl}(B)$. Hence $\text{grcl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$.

(iii) \rightarrow (ii).

Suppose that (iii) holds, let $B = f(A)$ where A is a subset of X . Then we get from (iii) $\text{grcl}(A) \subseteq \text{grcl}(f^{-1}(f(A))) \subseteq f^{-1}(\text{cl}(f(A)))$. Therefore $f(\text{grcl}(A)) \subseteq \text{cl}(f(A))$.

Definition: A function $f : X \rightarrow Y$ from a topological space X into a topological space Y is said to be perfectly generalized regular continuous (briefly perfectly gr-continuous) if $f^{-1}(V)$ is gr-clopen (gr-open and gr-closed) set in X for every open set V in Y .

Definition: A function $f : X \rightarrow Y$ from a topological space X into a topological space Y is said to be contra generalized regular continuous (briefly contra gr-continuous) if $f^{-1}(V)$ is gr-closed set in X for every open set V in Y .

Theorem: Let $f : X \rightarrow Y$ be a function. Then

- 1) If f is r-continuous, then f is gr-continuous.
- 2) If f is gr-continuous, then f is g-continuous.
- 3) If f is gr-continuous, then f is rg-continuous.
- 4) If f is gr-continuous, then f is gb-continuous.
- 5) If f is continuous, then f is rg-continuous.
- 6) If f is perfectly gr-continuous, then f is gr-continuous.
- 7) If f is perfectly gr-continuous, then f is rg-continuous.
- 8) If f is perfectly gr-continuous, then f is g-continuous.
- 9) If f is perfectly gr-continuous, then f is gb-continuous.
- 10) If f is perfectly gr-continuous, then f is contra gr-continuous.

Proof:

1) Let F be a closed set in Y , Since f is r-continuous, then by (1.5) no.3, $f^{-1}(F)$ is r-closed in X . Since every r-closed set is gr-closed, then $f^{-1}(F)$ is gr-closed in X .

Hence f is gr-continuous.

3) Let F be a closed set in Y , Since f is gr-continuous, then by (2.1), $f^{-1}(F)$ is gr-closed in X .

Since every gr-closed set is rg-closed, then $f^{-1}(F)$ is rg-closed in X .

Hence f is rg-continuous.

5) Let F be a closed set in Y , Since f is continuous, then $f^{-1}(F)$ is closed in X .

Since every closed set is rg-closed, then $f^{-1}(F)$ is rg-closed in X .

Hence f is rg-continuous.

9) Let U be an open set in Y , Since f is perfectly gr-continuous, then by (2.5), $f^{-1}(U)$ is gr-

closed and gr-open in X . Since every gr-open set is gb-open, then $f^{-1}(U)$ is gb-open in X .

Hence f is gb-continuous.

Similarly, we can prove (2),(4),(6),(7),(8) and (10).



Remarks :

1) Continuous functions and gr-continuous functions are in general independent .Consider the following examples:-

Examples

i) Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}, \{b, c\}, \{b\}\}$ and $\tau' = \{X, \phi, \{a, b\}\}$.

Let $f : (X, \tau) \rightarrow (X, \tau')$ be a function defined by : $f(a) = a, f(b) = b$ and $f(c) = c$.

It is clear that f is continuous ,but f is not gr-continuous ,since $\{c\}$ is closed in (X, τ') ,but $f^{-1}(\{c\}) = \{c\}$ is not gr-closed in (X, τ) .

ii) Let $X = \{a, b, c\}$, $\tau = \{X, \phi\}$ and $\tau' = \{X, \phi, \{a\}\}$.

Let $f : (X, \tau) \rightarrow (X, \tau')$ be a function defined by : $f(a) = a, f(b) = b$ and $f(c) = c$.

It is clear that f is not continuous ,but f is gr-continuous ,since $f^{-1}(\phi) = \phi, f^{-1}(X) = X$ and $f^{-1}(\{b, c\}) = \{b, c\}$ are gr-closed in (X, τ) .

2) The converse of ((2.7),no.1) in general is not true .Consider the following examples:-

Let $X = \{a, b, c, d\}$, $Y = \{p, q\}$, $\tau = \{X, \phi, \{c, d\}\}$ and $\tau' = \{Y, \phi, \{p\}\}$.

Let $f : (X, \tau) \rightarrow (Y, \tau')$ be a function defined by : $f(a) = f(b) = f(d) = q$ and $f(c) = p$.

f is gr-continuous ,since $f^{-1}(\phi) = \phi, f^{-1}(Y) = X$ and $f^{-1}(\{q\}) = \{a, b, d\}$ are gr-closed in (X, τ) . But f is not r-continuous ,since $\{q\}$ is closed in (Y, τ') ,but $f^{-1}(\{q\}) = \{a, b, d\}$ is not r-closed in (X, τ) .

3) The converse of ((2.7),no.2,3,4) in general is not true .In (1,(i)), f is g-continuous (resp. rg-continuous, gb-continuous) since f is continuous, but f is not gr-continuous .

4) The converse of ((2.7),no.5) in general is not true .In (1,(ii)), f is rg-continuous ,but f is not continuous .

5) The converse of ((2.7),no.6,7,8,9) in general is not true .In (2), f is gr-continuous (resp. rg-continuous, g-continuous, gb-continuous), but f is not perfectly gr-continuous ,since $\{p\}$ is open in (Y, τ') ,but $f^{-1}(\{p\}) = \{c\}$ is gr-open, but not gr-closed in (X, τ) .

6) The converse of ((2.7),no.10) in general is not true .Consider the following examples:-

Let $X = Y = \{a, b\}$ and $\tau = \tau' = \{X, \phi, \{a\}\}$.

Let $f : (X, \tau) \rightarrow (X, \tau')$ be a function defined by : $f(a) = b$ and $f(b) = a$.

f is contra gr-continuous ,since $f^{-1}(\phi) = \phi, f^{-1}(Y) = X$ and $f^{-1}(\{a\}) = \{b\}$ are gr-closed in (X, τ) .But f is not perfectly gr-continuous ,since $\{a\}$ is open in (X, τ') ,but $f^{-1}(\{a\}) = \{b\}$ is gr-closed, but not gr-open in (X, τ) .

7) Continuous functions and contra gr-continuous functions are in general independent .

Consider the following examples:-

i) It is clear that In (1,(i)), f is continuous ,but f is not contra gr-continuous .

ii) It is clear that In (6), f is contra gr-continuous ,but f is not continuous .

8) Contra gr-continuous functions and r-continuous functions are in general independent .

Consider the following examples:-

i) It is clear that In (6), f is contra gr-continuous ,but f is not r-continuous .

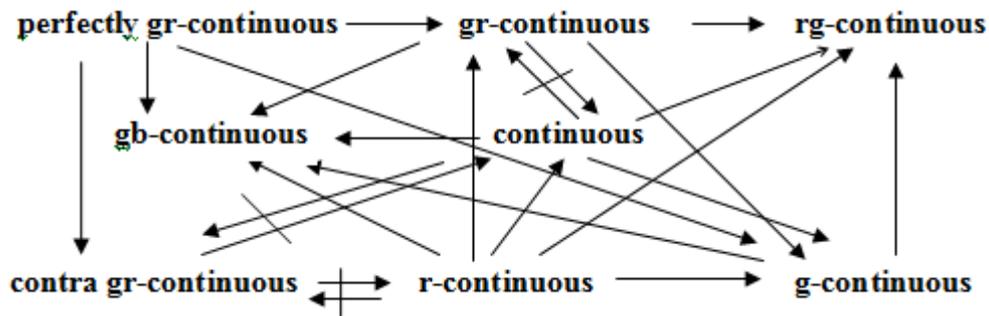
ii) Let $X = Y = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and $\tau' = \{Y, \phi, \{a\}\}$.

Let $f : (X, \tau) \rightarrow (Y, \tau')$ be a function defined by : $f(a) = a, f(b) = b, f(c) = c$ and $f(d) = d$.

f is r-continuous ,since $f^{-1}(\phi) = \phi, f^{-1}(Y) = X$ and $f^{-1}(\{b, c, d\}) = \{b, c, d\}$ are r-closed in (X, τ) . But f is not contra gr-continuous ,since $\{a\}$ is open in (Y, τ') ,but $f^{-1}(\{a\}) = \{a\}$ is not gr-closed in (X, τ) .



Thus we have the following diagram:



Definition: A topological space (X, τ) is called a T_{gr} - space if every rg-closed set is gr-closed set .

Examples:

1) In Remarks ((2.8) no.1(ii)), (X, τ) is a T_{gr} - space, since rg-closed set $= \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\} = \text{gr-closed}$.

2) In Remarks ((1.2) no.1(ii)), (X, τ) is not a T_{gr} - space, since $\{c\}$ is rg-closed set ,but not gr-closed .

Theorem: Let $f : X \rightarrow Y$ be a function such that X is a T_{gr} - space, then:-

- i) Every continuous function is gr-continuous .
- ii) Every rg-continuous function is gr-continuous .
- iii) Every g-continuous function is gr-continuous .

Proof: i) Let F be a closed set in Y , Since f is continuous, then $f^{-1}(F)$ is closed in X .

Since every closed set is rg-closed ,then $f^{-1}(F)$ is rg-closed in X .

Since X is a T_{gr} - space ,then $f^{-1}(F)$ is gr-closed in X .

Hence f is gr-continuous .

Similarly, we can prove (ii) and (iii) .

Generalized Regular Irresolute Functions

Definition: A function $f : X \rightarrow Y$ from a topological space X into a topological space Y is called a generalized regular irresolute (briefly gr-irresolute) if $f^{-1}(V)$ is gr-closed set in X for every gr-closed set V in Y .

Theorem: A function $f : X \rightarrow Y$ from a topological space X into a topological space Y is gr-irresolute iff $f^{-1}(V)$ is gr-open set in X for every gr-open set V in Y .

Proof: It is Obvious .

Definition: A function $f : X \rightarrow Y$ from a topological space X into a topological space Y is said to be contra generalized regular irresolute (briefly contra gr-irresolute) if $f^{-1}(V)$ is gr-closed set in X for every gr-open set V in Y .

Remarks :

1) gr-irresolute functions and gr-continuous functions are in general independent .Consider the following examples:-

**Examples:**

i) Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}\}$ and $\tau' = \{X, \phi\}$.

Let $f : (X, \tau) \rightarrow (X, \tau')$ be a function defined by : $f(a) = a$, $f(b) = b$ and $f(c) = c$.

It is clear that f is gr-continuous, but f is not gr-irresolute, since $\{a\}$ is gr-closed in (X, τ') , but $f^{-1}(\{a\}) = \{a\}$ is not gr-closed in (X, τ) .

ii) Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}\}$ and $\tau' = \{X, \phi, \{a, b\}, \{b, c\}, \{b\}\}$.

Let $f : (X, \tau) \rightarrow (X, \tau')$ be a function defined by : $f(a) = a$, $f(b) = b$ and $f(c) = c$.

f is gr-irresolute, since $f^{-1}(\phi) = \phi$, $f^{-1}(X) = X$ and $f^{-1}(\{a, c\}) = \{a, c\}$ are gr-closed in (X, τ) .

But f is not gr-continuous, since $\{a\}$ is closed in (X, τ') , but $f^{-1}(\{a\}) = \{a\}$ is not gr-closed in (X, τ) .

Theorem: Let $f : X \rightarrow Y$ be a function such that X is a T_{gr} -space, then:-

i) Every rg-irresolute function is gr-irresolute.

ii) Every g-irresolute function is gr-irresolute.

Proof:

i) Let F be a gr-closed set in Y , then by (1.2) no.3, F is rg-closed in Y .

Since f is rg-irresolute, then by (1.5) no.6, $f^{-1}(F)$ is rg-closed in X .

Since X is a T_{gr} -space, then $f^{-1}(F)$ is gr-closed in X .

Hence f is gr-irresolute.

ii) Let F be a gr-closed set in Y , then by (1.2) no.2, F is g-closed in Y .

Since f is g-irresolute, then by (1.5) no.5, $f^{-1}(F)$ is g-closed in X .

Since every g-closed set is rg-closed, then $f^{-1}(F)$ is rg-closed in X .

Since X is a T_{gr} -space, then $f^{-1}(F)$ is gr-closed in X .

Hence f is gr-irresolute.

Theorem: If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are functions, then:-

1) If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are both gr-irresolute functions, then $g \circ f : X \rightarrow Z$ is gr-irresolute.

2) If $f : X \rightarrow Y$ is contra gr-irresolute and $g : Y \rightarrow Z$ is gr-irresolute, then $g \circ f : X \rightarrow Z$ is contra gr-irresolute.

3) If $f : X \rightarrow Y$ is gr-irresolute and $g : Y \rightarrow Z$ is gr-continuous, then $g \circ f : X \rightarrow Z$ is gr-continuous.

4) If $f : X \rightarrow Y$ is gr-continuous and $g : Y \rightarrow Z$ is r-continuous, then $g \circ f : X \rightarrow Z$ is gr-continuous.

5) If $f : X \rightarrow Y$ is gr-continuous and $g : Y \rightarrow Z$ is continuous, then $g \circ f : X \rightarrow Z$ is gr-continuous.

6) If $f : X \rightarrow Y$ is contra gr-irresolute and $g : Y \rightarrow Z$ is gr-continuous, then $g \circ f : X \rightarrow Z$ is contra gr-continuous.

Proof:

1) Let U be a gr-open set in Z , since g is gr-irresolute, then $g^{-1}(U)$ is gr-open in Y , since f is gr-irresolute, then $f^{-1}(g^{-1}(U))$ is gr-open in X . Since $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$, then $(g \circ f)^{-1}(U)$ is a gr-open set in X . Thus $g \circ f$ is gr-irresolute.

2) Let U be a gr-open set in Z , since g is gr-irresolute, then $g^{-1}(U)$ is gr-open in Y , since f is contra gr-irresolute, then $f^{-1}(g^{-1}(U))$ is gr-closed in X . Since $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$, then



$(g \circ f)^{-1}(U)$ is a gr-closed set in X . Thus $g \circ f$ is contra gr-irresolute.

3) Let F be a closed set in Z , since g is gr-continuous, then $g^{-1}(F)$ is gr-closed in Y , since f is gr-irresolute, then $f^{-1}(g^{-1}(F))$ is gr-closed in X . Since $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$, then $(g \circ f)^{-1}(F)$ is a gr-closed set in X . Thus $g \circ f$ is gr-continuous.

4) Let F be a closed set in Z , since g is r-continuous, then $g^{-1}(F)$ is r-closed in Y , since every r-closed set is closed, then $g^{-1}(F)$ is closed in Y , since f is gr-continuous, then $f^{-1}(g^{-1}(F))$ is gr-closed in X . Since $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$, then $(g \circ f)^{-1}(F)$ is a gr-closed set in X . Thus $g \circ f$ is gr-continuous.

5) Let F be a closed set in Z , since g is continuous, then $g^{-1}(F)$ is closed in Y , since f is gr-continuous, then $f^{-1}(g^{-1}(F))$ is gr-closed in X . Since $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$, then $(g \circ f)^{-1}(F)$ is a gr-closed set in X . Thus $g \circ f$ is gr-continuous.

6) Let U be an open set in Z , since g is gr-continuous, then $g^{-1}(U)$ is gr-open in Y , since f is contra gr-irresolute, then $f^{-1}(g^{-1}(U))$ is gr-closed in X . Since $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$, then $(g \circ f)^{-1}(U)$ is a gr-closed set in X . Thus $g \circ f$ is contra gr-continuous.

References

1. Arya, S. P. and Gupta, R. (1974) On strongly continuous functions, Kyungpook Math. J., 14:131-143.
2. Palaniappan, N. and Rao, K.C. (1993) Regular Generalized Closed Sets, Kyungpook Math. J., 33: (2) 211-219.
3. Bhattacharya, S. (2011) On Generalized Regular Closed Sets, Int. J. Contemp. Math. Sciences, 6: (3) 145-152.
4. Levine, N. (1970) Generalized Closed Sets In Topology, Rend. Circ. Math. Palermo, 19: (2) 89-96.
5. Stone, M. (1937) Applications of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41: 375-381.
6. Andrijevic, D. (1996) On b-open sets, Mat. Vesnik, 48: (1-2) 59-64.
7. Ganster, M. and Steiner, M., 2007, On $b\tau$ -closed sets, Applied General Topology, 8: (2) 243-247.
8. Balachandran, K.; Sundaram, P. and Maki, H. (1991) On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 12: 5-13.
9. Al-omari, A. and Noorani, M.S., 2009, On Generalized b-closed sets, Bull. Malays. Math. Sci. Soc., 2: (32) 19-30.



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استلم البحث في: 30 كانون الثاني 2012 قبل البحث في: 21 ايار 2012

الخلاصة

في هذا البحث قدمنا نوعا جديدا من الدوال أسميناها بالدوال المستمرة المنتظمة المعممة (generalized regular continuous functions) هذه الدوال اضعف من الدوال المستمرة المنتظمة (regular continuous functions) وأقوى من الدوال المستمرة المعممة المنتظمة (regular generalized continuous functions). كذلك درسنا بعض المكافآت والخواص الأساسية للدوال المستمرة المنتظمة المعممة. فضلاً عن ذلك درسنا أنواعا أخرى من الدوال المستمرة المنتظمة المعممة ومن ثم درسنا العلاقة بينها .

الكلمات المفتاحية: الدوال المستمرة المنتظمة المعممة ، الدوال المستمرة المنتظمة ، الدوال المستمرة المعممة المنتظمة .