Numerical Simulation to Study the Effects of Riemann Problems on the Physical Properties of the Astrophysics Gas Dynamics

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Abstract

In this work we run simulation of gas dynamic problems to study the effects of Riemann problems on the physical properties for this gas.

We studied a normal shock wave travels at a high speed through a medium (shock tube). This would cause discontinuous change in the characteristics of the medium, such as rapid rise in velocity, pressure, and density of the flow.

When a shock wave passes through the medium, the total energy is preserved but the energy which can be extracted as work decreases and entropy increases.

The shock tube is initially divided into a driver and a driven section by a diaphragm. The shock wave is created by increasing the pressure in the driver section until the diaphragm bursts, sending normal shock waves down the shock tube into the low pressure driven section and at the same time sending an expansion wave into the high pressure driver section.

Keywords: Riemann problems, numerical simulation, real gas flow

Introduction

From the solution of Riemann problems one finds directly how much velocity, density, and pressure flows into a cell from the interface under consideration.
The initial value problem for any discontinuity is known as the Riemann problem.

$$W(t=0) = \begin{cases} W_{left} & x < x_o \\ W_{right} & x > x_o \end{cases}$$

Where W represents the values of density, velocity and pressure. They can be thought of two constant states separated by a diaphragm at x_0 which is removed at t = 0
and x_0 is the interface between two meshes as it clear in fig. 1.

The initial condition for the Riemann problems is set by specifying the left and right values of the density, velocity and pressure.

Riemann problem for the Euler equations leads to two types of waves spreading. They are shock and expansion waves. The possible Riemann configurations are thus:

1. One shock and one expansion wave.
2. Two shock waves.
3. Two expansion waves.

In each of the above cases there will be a contact discontinuity in between the two waves (due to the fact that we started with an initial discontinuity). [1]

The collision between two flows, leading to the formation of three discontinuity as shown in fig.2.

The first Riemann problem we will consider is the so-called shock tube. The shock tube is a device in which a normal shock wave is produced by the sudden bursting of a diaphragm separating a gas at high pressure from one at lower pressure.

When the diaphragm bursts a shock wave forms almost instantaneously and propagates into the driven section, while simultaneously an expansion wave propagates, in the opposite direction, into the driver section. The propagation of the shock front and expansion fan changes the gas pressure, and density, and sets the gas in motion relative to the shock-tube walls. The strength of the shock wave and expansion wave thus produced depends on the initial pressure ratio across the diaphragm and on the physical properties of the gases in the driver and the driven sections. [2]

Jump Conditions Across a Standing Normal Shock Wave; Relationship Between Laboratory Fixed and Shock Fixed Coordinates.

We first consider the case of a standing normal shock wave in a tube, which is clear in fig. 1.
The conservation of mass, momentum, and energy for the flow through this wave are given by:

$$\rho_1 u_1 = \rho_2 u_2 \qquad (1)$$

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2 \qquad (2)$$

$$\frac{1}{2}u_1^2 + \varepsilon_1 + \frac{P_1}{\rho_1} = \frac{1}{2}u_2^2 + \varepsilon_2 + \frac{P_2}{\rho_2} \qquad (3)$$

Where: ρ_1, u_1, p_1 are the pre-shock density, velocity, and pressure and ρ_2, u_2, p_2 are the post shock density, velocity, and pressure repectively.
For an ideal gas pressure and internal energy are related by:

$$\rho\varepsilon = \frac{P}{(\gamma - 1)} \qquad (4)$$

ε is the specific internal energy.
γ is the specific heat capacity.

The entropy related quantity is $\xi = P\rho^{-\gamma}$ (5)

The ratio of the shock parameters, are known as the Shock jump or Rankine –Hugoniot conditions (in the frame of Shock) and express by, [3, 4]:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2} \qquad (6)$$

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{(\gamma+1)(M_1^2 - 1)} \qquad (7)$$

Where *M* is the Mach number of the flow and it define by:

$$M_1 = \frac{|u|}{c_S} \qquad (8)$$

C_S is the sound speed and it is equal to:

$$C_s = \sqrt{\gamma \frac{P}{\rho}} \tag{9}$$

Numerical Results and Discussions

In this work we applied numerical software to analyses Riemann problems of the Shock – tube problems.
We run the simulation with the initial conditions in table 1.
A stability condition is required and, assuming a constant grid size (Δx), the time step (Δt) is calculated by using:

$$\Delta t = \frac{N_{CFL}\,\Delta x}{\max(|\lambda_i|)} \tag{10}$$

Where $(\max|\lambda_i|)$ is the maximum wave speed and N_{CFL} the required Courant (CFL) number. [5]
The numerical domain size (Δx) of 1 is divided into 1000 computational cells of length 0.001.
The computation has been performed using CFL number of 0.5.

Shock Tube Problems Results

The Sod shock tube is a Riemann problem used as a standard test problem, but in this case we did not exactly do the classical Sod problem because we do not consider any combustion reactions here, but are instead interested in the pattern of discontinuities.

We started with the initial conditions shown in table 1, and the specific heat capacity is chosen to be ($\gamma = 5/2$).
Applying the initial conditions and running the program leads to solution consisting of a shock wave propagates into the region of lower pressure, across which the density and pressure jump to higher values and all of the state variables are discontinuous. This is followed by a contact discontinuity, across which the density is again discontinuous but the velocity and pressure are constant. The third wave moves into the opposite direction and have a very different structure. This wave is called an expansion wave, and across which the materials streams in with a high density, low velocity, and high pressure.
Results of solution of Riemann problems for Euler equations is shown in figure 3, and figure 5. On the plot in figure 3 one can see the density jumps through the contact discontinuity while the speed of the flow and pressure are the same.
In the same figure, the expansion wave moves to the left into the high pressure region, while the shock and contact discontinuity move to the right into the low pressure region.

Using the shock jump conditions for the pre- and post shock states, one can calculate the Mach number characterizing the strength of the shock in the shock frame 1.48, and the shock velocity in the lab frame is 1.8 cm s^{-1}.
The time evolution of the right travelling shock and the left expansion wave shown in figure 4. In this figure the density profile is plotted for three instants in time.

The contact discontinuity in figure 5, travelling slowly to the right with speed of 0.8 cm s^{-1}, Shock moves fast to the right with speed of 1.83 cm s^{-1}, and the expansion wave goes to the left with speed of - 1.28 cm $\mathbf{s^{-1}}$.
If we compare the value for the shock here, it is close to the previous value. So, the numerical solution is close to the exact solution in case of shock tube problem.

Figure 5, illustrates results of Riemann problem of the shock tube for the Mach number μ in the lab frame, specific internal energy, total energy density, and entropy related quantity. It is clear that there is a jump in the Mach number, specific internal energy density, the total energy density, and the entropy related quantity across the shock and at the contact discontinuity.

Conclusions

This work discusses the propagation of shock waves through a shock tube. It was found that the gas to the left and right of the diaphragm is initially at rest. The pressure and density are discontinuous across the diaphragm. At t = 0, the diaphragm is broken. Three types of discontinuity then propagate through the gas:

• Contact discontinuity: The pressure p and velocity u are continuous, but the density ρ, Mach number, specific internal energy density, the total energy density, and the entropy related quantity are discontinuous.

• Shock waves: All quantities p, u, ρ, Mach number, specific internal energy density, the total energy density, and the entropy related quantity are in general discontinuous across the shock front.

• Expansion wave: which is basically the reverse of a shock wave.

It was found also that the numerical calculations of the velocities of the three discontinuities give similar speed of the 3 discontinuities by looking at a range of outputs (figure 4).

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Regions	Density ($g\ cm^{-1}\ s^{-1}$)	Velocity ($cm\ s^{-1}$)	Pressure ($g\ cm^{-1}\ s^{-2}$)
Right	0.125	0	0.1
Left	1.0	0	1.0

Table(1): Initial conditions for shock tube problems

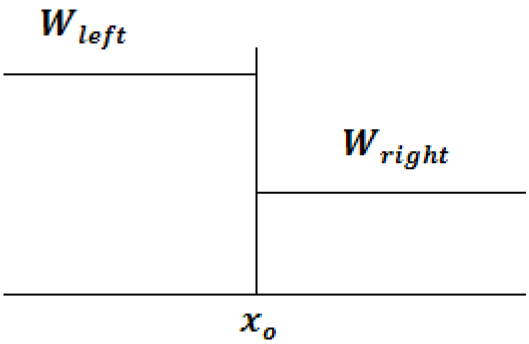
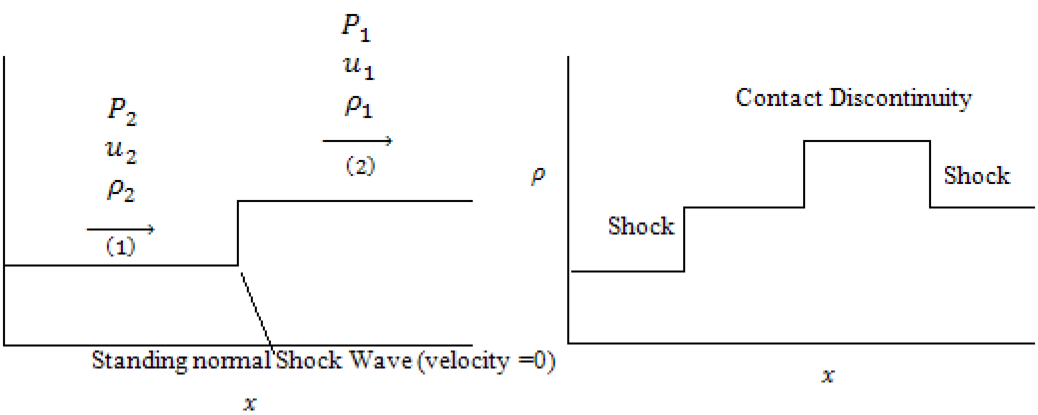


Fig. (1): The Riemann Problem



Fig(2): Collision of two flows

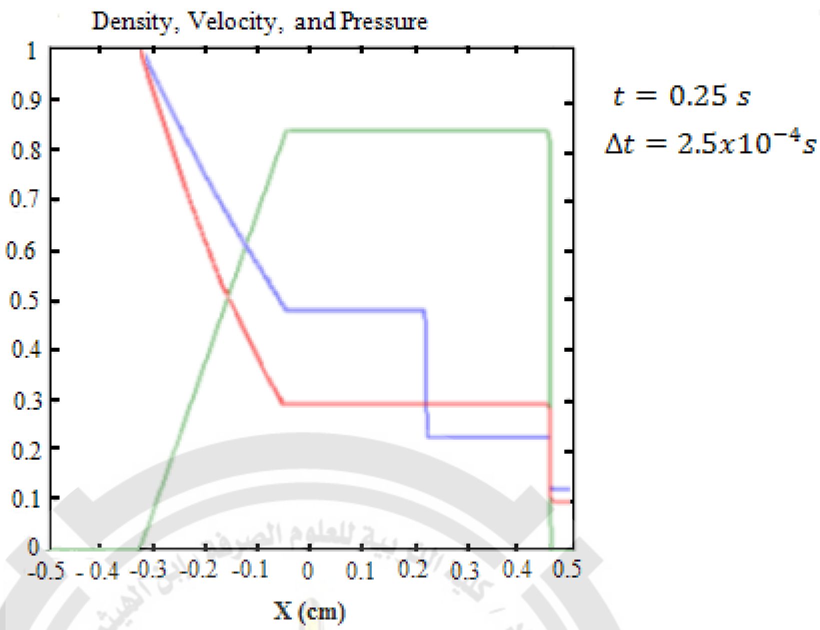


Fig.(3): Results of the shock tube problem. The figure is show the density profile in blue color, velocity profile in green color, and the pressure profile in red color. The velocity and pressure are continuous at the contact discontinuity. The size of the time step was controlled by the CFL condition with CFL number of 0.5

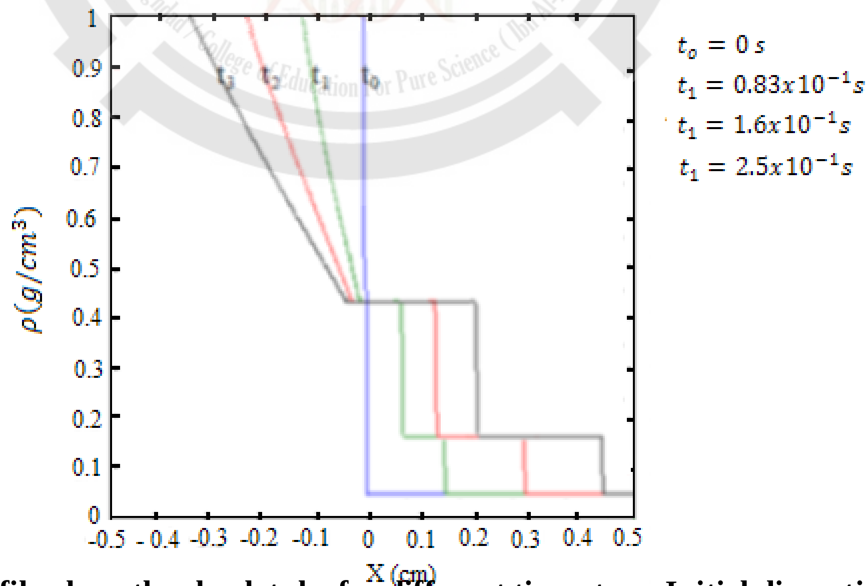


Fig.(4): Density profile along the shock tube for different time steps. Initial discontinuity at x = 0

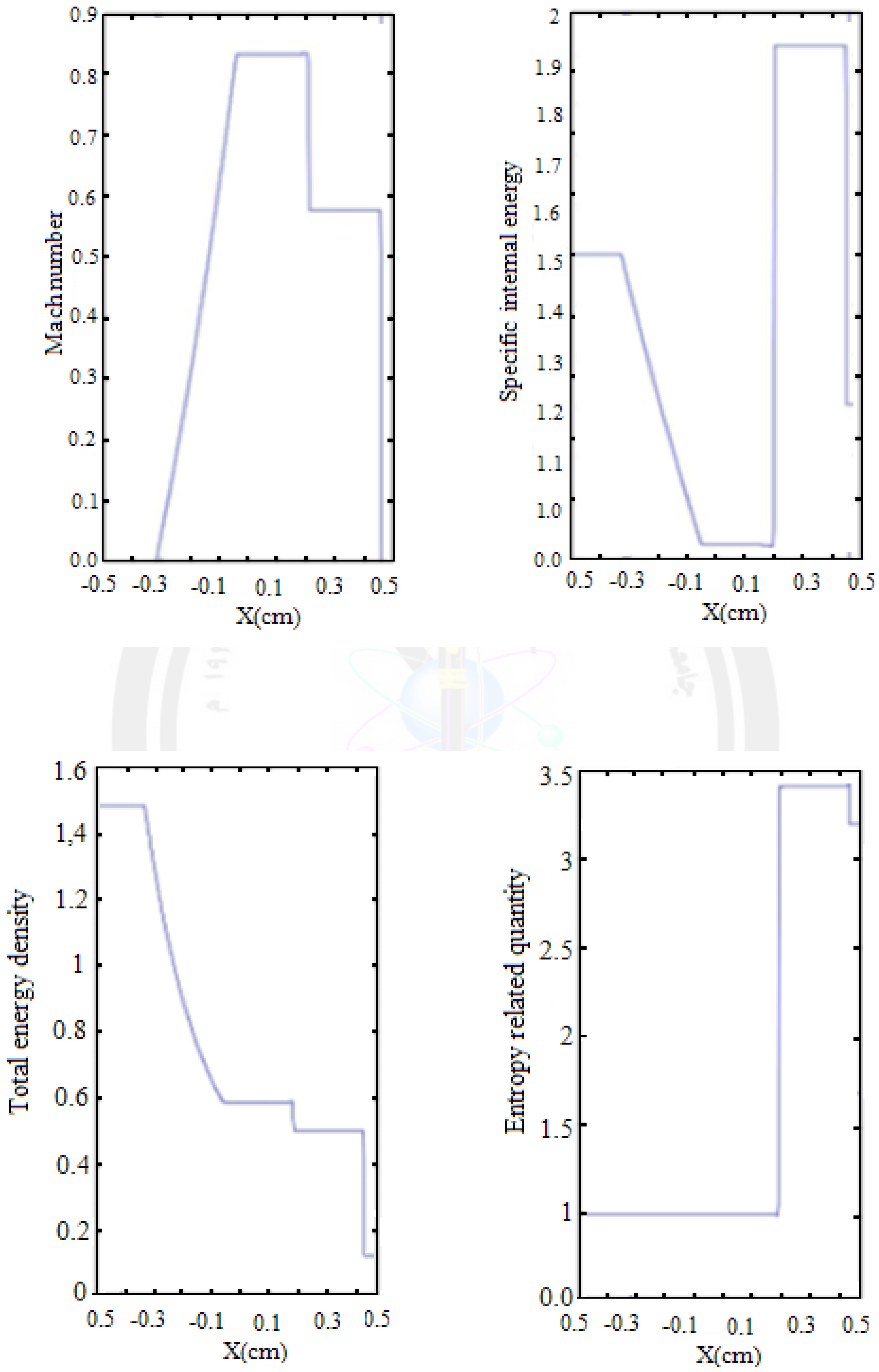


Fig.(5): The Mach number, specific internal energy, total energy density, and entropy related quantity, as a function of position

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تحليلات عددية لدراسة تأثيرات مشاكل ريمان في الخصائص الفيزيائية للغاز الداينميكي الفيزيائي الفلكي

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استلم البحث في 26 ايلول 2011 قبل البحث في 26 شباط 2012

الخلاصة

في هذا البحث نفذ برنامج تحليل عددي لمشاكل الغاز الداينميكي لدراسة تأثيرات مشاكل ريمان في الخصائص الفيزيائية لهذا الغاز.

لقد درست موجة اصطدام اعتيادية التي تسمى (Shock wave) منتشرة بسرعة عالية خلال وسط معين مثل (Shock tube). ما يسبب تغيير في خصائص ذلك الوسط ،اذ يحدث ارتفاع سريع لقيم السرعة، والكثافة، و الضغط. و عند مرور هذه لموجة خلال الوسط فان اجمالي الطاقة لايتبدد و لكن كمية الطاقة التي تتحول الى شغل هي التي سوف تتبدد و يحدث زيادة في عشوائية النظام ايضا".

كما ان زيادة الضغط داخل انبوب الاصطدام الذي يتسبب في تحطم الحاجز الذي يفصل بين جزئي الانبوب مولدا موجات اصطدام اعتيادية منتشرة الى مناطق الضغط العالي و موجات اخرى تنتقل الى مناطق الضغط الواطئ وتسمى (expansion waves).

الكلمات المفتاحية: مشاكل ريمان، تحليلات عددية، جريان الغاز الحقيقي