Weak N-open sets

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Abstract

In this paper we introduce new class of open sets called weak N-open sets and we study the relation between N-open sets , weak N-open sets and some other open sets. We prove several results about them.

Keywords: N-open, semi-open, α -open, b-open, β-open, ω -closed.

Introduction

Let (X,τ) be a topological space (or simply, a space) and $A \subseteq X$. Then the closure of A and the interior of A will be denoted by $cl(A)$ and $\mathrm{int}(A)$, respectively. A subset $A \subseteq X$ is called semi-open [1] if there exists an open set $O \in \tau$ such that $O \subseteq A \subseteq cl(O)$. Clearly A is semi-open if and only if $A \subseteq cl(\mathrm{int}(A))$. The complement of a semi-open set is called semi-closed [1]. A is called preopen [2] (respt. α-open [3],b-open[4] ,β-open[5]) if $A \subseteq \mathrm{int}(cl(A))$ (respt. $A \subseteq \mathrm{int}(cl(\mathrm{int}(A)))$, $A \subseteq cl(\mathrm{int}(A)) \bigcup \mathrm{int}(cl(A))$, $A \subseteq cl(\mathrm{int}(clA)))$). A space (X,τ) is called anti-locally countable [8] if every non-empty open subset is uncountable. In [6], the concept of ω-closed subsets was explored where a subset A of a space (X,τ) is ω-closed if it contains all of its condensation points (a point x is a condensation point of A if $A \cap U$ is uncountable for every open U containing x). The complement of an ω-closed set is called ω-open or equivalently for each $x \in A$, there exists $U \in \tau$ such that $x \in U$ and $U - A$ is countable. ω-closure and ω-interior, that can be defined in an analogous manner to $cl(A)$ and $\mathrm{int}(A)$, will be denoted by $cl_{\omega}(A)$ and $\mathrm{int}_{\omega}(A)$, respectively. For several characterizations of ω-closed subsets, see [7,8].

Weak N-open sets

Definition[8,9] 2.1: A subset A of a topological space (X, τ) is called:

1) pre-ω-open if $A \subseteq \mathrm{int}_{\omega}(cl(A))$.
2) α-ω-open if $A \subseteq \mathrm{int}_{\omega}(cl(\mathrm{int}_{\omega}(A)))$.
3) b-ω-open if $A \subseteq \mathrm{int}_{\omega}(cl(A)) \bigcup cl(\mathrm{int}_{\omega}(A))$.
4) β-ω-open if $cl(\mathrm{int}_{\omega}(cl(A)))$.
5) N-open subset of X if for any x in A there exists an open set U containing x such that U\A is finite.

Remark[9] 2.2:
Every open set is N-open and every N-open set is ω-open.
The converse of above remark may be not true in general as seen in the following examples.
Examples 2.3:
(1) Let N be the set of natural numbers with topology defined on it by
$\tau = \{U_i: U_i= \{i, i+1, i+2,\ldots\}, i \in N\} \bigcup \{\phi\}$, then $U_5 \bigcup \{3\}$ is N-open since for any $a \in U_5 \bigcup \{3\}$ there exists U_2 containing a and $U_2 \backslash (U_5 \bigcup \{3\})=\{2,4\}$ is finite, but $U_5 \bigcup \{3\}$ is not open.
(2) Let (R, τ_u) be usual topological space and Let Q be rational numbers then R\Q is ω-open Since for any x $\in$ R\Q there is open set (x-ε,x+ε) containing x and (x-ε,x+ε)\(R\Q) is countable, but not N-open, since every open set A containing x implies A\(R\Q) is infinite.
The family of all N-open sets in a topological space (X, τ) will be denoted by NO(X), and it is clear form a topology τ^N on X which is finer than τ.
Remark 2.4: For any finite topological space τ^N= P(X) (that is any subset of X is N-open).
Definition [9] 2.5: Let (X,τ) be a topological space. And A$\subseteq$X, then the union of all N-open contained in A is called N-interior and we will denote it by int_N (A).
Note that the N-interior is the maximal N-open which contains in A.
Proposition 2.6: Let (X,τ) be a topological space. And A,B$\subseteq$X, then

(1) $\text{int}(A) \subseteq \text{int}_N(A) \subseteq \text{int}_\omega(A) \subseteq A$
(2) A is N-open iff $\text{int}_N(A) = A$.
(3) $\text{int}_N(X) = X$ and $\text{int}_N(\phi) = \phi$
(4) $\text{int}_N(\text{int}_N(A)) = \text{int}_N(A)$.
(5) If $A \subseteq B$, then $\text{int}_N(A) \subseteq \text{int}_N(B)$.
(6) $\text{int}_N(A) \cup \text{int}_N(B) \subseteq \text{int}_N(A \cup B)$.
(7) $\text{int}_N(A \cap B) = \text{int}_N(A) \cap \text{int}_N(B)$.

Proof
As in the usual case
By the following example we note that in general $A \not\subset \text{int}_N(A)$ and
$\text{int}_N(A \cup B) \not\subset \text{int}_N(A) \cup \text{int}_N(B)$

Example 2.7: Let (R, τ_u) be the usual topological space and Let A=(2,3] and B=(3,4], then $\text{int}_N(A) = (2,3)$, $\text{int}_N(B) = (3,4)$, hence $A \not\subset \text{int}_N(A)$ and $\text{int}_N(A \cup B) = (2,4)$**,** and $\text{int}_N(A) \cup \text{int}_N(B) = (2,4) \setminus \{3\}$, therefore $\text{int}_N(A \cup B) \not\subset \text{int}_N(A) \cup \text{int}_N(B)$.

Definition[9] 2.8: Let (X,τ) be a topological space. And $A \subseteq X$, then the intersection of all N-closed sets containing A is called N-closure of A and we will denote it by $cl_N(A)$.

Note that the N-closure is the smallest N-closed containing A.

Proposition 2.9: Let (X,τ) be a topological space. And $A, B \subseteq X$, then
(1) $A \subseteq cl_\omega(A) \subseteq cl_N(A) \subseteq cl(A)$.
(2) A is N-closed iff $cl_N(A) = A$.
(3) $cl_N(X) = X$ and $cl_N(\phi) = \phi$.
(4) $cl_N(cl_N(A)) = cl_N(A)$.
(5) If $A \subseteq B$, then $cl_N(A) \subseteq cl_N(B)$.
(6) $cl_N(A \cup B) = cl_N(A) \cup cl_N(B)$.
(7) $cl_N(A \cap B) \subseteq cl_N(A) \cap cl_N(B)$.
(8) $\text{int}_N(X \setminus A) = X \setminus cl_N(A)$.
(9) $cl_N(X \setminus A) = X \setminus \text{int}_N(A)$.

Proof
As in the usual case

Proposition [9] 2.10: Let (X,τ) be a topological space. And $A \subseteq X$, then $x \in cl_N(A)$ iff for every N-open set U containing x, then $A \cap U \neq \phi$.
The following are new modified definition.

Definition 2.11: Let (X,τ) be a topological space. A subset A of X is called
(1) α - N-open if A$\subseteq \text{int}_N(cl(\text{int}_N(A)))$.
(2) pre-N-open if A$\subseteq \text{int}_N(cl((A))$.
(3) b- N-open if A$\subseteq \text{int}_N(cl(A)) \cup cl(\text{int}_N(A))$.
(4) β - N-open if A $\subseteq cl(\text{int}_N(cl(A)))$.
The complement of α - N-open (resp. pre-N-open, b- N-open, β - N-open) set is called α - N-closed (resp. pre-N- closed, b- N- closed, β - N- closed)

Proposition 2.12: In any topological space the following are satisfied
(1) Every N-open is α - N-open.
(2) Every α -N-open is pre-N-open.
(3) Every pre-N-open is b-N-open.
(4) Every b-N-open is β -N-open.

Proof (1) suppose that A is N-open, then by Proposition (2.9(2)) A=$\text{int}_N(A) \subseteq cl(\text{int}_N(A))$ and by Proposition(2.6(4,5)) A$\subseteq \text{int}_N(cl(\text{int}_N(A)))$, hence A is α - N-open.
(2) Suppose that A is α -N-open, then A$\subseteq \text{int}_N(cl(\text{int}_N(A))) \subseteq \text{int}_N(cl(A))$ (by Proposition(2.6(1))), hence A is pre-N-open.
(3) Suppose that A is pre-N-open, then A$\subseteq \text{int}_N(cl(A)) \subseteq \text{int}_N(cl(A)) \cup (cl(\text{int}_N(A))$ therefore A is b-N-open.
(4) Suppose that A is b-N-open, then A$\subseteq \text{int}_N(cl(A)) \cup (cl(\text{int}_N(A))$, if A$\subseteq \text{int}_N(cl(A))$, then A$\subseteq cl(\text{int}_N(cl(A)))$, and if A$\subseteq \text{int}_N(cl(A))$, then A$\subseteq cl(\text{int}_N(A)) \subseteq cl(\text{int}_N(cl(A)))$(by Proposition(2.9(1))), that is A is β -N-open.
The converse of proposition (2.12) may be not true in general to show that see the following examples

Examples 2.13: (1) Let X be infinite set and A,B,C and D be subsets of X such that each of them is infinite and the collection {A,B,C,D}be a partition of X, define the topology on X by $\tau=\{\phi$, X,A,B,{A,B},{A,B,C}},where {A,B,C}=A$\cup$B$\cup$C and that similarly for {A,B},{A,B,D}and {B,C,D}.
Then {A,B,D} is α -N-open but not N-open and {B,C,D} is b-N-open but not pre-N-open.
(2) In example (2.7) let A= Q$\cap$[0,1], then A is β -N-open but not b-N-open.
(3) In example (2.7) let A= Q, then A is pre-N-open but not α -N-open

Proposition 2.14: In any topological space the following are satisfied
(1) Every α -open set is α -N-open, and every α -N-open set is α - ω -open.
(2) Every pre-open set is pre-N-open, and every pre-N-open set is pre- ω -open.
(3) Every b-open set is b-N-open, and every b-N-open set is b- ω -open.
(4) Every β -open set is β -N-open, and every β -N-open set is β - ω -open.

Proof (1) suppose that A is α -open set, that is A$\subseteq$ int(cl(int (A))) and by proposition (2.9(1)) We have A$\subseteq$ int(cl(int (A)))$\subseteq \text{int}_N(cl(\text{int}_N(A)))$,then A is α -N-open and if A is α -N-open,
Also by proposition (2.9(1)) we have A$\subseteq \text{int}_N(cl(\text{int}_N(A))) \subseteq \text{int}_\omega(cl(\text{int}_\omega(A)))$, then A is α - ω -open.
similar proof for the other cases.

Remark 2.15: The following examples show that the converse of some points of proposition (2.14) may be not true in general

Examples 2.16: (1) In example (2.3(1)) Let A={1,2,3} ,then A is α-ω-open (since X=N is countable) but not β-N-open since U_4 is open and X\$U_4$=A then A is closed hence cl(A)=A and int_N (A)=ϕ since $1\in$A and only open set containing 1 is X and X\A not finite hence A is not N-open and does not contain N-open set unless ϕ, hence A$\not\subset cl(\text{int}_N(cl(\text{A})))=\phi$, therefore A is not β-N-open.

(2) Let X={1,2,3} and τ={X,ϕ, {a},{b},{a,b}}. Then {c} is an α-N-open but not β-open.

Lemma [8] 2.17: Let (X,τ) be a topological space, then the following properties hold:
1. Every ω-open set is α-ω-open.
2. Every α-ω-open set is pre-ω-open.
3. Every pre-ω-open set is b-ω-open.
4. Every b-ω-open set is β-ω-open.

The following diagram explains the relation among the above concepts.

$$\begin{tikzcd}
\text{open} \arrow[r, shift left] \arrow[d, shift left] & \alpha\text{-open} \arrow[l, shift left, "/" marking] \arrow[r, shift left] \arrow[d, shift left] & \text{pre-open} \arrow[l, shift left, "/" marking] \arrow[r, shift left] \arrow[d, shift left] & \text{b-open} \arrow[l, shift left, "/" marking] \arrow[r, shift left] \arrow[d, shift left] & \beta\text{-open} \arrow[l, shift left, "/" marking] \arrow[d, shift left] \\
\text{N-open} \arrow[u, shift left, "\backslash" marking] \arrow[r, shift left] \arrow[d, shift left] & \alpha\text{-N-open} \arrow[u, shift left, "\backslash" marking] \arrow[l, shift left, "/" marking] \arrow[r, shift left] \arrow[d, shift left] & \text{pre-N-open} \arrow[u, shift left, "\backslash" marking] \arrow[l, shift left, "/" marking] \arrow[r, shift left] \arrow[d, shift left] & \text{b-N-open} \arrow[u, shift left, "\backslash" marking] \arrow[l, shift left, "/" marking] \arrow[r, shift left] \arrow[d, shift left] & \beta\text{-N-open} \arrow[u, shift left, "\backslash" marking] \arrow[l, shift left, "/" marking] \arrow[d, shift left] \\
\omega\text{-open} \arrow[u, shift left, "\backslash" marking] \arrow[r, shift left] & \alpha\text{-}\omega\text{-open} \arrow[u, shift left, "\backslash" marking] \arrow[l, shift left, "/" marking] \arrow[r, shift left] & \text{pre-}\omega\text{-open} \arrow[u, shift left, "\backslash" marking] \arrow[l, shift left, "/" marking] \arrow[r, shift left] & \text{b-}\omega\text{-open} \arrow[u, shift left, "\backslash" marking] \arrow[l, shift left, "/" marking] \arrow[r, shift left] & \beta\text{-}\omega\text{-open} \arrow[u, shift left, "\backslash" marking] \arrow[l, shift left, "/" marking]
\end{tikzcd}$$

Definition 2.18[8]: A topological space (X,τ) is called door space if every subset of X is open or closed.

Proposition 2.19: If (X,τ) is door space , then every pre-N-open set is N-open.

Proof : Let A be pre-N-open. If A is open then it is N-open. Otherwise A is closed, then A$\subseteq \text{int}_N(cl(\text{A}))=\text{int}_N(\text{A})\subseteq$A, then by proposition(2.6(2)) A is N-open.

Proposition 2.20: Let (X,τ) be a topological space. And Let A be b-N-open such that int_N(A)=ϕ, then A is pre-N-open.

Proof It is clear

Lemma 2.21[10]: Let (X,τ) be a topological space. And U be an open set of X, then cl(U$\cap$A)= cl(U$\cap cl$(A)) and hence U$\cap cl$(A)$\subseteq cl$(U$\cap$A) for any subset A of X.

Proposition 2.22: A subset U of a topological space (X,τ) is pre-N-open set iff there exists a pre-N-open set A such that U$\subseteq$A$\subseteq cl$(U).

Proof since U$\subseteq$A$\subseteq \text{int}_N(cl(\text{A}))$, also cl(A) $\subseteq cl(cl(\text{U}))=cl$(U), and by proposition(2.6(5)) we have $\text{int}_N(cl(\text{A}))\subseteq \text{int}_N(cl(\text{U}))$, that is U$\subseteq \text{int}_N(cl(\text{U}))$, hence U is pre-N-open set.

Conversely: suppose that U is pre-N-open. If we take A=U, then A is pre-N-open set such that U$\subseteq$A$\subseteq cl$(U).

Proposition 2.23: A subset A of a topological space (X,τ) is semi open iff A is β-N-open and $\operatorname{int}_N(cl(A)) \subseteq cl(\operatorname{int}(A))$.

Proof Let A be semi open set, then $A \subseteq cl(\operatorname{int}(A)) \subseteq cl(\operatorname{int}_N(A)) \subseteq cl(\operatorname{int}_N(cl(A)))$ (by proposition (2.9(1)), hence A is β-N-open. Now, $cl(A) \subseteq cl(\operatorname{int}(A))$, then $\operatorname{int}_N(cl(A)) \subseteq cl(\operatorname{int}(A))$ (by proposition (2.9(1)).

Conversely: $A \subseteq cl(\operatorname{int}_N(cl(A))) \subseteq cl(cl(\operatorname{int}(A))) = cl(\operatorname{int}(A))$, hence A is semi open set.

Proposition 2.24: In any topological space the intersection of a β-N-open set and open set is β-N-open.

Proof Let Ube an open set and A be a β-N-open since every open set is N-open, then by lemma (2.21) we have $U \cap A \subseteq U \cap cl(\operatorname{int}_N(A)) \subseteq cl(U \cap cl(\operatorname{int}_N(cl(A)))$

$$= cl(\operatorname{int}_N(U) \cap \operatorname{int}_N(cl(A)))$$

$$= cl(\operatorname{int}_N(U \cap cl(A))) \quad \text{(by proposition 2.9(7))}$$

That is $U \cap A \subseteq U \cap cl(\operatorname{int}_N(A)) \subseteq cl(\operatorname{int}_N(cl(U \cap A)))$.

Hence $U \cap A$ is β-N-open.

Proposition 2.25: In any topological space the intersection of a b-N-open set and an open set is b-N-open.

Proof Let A be a b-N-open and U be an open set, then

$$U \cap A \subseteq U \cap [\operatorname{int}_N(cl(A)) \cup cl(\operatorname{int}_N(A))] = [U \cap \operatorname{int}_N(cl(A))] \cup [U \cap cl(\operatorname{int}_N(A))]$$

$$= [\operatorname{int}_N(U) \cap \operatorname{int}_N(cl(A))] \cup [U \cap cl(\operatorname{int}_N(A))]$$

$$\subseteq [\operatorname{int}_N(U \cap cl(A))] \cup [cl(U \cap \operatorname{int}_N(A))]$$

$$\subseteq \operatorname{int}_N(cl(U \cap A)) \cup cl(\operatorname{int}_N(U \cap A)).$$

Hence $U \cap A$ is b-N-open.

Proposition 2.26: In any topological space the intersection of a α-N-open set and an open set is α-N-open.

Proof: Let A be an α-N-open set and U be an open set, then

$$U \cap A \subseteq \operatorname{int}_N(U) \cap \operatorname{int}_N(cl(\operatorname{int}_N(A))) \subseteq \operatorname{int}_N(U \cap cl(\operatorname{int}_N(A)))$$

$$\subseteq \operatorname{int}_N(cl(U \cap \operatorname{int}_N(A))$$

$$\subseteq \operatorname{int}_N(cl(\operatorname{int}_N(U \cap A))).$$

Therefore $U \cap A$ is α-N-open.

Remark 2.27: The intersection of two pre-N-open (resp. b-N-open, β-N-open) set need not be pre-N-open (resp. b-N-open, β-N-open) in general to show that for pre-N-open see the following example.

Example 2.28: In example (2.7) let A=Q and B= (R\Q) $\cup \{1\}$. Then A and B are pre-N-open , b-N-open and β-N-open sets but $A \cap B = \{1\}$ is not pre-N-open set since $\operatorname{int}_N(cl(\{1\})) = \phi$

$A \cap B$ is not b-N-open since $\operatorname{int}_N(cl(\{1\})) \cup cl(\operatorname{int}_N(\{1\})) = \phi \cup \phi = \phi$.

$A \cap B$ is not β-N-open since $cl(\operatorname{int}_N(cl(\{1\}))) = cl(\operatorname{int}_N(\{1\})) = cl(\phi) = \phi$.

Proposition 2.29: In any topological space the union of any family of b-N-open (resp. pre-N-open, β-N-open) set is b-N-open (resp. pre-N-open, β-N-open).

Proof: Let $\{A_\alpha\}_{\alpha\in\Lambda}$ be a family of b-N-open sets, since $A_\alpha \subseteq \operatorname{int}_N(cl(A_\alpha)) \cup cl(\operatorname{int}_N(A_\alpha))$ $\forall \alpha \in \Lambda$, then $\bigcup_{\alpha\in\Lambda} A_\alpha \subseteq \bigcup_{\alpha\in\Lambda} [\operatorname{int}_N(cl(A_\alpha)) \cup cl(\operatorname{int}_N(A_\alpha))]$

$$\subseteq [\bigcup_{\alpha\in\Lambda} \text{int}_N (cl(A_\alpha))] \cup [\bigcup_{\alpha\in\Lambda} cl(\text{int}_N (A_\alpha))]$$
$$\subseteq \text{int}_N (\bigcup_{\alpha\in\Lambda} (cl(A_\alpha))) \cup cl(\bigcup_{\alpha\in\Lambda} (\text{int}_N (A_\alpha)))$$
$$\subseteq \text{int}_N (cl(\bigcup_{\alpha\in\Lambda} (A_\alpha))) \cup cl(\text{int}_N (\bigcup_{\alpha\in\Lambda} (A_\alpha))).$$

Hence $\bigcup_{\alpha\in\Lambda} A_\alpha$ is b-N-open.(A similar proof for the other cases).

Contra N-continuous

Definition 3.1: A function f from a topological space (X,τ) into a topological space (Y, τ') is Called:

1- Contra-continuous if the inverse image of each open subset of Y is closed subset of X [11].
2- ω-continuous if the inverse image of each open subset of Y is ω-open subset of X [6].
3- Contra-ω-continuous if the inverse image of each open subset of Y is ω-closed subset of X [13].
4- N-continuous if the inverse image of each open subset of Y is N-open subset of X [9].
5- Contra N-continuous inverse image of each open subset of Y is N-closed subset of X.

Since every closed set is N-closed, then every contra continuous is contra-N-continuous, and since every N-closed is ω-closed, then every contra-N-continuous is co-ω-continuous. But in general: the converse of above may be not true, show the following examples

Examples 3.2:

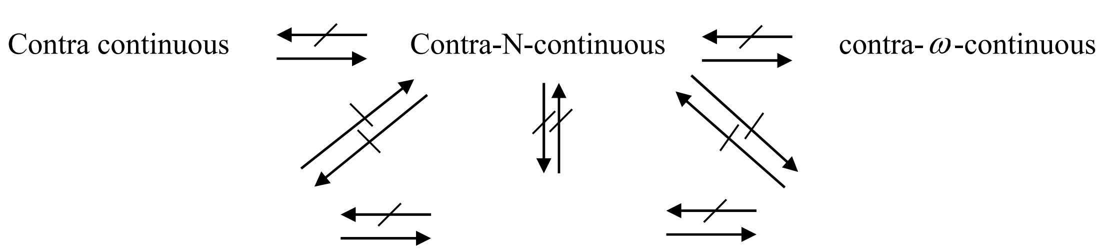
1- Let *f* be the identity function from the set of natural numbers with indiscrete topology onto itself with the discrete topology, then *f* is contra-N-continuous but not contra-continuous.
2- Let *f* be the identity function from the set of rational numbers with indiscrete topology onto itself with the discrete topology, then *f* is ω-continuous and contra-ω-continuous but *f* is not contra-N-continuous.
3- Let X={1,2,3}, Y={a,b}, $\tau_X = \{X, \phi, \{2\}\}$ and $\tau_Y = \{Y, \phi, \{b\}\}$.
Define *f* from X into Y by *f*(1)=a, *f*(2)=*f*(3)=b, then *f* is contra-N-continuous but not continuous.

4 -Consider the two functions *f* and *g* from usual topological space into space $Y=\{0,1\}$ with
topology defined by $\tau_Y = \{Y, \phi, \{0\}\}$ defined by

$$f(x)=\begin{cases} 0 & x\in R\setminus A \\ 1 & x\in A \end{cases} \qquad g(x)=\begin{cases} 1 & x\in R\setminus A \\ 0 & x\in A \end{cases}$$ where A is finite set in R.

Then *f* is continuous but not contra-N-continuous and g is contra- N-continuous but not ω-continuous

The following diagram explains the relation among the above concepts.



Continuous N-continuous ω-continuous

Definition [13] 3.3: Let A be a subset of a topological space (X,τ). The kernel of A is the set defined as $\ker(A) = \bigcap\{U \in \tau : A \subseteq U\}$

Lemma [13] 3.4: The following properties hold for subsets A and B of a topological space (X,τ)

1- $x \in \ker(A)$ iff $A \cap F = \phi$ for any closed subset F containing x.

2- $A \subseteq \ker(A)$ and $A = \ker(A)$ if A is open in X.

3- If $A \subseteq B$, then $\ker(A) \subseteq \ker(B)$.

Theorem 3.5: Let f be a function from topological (X,τ) into a topological space (Y,σ). Then the following are equivalent:-

1- f is contra N-continuous.

2- For every closed subset F of Y, f^{-1}(F) is N-open.

3- For each $x \in X$ and each closed subset F of Y containing $f(x)$, there exists an N-open U containing x such that $f(U) \subseteq F$.

4- $f(cl_N(A)) \subseteq \ker(f(A))$ for all $A \subseteq$ X.

5- $cl_N(f^{-1}(B)) \subseteq f^{-1}(\ker(B))$ for all $B \subseteq Y$.

Proof ($1 \Rightarrow 2$) Let F$\subseteq$Y closed set. Then $Y \setminus F$ is open and since f is contra N-continuous, then $f^{-1}(Y \setminus F)$ is N-closed in X, but $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$, that is, $f^{-1}(F)$ is N-open.

($2 \Rightarrow 3$)Let $x \in X$ and $F \subseteq Y$ such that $f(x) \in F$, we have $x \in f^{-1}(F)$ which is N-open set Put $U = f^{-1}(F)$, then we have $f(U) = f(f^{-1}(F)) \subseteq F$, hence $f(U) \subseteq F$.

($3 \Rightarrow 4$)Suppose that, there exists $y \in f(cl_N(A))$ and $y \notin \ker(f(A))$ for some subset A of X, then $y = f(x)$ for some $x \in cl_N(A))$, hence there exists a closed set $F \subseteq Y$ such that $y \in F$ and $F \cap f(A) = \phi$, thus by (3) there exist $U \in \tau^N$ such that $f(U) \subseteq F$, then $U = f^{-1}(f(U)) \subseteq f^{-1}(F)$, hence $U \cap A = \phi$ and $x \notin cl_N(A)$, that is $y = f(x) \notin f(cl_N(A))$ Which is contradiction.

($4 \Rightarrow 5$) Let $B \subseteq Y$. Then $\ker(f(f^{-1}(B)) \subseteq \ker(B)$ and by (4) we have $f(cl_N(f^{-1}(B))) \subseteq \ker(f(f^{-1}(B))) \subseteq \ker(B)$, hence $cl_N(f^{-1}(B)) \subseteq f^{-1}(\ker(B))$.

($5 \Rightarrow 1$) Let V be any open of Y, then by lemma and the assumption $cl_N(f^{-1}(V)) \subseteq f^{-1}(\ker(V)) = f^{-1}(V)$, therefore $cl_N(f^{-1}(V)) \subseteq f^{-1}(V)$. Hence $f^{-1}(V) = cl_N(f^{-1}(V))$ and then f is contra N-continuous.

References

1. Levine, N. (1963); Semi-open sets and semi-continuity in topological spaces, Math. Monthly., 70, 36-41.

2.Mashhour, A.S.; Abd El-Monsef, M.E. and El.Deeb, S.N. (1982), on pre-continuous and week precontinuous mappings, Proc. Math and phys. Soc. Egypt., 53, 47-53.

3. Njastad, O. (1965); On some classes of nearly open sets, Pacific J. Math., 15, 961-970.

4. Andrijević, D. (1996), On b-open sets, Math.. Vesik., 48, 59-64.

5. Abd El-Monsef, M., El-Deeb S.; and Mahmoud R. (1983), β-open sets and β-continuous mappings, Ball.Fac. Sci. Assuit Univ., 12, 77-90.

6. Hdeib, H. (1989), ω-Continuous functions, Dirasat., 16, 136-142.

7. Al-Hawary, T. (2004), ω–generalized closed sets, international Journal of Applied Mathematics., 16(3), 341-353.

8. Noiri, T.; Al-Omari A. and Noorani S. (2009), Weak forms of ω-open sets and decompositions of continuity, Eur. J. Pure Appl. Math., 2, 73-84.

9. Al-Omari A. and Noorani M. S. Md. (2009), New characterization of compact space, proceedings of the 5th Asian Mathematical conference, Malaysia.

10. Engelking, R. (1989); General Topology, Heldermann Veriag Berlin, 2nd edition.

11. Dontchev, J. (1996); Contra-continuous function and strongly S-closed spaces, Internat. J. Math. And Math. Sci., 19(2), 303-310.

12 Al-Hawary, T. (2007), ω-continuous like mappings, Al-Manarah., 13(6), 135-147.

13. Mrsevic, M. (1978);On pairwise R and pairwise RI bitopological spaces, Bull. Math. Soc. Sci. Math. R.S. Roumanie., 30, 141-148.

المجموعات المفتوحة الضعيفة من النمط N

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الخلاصة

في هذا البحث نقدم فئة جديدة من المجموعات المفتوحة تسمى المجموعات المفتوحة من النمط N والمجموعات المفتوحة الضعيفة N وقد تمت دراسة العلاقة بين هذه المجموعات المفتوحة ومجموعات مفتوحة أخرى. وتم اثبات عدة نتائج عنهم.

الكلمات المفتاحية: المجموعات المفتوحة من النمط N, المجموعات شبه المفتوحة, المجموعات المفتوحة من النمط α, المجموعات المفتوحة من النمط β, المجموعات المفتوحة من النمط ω, المجموعات المفتوحة من النمط b.