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On Double Stage Shrinkage-Bayesian Estimator for the Scale Parameter of Exponential Distribution

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Abstract

This paper is concerned with Double Stage Shrinkage Bayesian (DSSB) Estimator for lowering the mean squared error of classical estimator $\hat{\theta}$ for the scale parameter (θ) of an exponential distribution in a region (R) around available prior knowledge (θ_0) about the actual value (θ) as initial estimate as well as to reduce the cost of experimentations.

In situation where the experimentations are time consuming or very costly, a Double Stage procedure can be used to reduce the expected sample size needed to obtain the estimator.

This estimator is shown to have smaller mean squared error for certain choice of the shrinkage weight factor $\psi(\cdot)$ and for acceptance region R.

Expression for Bias, Mean Square Error (MSE), Expected sample size [E(n/θ,R)], Expected sample size proportion [E(n/θ,R)/n], probability for avoiding the second sample [$p(\hat{\theta}_1 \in R)$] and percentage of overall sample saved [$\frac{n_2}{n} p[\hat{\theta}_1 \in R) * 100$] for the proposed estimator are derived.

Numerical results and conclusions are established when the consider estimator (DSSB) are testimator of level of significance α.

Comparisons with the classical estimator as well as with some existing studies were made to show the usefulness of the proposed estimator.

Key Words: Exponential distribution, Maximum likelihood estimator, Bayesian estimator, Double stage shrinkage estimator, Mean square error, Relative Efficiency.

Introduction

1.1 The Model:

Exponential distribution is one of the most useful and widely exploited model, Epstein [1] remarks that the exponential distribution plays as important a role in life experiments as the part played by the normal distribution in agricultural experiments. It is applied in a very wide variety of statistical procedures. Among the most prominent applications are those in the field of life testing and reliability theory. The scale parameter (θ) is known as mean life time. The maximum likelihood estimator (MLE; $\hat{\theta}$) is the sample mean which is the minimum variance unbiased estimator.

The one parameter exponential distribution has the following probability density function (p.d.f.)

$$f(t;\beta) = \begin{cases} \beta \exp(-\beta t) & , t \geq 0, \beta > 0 \\ 0 & , \text{o.w.} \end{cases} \qquad \ldots(1)$$

Also, the above p.d.f can be written as below:-

$$f(t;\theta)=\begin{cases}\frac{1}{\theta}\exp(\frac{-t}{\theta}) & ,t\geq 0,\ \theta>0\\ 0 & ,\text{o.w.}\end{cases} \qquad \ldots(2)$$

where θ is the average or the mean life or mean time to failure (MTTF) and it also acts as scale parameter, while $\beta = 1/\theta$ is called the hazard rate or the mean arrival rate (MAR), see [1].

Furthermore, the Reliability function R(t) is defined as:
$R(t) = \exp(-t/\theta)$, $t > 0$, $\theta > 0$.

Note that the maximum likelihood estimator $\hat{\theta}$ of the scale parameter θ of the mentioned distribution is $\bar{t}=\frac{\sum_{i=1}^{n} t_i}{n}$.

1.2 Bayesian Estimator [2]

Consider the one parameter exponential distribution

$$f(t;\theta)=\begin{cases}\frac{1}{\theta}\exp(\frac{-t}{\theta}) & ,\text{for } t\geq 0,\ \theta>0\\ 0 & ,\text{o.w.}\end{cases}$$

Based on the rule proposed by Jeffery, one can get the prior distribution of θ [$g(\theta)$] as below,

$g(\theta) \propto \sqrt{I(\theta)}$, where $I(\theta)$ is fisher information such that

, see [2] $I(\theta) = -n\,E\left(\frac{\partial^2 \ln f(t,\theta)}{\partial\theta^2}\right)=\frac{n}{\theta^2}$

$$\therefore\ g(\theta) \propto \frac{\sqrt{n}}{\theta} \Rightarrow g(\theta) = k\frac{\sqrt{n}}{\theta}$$

$$L(t_1, t_2,\ldots,t_n) = \prod_{i=1}^{n} f(t_i \mid \theta) = \frac{1}{\theta^n}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)$$

The joint probability density function $H(t_1, t_2,\ldots,t_n,\theta)$ is given by

$$\begin{aligned} H(t_1, t_2,\ldots,t_n,\theta) &= \prod_{i=1}^{n} f(t_i,\theta)\, g(\theta)\\ &= L(t_1, t_2,\ldots,t_n \mid \theta)\, g(\theta)\\ &= \frac{1}{\theta^n}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)\frac{k\sqrt{n}}{\theta}\end{aligned}$$

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$$= \frac{k\sqrt{n}}{\theta^{n+1}} \exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) \qquad \ldots(3)$$

the marginal probability density function of $(t_1, t_2,\ldots,t_n)$ is given by

$$p(t_1, t_2,\ldots,t_n) = \int_{\theta} H(t_1, t_2,\ldots,t_n,\theta)\, d\theta$$

$$= \int_0^{\infty} \frac{k\sqrt{n}}{\theta^{n+1}} \exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) d\theta$$

$$= \frac{\left(k\sqrt{n}\right)(n-1)!}{\left(\sum_{i=1}^{n} t_i\right)^n}$$

And the condition probability density function of θ given the data $(t_1, t_2,\ldots,t_n)$ is given by

$$\pi(\theta \mid t_1, t_2,\ldots,t_n) = \frac{H(t_1,t_2,\ldots,t_n,\theta)}{p(t_1,t_2,\ldots,t_n)}$$

$$= \frac{\exp\left[-\frac{\sum_{i=1}^{n} t_i}{\theta}\right]\left[\sum_{i=1}^{n} t_i\right]^n}{\theta^{n+1}\,(n-1)!}$$

using squared error loss function

$$L(\hat{\theta},\theta) = c(\hat{\theta}-\theta)^2$$

We can give Risk function, such that

$$R(\hat{\theta},\theta) = E[L(\hat{\theta},\theta)]$$

$$= \int_0^{\infty} L(\hat{\theta},\theta)\,\pi(\theta \mid t_1,t_2,\ldots,t_n)\, d\theta$$

$$= \int_0^\infty c(\hat{\theta}-\theta)^2 \frac{\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)\left(\sum_{i=1}^{n} t_i\right)^n}{\theta^{n+1}\,(n-1)!}\,d\theta$$

$$= c\hat{\theta}^2 - 2c\hat{\theta}\int_0^\infty \frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!}\,\theta^{-n}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)d\theta + f(\theta)$$

$$\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 2c\hat{\theta} - 2c\frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!}\int_0^\infty \theta^{-n}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)d\theta + \text{zero}$$

Let $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 0$, then

$$\hat{\theta}_B = \frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!}\int_0^\infty \theta^{-n}\exp\left[-\frac{\sum_{i=1}^{n} t_i}{\theta}\right]d\theta$$

$$= \frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!}\int_0^\infty \left[\frac{\sum_{i=1}^{n} t_i}{y}\right]^{-n}\exp(-y)\left(\frac{\sum_{i=1}^{n} t_i}{y^2}\right)dy$$

$$= \frac{\left(\sum_{i=1}^{n} t_i\right)}{(n-1)!}\int_0^\infty y^{n-2}\exp(-y)\,dy$$

$$\hat{\theta}_B = \frac{\left(\sum_{i=1}^{n} t_i\right)(n-2)!}{(n-1)!}$$

$$\therefore\ \hat{\theta}_B = \frac{\sum_{i=1}^{n} t_i}{n-1} \qquad \text{(Bayes estimator)} \qquad \ldots(5)$$

where $E(\hat{\theta}_B) = \frac{n}{n-1}\theta$

$$\text{Biased}(\hat{\theta}_B) = E(\hat{\theta}_B) - \theta = \frac{\theta}{n-1} \qquad \ldots(6)$$

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$$MSE(\hat{\theta}_B) = E(\hat{\theta}_B - \theta)^2 = \frac{(n+1)}{(n-1)^2}\theta^2$$ …(7)

1.3 Double Stage Shrinkage Estimator [3], [4], [5], [6]

A Double Stage Shrinkage Estimator is defined as follows

Let x_{1i}; i = 1, 2, …, n_1 be a random sample of n_1 from exponential distribution and $\hat{\theta}_1$ be a classical estimator (MLE) of θ based on n_1 observation. Construct a preliminary test region (R) in the parameter space based on prior estimate θ_0 and an appropriate criterion.

If $\hat{\theta}_1 \in R$ shrink $\hat{\alpha}_1$ towards α_0 by shrinkage weight factor $\psi(\hat{\theta}_1)$; $0 \le \psi(\hat{\theta}_1) \le 1$ and use the shrinkage estimator $\psi(\hat{\theta}_1)\hat{\theta}_1 + (1 - \psi(\hat{\theta}_1))\theta_0$, to estimate θ, see [5].

If $\hat{\theta}_1 \notin R$, obtain x_{2i}; i = 1, 2, …, n_2, an additional sample of size n_2 and use a pooled estimator $\hat{\alpha}_p$ of α based on combined sample of size $n = n_1 + n_2$,

i.e.; $\hat{\theta}_p = \dfrac{n_1\hat{\theta}_1 + n_2\hat{\theta}_2}{n}$.

Thus, the Double Stage Shrinkage Estimator (DSSE) will be

$$\tilde{\theta} = \begin{cases} \psi(\hat{\theta}_1)\hat{\theta}_1 + (1-\psi(\hat{\theta}_1))\theta_0 & ;\text{if } \hat{\theta}_1 \in R \\ \hat{\theta}_p & ;\text{if } \hat{\theta}_1 \notin R \end{cases}$$
…(8)

To motivation of this study was provided by the work of [3], [4], [5], [6], [7], [8] and [9] and others.

The aim of this paper is to employ Bayesian estimator which is defined in (5) in double stage shrinkage estimator (DSSE) which is defined in (8) to estimate the scale parameter (θ) of exponential distribution.

The expressions for Bias, Mean Square Error [MSE], Relative Efficiency [R.Eff(·)], Expected sample size, Expected sample size proportion, probability for avoiding the second sample and percentage of overall sample saved are derived and obtained for the proposed estimator.

Numerical results and conclusions due mentioned expressions including some constants are performed and displayed in annexed tables.

Comparisons between the proposed estimator with the classical estimator ($\hat{\theta}$) and with some of the last studies are demonstrated.

2. Double Stage Shrinkage-Bayesian Estimator

This section is concerned with pooling approach between shrinkage estimation that uses a prior information about unknown parameter as initial value and Bayesian estimation that uses a prior information about unknown parameter as a prior distribution for the scale parameter (θ) of exponential distribution using different shrinkage weight factors as well as pretest region R when a prior information about (θ) is available as initial value (θ_0).

A proposed Double Stage Shrinkage-Bayesian Estimator (DSSBE) has the following form

$$\tilde{\theta}_{SB} = \begin{cases} \psi(\hat{\theta}_1)\hat{\theta}_{1B} + (1-\psi(\hat{\theta}_1))\theta_0 & ;\text{if } \hat{\theta}_1 \in R \\ \hat{\theta}_p = \dfrac{n_1\hat{\theta}_1 + n_2\hat{\theta}_2}{n} & ;\text{if } \hat{\theta}_1 \notin R \end{cases}$$

…(9)

where $\hat{\theta}_{1B}$ represent to Bayes estimator for θ on n_1 observation, R is suitable region (say pretest region) and $\psi(\hat{\theta}_1)$; $0 \le \psi(\hat{\theta}_1) \le 1$ is shrinkage weight factor which may be a function of $\hat{\theta}_1$ or constant, see [3], [4] and [9].

2.1 DSSBE($\tilde{\theta}_{SB}$) Using Constant Shrinkage Weight Factor

Using the form (9), the proposed DSSBE $\tilde{\theta}_{SB}$ has the following forms:

$$\tilde{\theta}_{SB} = \begin{cases} \hat{\theta}_{1B} & ,\text{ if } \hat{\theta}_1 \in R \\ \hat{\theta}_p & ,\text{ if } \hat{\theta}_1 \notin R \end{cases}$$

…(10)

i.e. $\psi_1(\hat{\theta}_1) = 0$ (constant).

where R is pretest region of acceptance of size α for testing the hypothesis $H_0: \theta = \theta_0$ Vs. the hypothesis $H_A: \theta \ne \theta_0$ using test statistic $T(\hat{\theta}_1|\theta) = \dfrac{2n_1\hat{\theta}_1}{\theta_0}$

In that,

$$R = \left[\frac{\theta_0}{2n_1}X^2_{1-\alpha/2,2n_1}, \frac{\theta_0}{2n_1}X^2_{\alpha/2,2n_1}\right]$$

…(11)

Assume that, R=[*a*,*b*], *a* < *b*.

i.e. $a = \dfrac{\theta_0}{2n_1}X^2_{1-\alpha/2,2n_1}$ and $b = \dfrac{\theta_0}{2n_1}X^2_{\alpha/2,2n_1}$

…(12)

where $X^2_{1-\alpha/2,2n_1}$ and $X^2_{\alpha/2,2n_1}$ are respectively lower and upper $100(\alpha/2)$ percentile point of Chi-square distribution with degree of freedom ($2n_1$).

The expression for Bias is given below

$$\text{Bias}(\tilde{\theta}_{SB}|\theta, R) = E(\tilde{\theta}_{SB}) - \theta$$

$$= \int_R (\hat{\theta}_{1B} - \theta_0) f(\hat{\theta}_1;\theta)\,d\hat{\theta}_1 + \int_{\bar{R}} (\hat{\theta}_p - \theta) f(\hat{\theta}_1;\theta)\,d\hat{\theta}_1$$

where $\bar{R}$ is the complement region of R in real space and $f(\hat{\theta}_1;\theta)$ is a p.d.f. of $\hat{\theta}_1$ which has the following form:-

$$f(\hat{\theta}_1;\theta) = \begin{cases} \dfrac{[\hat{\theta}_1]^{n_1-1}\exp[-n_1\hat{\theta}_1/\theta]}{\Gamma(n_1)(\theta/n_1)^{n_1}} & ,\text{for } 0 < \hat{\theta}_1 < \infty \\ 0 & ,\text{o.w} \end{cases}$$

we conclude:

…(13) $$\text{Bias}(\tilde{\theta}_{SB}|\theta,R)=\theta\left\{\frac{n_1}{n_1-1}J_1(a^*,b^*)-J_0(a^*,b^*)-\frac{1}{1+u}\left[J_1(a^*,b^*)-J_0(a^*,b^*)\right]\right\}$$

where,

$\lambda=\theta_0/\theta$, $y=n_1\hat{\theta}_1|\theta$, $u=n_2/n_1$, $n=n_1+n_2$, $J_l(a^*,b^*)=\frac{1}{n_1^l\Gamma(n_1)}\int_{a^*}^{b^*}y^l y^{n_1-1}e^{-y}dy$ …(14)

and $a^*=\lambda\chi^2_{1-\alpha/2,2n_1}$, $b^*=\lambda\chi^2_{\alpha/2,2n_1}$ …(15)

The Bias ratio B(·) of $\tilde{\theta}_{SB}$ is defined as below …(16)

$$B(\tilde{\theta}_{SB}|\theta,R)=\frac{\text{Bias}(\tilde{\theta}_{SB}|\theta,R)}{\theta}$$

See [6] and [9].

The expression of mean square error [MSE] of $\tilde{\theta}_{SB}$ is as follows ….17

$$\begin{aligned}\text{MSE}(\tilde{\theta}_{SB}|\theta,R)&=E(\tilde{\theta}_{SB}-\theta)^2\\&=\theta^2\left\{\left(\frac{n_1}{n_1-1}\right)^2J_2(a^*,b^*)-2\frac{n_1}{n_1-1}J_1(a^*,b^*)+J_0(a^*,b^*)+\right.\\&\left[\left(\frac{1}{1+u}\right)^2\frac{1}{n_1}+\left(\frac{u}{1+u}\right)^2\frac{1}{n_1u}\right]-\left(\frac{1}{1+u}\right)^2\left[J_2(a^*,b^*)-2J_1(a^*,b^*)+J_0(a^*,b^*)\right]-\\&\left.\left(\frac{u}{1+u}\right)^2\frac{1}{n_1u}J_0(a^*,b^*)\right\}\end{aligned}$$

The Relative Efficiency of estimator $\tilde{\theta}_{SB}$ with respect to classical estimator ($\hat{\theta}_1$) is defined as below

…(18) $$\text{R.Eff}(\tilde{\theta}_{SB}|\theta,R)=\frac{\text{MSE}(\hat{\theta})}{\text{MSE}(\tilde{\theta}_{SB}|\theta,R)[E(n|\theta,R)/n]}$$

where E(n|α,R) is the Expected sample size, which is defined as:

$$E(n|\theta,R)=n\left[1-\frac{u}{1+u}J_0(a^*,b^*)\right]$$

See for example [3], [10], [11], [12] and [13].

As well as, the Expected sample size proportion E(n|α,R)/n equal to

…(19) $$1-\frac{u}{1+u}J_0(a^*,b^*)$$

See [6] and [9].

Also, we have to define the percentage of the overall sample saved (P.O.S.S) of $\tilde{\theta}_{SB}$ as:

$$\text{P.O.S.S.} = \frac{n_2}{n} J_0(a^*, b^*) * 100 \quad \ldots(20)$$

See [6] and [8].

And, finally, $p(\hat{\theta}_1 \in R)$ represent the probability of avoiding the second sample.

3. Conclusions and Numerical Results

The computations of Relative Efficiency [R.Eff(·)] and Bias Ratio [B(·)], Expected sample size [E(n|θ,R)], Expected sample size proportion [E(n|θ,R)/n], Percentage of the overall sample saved (P.O.S.S.) and probability of a voiding the second sample $p(\hat{\theta}_1 \in R)$ were used for the estimator $\tilde{\theta}_{SB}$. These computations (using Mat.LAB programs) were performed for n_1 = 4, 6, 8, 10, 12, 16, $u = (n_2/n_1)$ = 0.5, 1, 2, 3, 9, 12, $\lambda = (\theta_0/\theta)$ = 0.25(0.25)2, α = 0.01,0.05,0.1.

Some of these computations are given in tables (1)-(6).

The observation mentioned in the tables leads to the following results:

i. The Relative Efficiency [R.Eff(·)] of $\tilde{\theta}_{SB}$ are adversely proportional with small value of α especially when $\lambda = 1$, i.e. $\alpha = 0.01$ yield highest efficiency.

ii. The Relative Efficiency [R.Eff(·)] of $\tilde{\theta}_{SB}$ has maximum value when $\theta=\theta_0(\lambda=1)$, for each n_1 and α, and decreasing otherwise $(\lambda \neq 1)$. This feature showed the important usefulness of prior knowledge which gave higher effects of proposed estimator as well as the important role of shrinkage technique and its philosophy.

iii. Bias ratio [B(·)] of $\tilde{\theta}_{SB}$ are reasonably small when $\theta=\theta_0$ for each n_1, α, and increases otherwise. This property showed that the proposed estimator $\tilde{\theta}_{SB}$ is very closely to unbiasedness property especially when $\theta=\theta_0$.

iv. The Effective interval of $\tilde{\theta}_{SB}$ [the value of λ which makes R.Eff.(·) of $\tilde{\theta}_{SB}$ greater than one] is approximately [0.75,1.25].

v. Bias ratio [B(·)] of $\tilde{\theta}_{SB}$ are reasonably small with small value of u.

vi. R.Eff($\tilde{\theta}_{SB}$) is decreasing function with increasing of the first sample size n_1, for each α and λ.

vii. The Expected value of sample size of $\tilde{\theta}_{SB}$ is close to n_1, especially when $0.5 \le \lambda < 1$ and start faraway otherwise.

viii. Percentage of the overall sample saved $\left[\frac{n_2}{n} J_0(a^*, b^*) * 100\right]$ is increasing value with increasing value of $u(u = n_2/n_1)$ and decreasing value with increasing value of $\lambda \ge 0.5$.

ix. R.Eff($\tilde{\theta}_{SB}$) is an increasing function with respect to u. This property showed the effective of proposed estimator using small n_1 relative to n_2 (or large n_2) which gave higher efficiency and reduce the observation cost.

x. The considered estimator $\tilde{\theta}_{SB}$ is better that the classical estimator especially when $\theta \approx \theta_0$, this will gave the effective of $\tilde{\theta}_{SB}$ relative to $\hat{\theta}$ and also gave an important weight of prior knowledge, and the augmentation of efficiency may reach to tens times.

xi. The considered estimator $\hat{\theta}_{SB}$ is more efficient than the estimators introduced by [6] and [9] in the sense of higher efficiency.

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10.

Table (1): Showed Bias Ratio [B(·)] and Relative Efficiency [R.Eff.(·)] of $\hat{\theta}_{SB}$ w.r.t. u, n_1 and λ when α =0.01

u	n_1	λ	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	4	R.Eff.(·)	0.3037	0.7751	2.0388	3.8362	1.8481	0.7603	0.3935	0.2417
		B(·)	– 0.4257	– 0.4877	– 0.2489	– 0.0078	0.2163	0.4145	0.5809	0.7124
	6	R.Eff.(·)	0.2487	0.5995	1.9277	5.4715	1.6515	0.5951	0.3188	0.2177
		B(·)	– 0.3848	– 0.4906	– 0.2488	– 0.0142	0.1830	0.3256	0.4093	0.4416
	8	R.Eff.(·)	0.2110	0.4712	1.6296	5.6753	1.3781	0.5233	0.3254	0.2702
		B(·)	– 0.3351	– 0.4918	– 0.2477	– 0.0193	0.1493	0.2398	0.2643	0.2500
3	4	R.Eff.(·)	0.2927	0.8492	2.4385	5.8772	2.0368	0.6782	0.3121	0.1761
		B(·)	– 0.5147	– 0.4915	– 0.2473	– 0.0046	0.2205	0.4186	0.5830	0.7102
	6	R.Eff.(·)	0.2012	0.6156	2.0286	7.6321	1.4868	0.4362	0.2089	0.1332
		B(·)	– 0.4599	– 0.4928	– 0.2458	– 0.0076	0.1915	0.3319	0.4081	0.4285
	8	R.Eff.(·)	0.1543	0.4743	1.6192	7.4731	1.0937	0.3436	0.1978	0.1610

		B(×)	– 0.4023	– 0.4934	– 0.2436	– 0.0101	0.1591	0.2417	0.2516	0.2203
9	4	R.Eff.(×)	0.1863	0.8696	2.7523	10.5122	1.8503	0.4648	0.1855	0.0953
		B(×)	– 0.5647	– 0.4936	– 0.2460	– 0.0019	0.2245	0.4234	0.5877	0.7136
	6	R.Eff.(×)	0.1116	0.6019	1.9471	11.9403	1.0260	0.2380	0.1023	0.0614
		B(×)	– 0.5030	– 0.4941	– 0.2438	– 0.0031	0.1979	0.3377	0.4103	0.4244
	8	R.Eff.(×)	0.0790	0.4568	1.4155	10.4676	0.6523	0.1695	0.0911	0.0726
		B(×)	– 0.4414	– 0.4944	– 0.2410	– 0.0041	0.1660	0.2446	0.2462	0.2053
12	4	R.Eff.(×)	0.1555	0.8581	2.7448	12.2188	1.6977	0.3980	0.1538	0.0774
		B(×)	– 0.5723	– 0.4939	– 0.2458	– 0.0015	0.2252	0.4243	0.5885	0.7142
	6	R.Eff.(×)	0.0907	0.5891	1.8572	13.2757	0.8771	0.1935	0.0814	0.0484
		B(×)	– 0.5096	– 0.4943	– 0.2435	– 0.0024	0.1989	0.3387	0.4108	0.4239
	8	R.Eff.(×)	0.0633	0.4460	1.3121	11.2646	0.5400	0.1352	0.0718	0.0570
		B(×)	– 0.4474	– 0.4945	– 0.2406	– 0.0032	0.1671	0.2451	0.2454	0.2030

Table (2): Showed Expected Sample Size [E] and Expected Sample Size Proportion [Ep] of $\tilde{q}_{SB}$ w.r.t. u and l when $n_1 = 4$, a =0.01

u	l	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	EP	0.6017	0.5050	0.5098	0.5238	0.5451	0.5729	0.6057	0.6417
	E	4.0400	0.0400	4.0783	4.1906	4.3612	4.5835	4.8458	5.1338
3	EP	0.4025	0.2575	0.2647	0.2857	0.3177	0.3594	0.4086	0.462
	E	6.4406	4.1200	4.2348	4.5719	5.0835	5.7506	6.5374	7.4013
9	EP	0.2830	0.1090	0.1176	0.1429	0.1813	0.2313	0.2903	0.3551
	E	11.3218	4.3600	4.7043	5.7156	7.2506	9.2518	11.6122	14.2038
12	EP	0.264	0.0862	0.0950	0.1209	0.1603	0.2116	0.2721	0.3386
	E	13.7624	4.4800	4.9391	6.2875	8.3341	11.0024	14.1496	17.601

Table (3): Showed Expected Sample Size [E] and Expected Sample Size Proportion [Ep] of $\tilde{q}_{SB}$ w.r.t. u and l when $n_1 = 6$, a =0.01

u	l	0.25	0.5	0.75	1	1.25	1.5	1.75	2

1	EP	0.6457	0.5050	0.5151	0.5458	0.5954	0.6580	0.7251	0.7889
	E	7.7484	6.0600	6.1818	6.5492	7.1444	7.8965	8.7016	9.4664
3	EP	0.4685	0.2575	0.2727	0.3187	0.3930	0.4871	0.5877	0.6833
	E	11.2451	6.1800	6.5453	7.6477	9.4331	11.6896	14.1047	16.3991
9	EP	0.3623	0.1090	0.1273	0.1824	0.2717	0.3845	0.5052	0.6200
	E	21.7354	6.5400	7.6358	10.9432	16.2992	23.0687	30.3141	37.1973
12	EP	0.3459	0.0862	0.1049	0.1614	0.2530	0.3687	0.4925	0.6102
	E	26.9806	6.7200	8.1810	12.5909	19.7323	28.7583	38.4189	47.5964

Table (4): Showed Expected Sample Size [E] and Expected Sample Size Proportion [Ep] of $\tilde{q}_{SB}$ w.r.t. u and l when n_1 = 8, a =0.01

u	l	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	EP	0.6885	0.5050	0.5215	0.5743	0.6584	0.7532	0.8380	0.9022
	E	11.0165	8.0800	8.3445	9.1891	10.5337	12.0509	13.4077	14.4345
3	EP	0.5328	0.2575	0.2823	0.3615	0.4875	0.6298	0.7570	0.8532
	E	17.0495	8.2400	9.0336	11.5674	15.6010	20.1526	24.2232	27.3035
9	EP	0.4394	0.1090	0.1388	0.2338	0.3850	0.5557	0.7084	0.8239
	E	35.1483	8.7200	11.1008	18.7021	30.8029	44.4577	56.6697	65.9105
12	EP	0.4250	0.0862	0.1167	0.2141	0.3693	0.5443	0.7009	0.8194
	E	44.1980	8.9600	12.1344	22.2694	38.4039	56.6102	72.8930	85.2141

Table (5): Showed the Percentage of overall Sample Saved (P.O.S.S.) of $\tilde{q}_{SB}$ w.r.t. u, n_1 and l when a =0.01

u	n_1	l	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	4	POSS	39.8308	49.5000	49.0218	47.6172	45.4853	42.7058	39.4275	35.8280
	6		35.4302	49.5000	48.4854	45.4230	40.4637	34.1956	27.4869	21.1136
	8		31.1469	49.5000	47.8467	42.5680	34.1646	24.6822	16.7843	9.7843
	12		23.3572	49.5001	46.2829	35.4833	21.0689	9.8173	3.7270	1.1963
3	4	POSS	59.7462	74.2500	73.5327	71.4259	68.2279	64.0587	59.1412	53.7420
	6		53.1453	74.2500	72.7281	68.1344	60.6956	51.2934	41.2304	31.6704
	8		46.7203	74.2500	71.7700	63.8520	51.2469	37.0233	24.3024	14.6765

	12		35.0358	74.2501	69.4244	53.2249	31.6033	14.7260	5.5905	1.7944
9	4	POSS	71.6954	89.1000	88.2392	85.7110	81.8735	76.8704	70.9694	64.4904
	6		63.7743	89.1000	87.2737	81.7613	72.8347	61.5521	49.4764	38.0045
	8		56.0643	89.1000	86.1240	76.6224	61.4963	44.4279	29.1628	17.6118
	12		42.0430	89.1001	83.3093	63.8699	37.9240	17.6712	6.7086	2.1533
12	4	POSS	73.5338	91.3846	90.5017	87.9087	83.9728	78.8414	72.7892	66.1440
	6		65.4095	91.3846	89.5115	83.8578	74.7022	63.1304	50.7451	38.9790
	8		57.5019	91.3846	88.3323	78.5871	63.0731	45.5671	29.9106	18.0634
	12		43.1210	91.3848	85.4454	65.5076	38.8964	18.1243	6.8807	2.2085

Table (6): Showed the Probability of a Voiding Second Sample [Av] w.r.t. u, n_1 and l when a =0.01

u	n_1	l	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	4	AV	0.7966	0.9900	0.9804	0.9523	0.9097	0.8541	0.7885	0.7166
	6		0.7086	0.9900	0.9697	0.9085	0.8093	0.6839	0.5497	0.4223
	8		0.6229	0.9900	0.9569	0.8514	0.6833	0.4936	0.3240	0.1957
	12		0.4671	0.9900	0.9257	0.7097	0.4214	0.1963	0.0745	0.0239
3	4	AV	0.766	0.9900	0.9804	0.9523	0.9097	0.8541	0.7885	0.7166
	6		0.7086	0.9900	0.9697	0.9085	0.8093	0.6839	0.5497	0.4223
	8		0.6229	0.9900	0.9569	0.8514	0.6833	0.4936	0.3240	0.1957
	12		0.4671	0.9900	0.9257	0.7097	0.4214	0.1963	0.0745	0.0239
9	4	AV	0.7966	0.9900	0.9804	0.9523	0.9097	0.8541	0.7885	0.7166
	6		0.7086	0.9900	0.9697	0.9085	0.8093	0.6839	0.5497	0.4223
	8		0.6229	0.9900	0.9569	0.8514	0.6833	0.4936	0.3240	0.1957
	12		0.4671	0.9900	0.9257	0.7097	0.4214	0.1963	0.0745	0.0239
12	4	AV	0.7966	0.9900	0.9804	0.9523	0.9097	0.8541	0.7885	0.7166
	6		0.7086	0.9900	0.9697	0.9085	0.8093	0.6839	0.5497	0.4223
	8		0.6229	0.9900	0.9569	0.8514	0.6833	0.4936	0.3240	0.1957
	12		0.4671	0.9900	0.9257	0.7097	0.4214	0.1963	0.0745	0.0239

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حول مقدر التقلص –البيزي
ذو المرحلتين لمعلمة القياس للتوزيع الاسي

عباس نجم سلمان ، منى داود سلمان
قسم الرياضيات - كلية التربية ابن الهيثم - جامعة بغداد

استلم البحث في : 19 تشرين الأول 2011 قبل البحث في : 7 كانون الأول 2011

الخلاصة

يتعلق موضوع البحث بمقدر التقلص – البيزي ذي المرحلتين (DSSBE) لتقليل متوسط مربعات الخطأ (MSE) لمقدر الامكان الاعظم $\hat{q}$ لمعلمة القياس q للتوزيع الاسي عند المنطقة (R) حول المعلومات المسبقة q_0 المتوافرة حول المعلمة الحقيقية (q) بشكل تقدير ابتدائي فضلاً عن تقليل كلفة المعينة والتجارب.

عندما يكون استهلاك الوقت او كلفة المعاينة او التجارب عالياً جداً فان طريقة التقلص ذا المرحلتين تكون مناسبة للحصول على مقدر يقلل حجم العينة المتوقع ومن ثم التقليل من هذه الكلف. ومن خواص هذا المقدر ايضاً انه ذو متوسط مربعات خطأ (MSE) صغير لاسيما عند اختيار عامل تقلص موزون $y(x)$ ومنطقة قبول R بشكل مناسب.

اشتقت معادلات التحيز، ومتوسط مربعات الخطأ (MSE)، والكفاية النسبية [R.Eff(×)]، وحجم العينة المتوقع [E(n/q,R)]، وحجم العينة المتوقع النسبي [E(n,q,R)/n]، واحتمالية تجنب العينة الثانية $p(\hat{q}_1 Î R)$ ، ونسبة الادخار الكلي المئوية للعينة $[\frac{n_2}{n} p[\hat{q}_1 Î R) * 100]$ للمقدر المقترح (DSSBE).

أعطيت النتائج العددية والاستنتاجات للمقدر (DSSBE) المقترح عندما يكون المقدر المقترح هو مقدر الاختبار الأولي لمستوى معنوية a.

اجريت المقارنات مع المقدر الكلاسيكي وبعض المقدرات المقترحة في الدراسات الاخيرة لبيان فائدة المقدر المقترح.

الكلمات المفتاحية: التوزيع الاسي، مقدر الامكان الأعظم، المقدر البيزي، مقدر التقلص ذو المرحلتين، متوسط مربعات الخطأ، الكفاية النسبية.