مجموعات مفتوحة من النوع

L-pre and L-semi-p

سعاد جدعان جاسم

قسم الرياضيات ، كلية التربية – إبن الهيثم ، جامعة بغداد

استلم البحث في :24 تشرين الثاني 2010 قبل البحث في : 8 آب 2011

الخلاصة

الغرض من هذا البحث دراسة انواع جديدة من المجموعات المفتوحة في الفضاءات التبولوجية الثنائية .

Ibn Al-Haitham Journal for Pure and Applied Science | مجلة إبن الهيثم للعلوم الصرفة و التطبيقية
No. 1 Vol. 25 Year 2012 | 2012 السنة 25 المجلد 1 العدد

L- pre-open and L-semi-p-open Sets

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Received in: 24 November 2010 Accepted in : 8 February 2011

Abstract

The purpose of this paper is to study new types of open sets in bitopological spaces. We shall introduce the concepts of L- pre-open and L-semi-p-open sets.

Keywords : pre- open- set, semi- p- open - set, L-pre-open, L-semi-p-open

1-Introduction

Navalagi [1] introduced the concepts of pre-open and semi-P-open sets. A subset A of a topological space (X,τ) is said to be "pre-open" set if and only if $A \subseteq \operatorname{int} cl(A)$, the family of all pre open subsets of X is denoted by PO(X).The complement of a pre-open set is called pre-closed set, the family of all pre- closed subsets of X is denoted by PC(X) [1].The smallest pre- closed subset of X containing A is called "pre-closure of A" and is denoted by pre-cl(A)[2].

Let (X,τ) be a topological space, a subset A of X is said to be "semi-P-open" set if and only if there exists a pre-open subset U of X such that $U \subseteq A \subseteq pre-cl(U)$, the family of all semi –p-open subsets of X is denoted by SPO(X).The complement of a semi-p-open set is called "semi-p-closed" set, the family of all semi-p-closed subsets of X is denoted by SPC(x). The smallest semi-p-closed set containing A is called semi-p-closure of A denoted by semi-p-cl(A)[3].[2]shows that every open set is a pre-open and the union of any family of pre-open subsets of X is a pre-open set, but the intersection of any two pre-open subsets of X need not be apre-open set.[3] shows that every pre-open set is a semi –p-open and consequentiy every open set is a semi-p-open. Also she shows that the union of any family of semi-p-open subsets of X is a semi-p-open set,but the intersection of any two semi-p-open subsets of X need not be a semi-p-open set.

The concepts of bitopological space was initiated by Kelly[4].A set X equipped with two topologies τ_1 and τ_2 is called a bitopological space denoted by (X,τ_1,τ_2).

L-open set was studied by Al-swid[5], asubset G of a bitopological space (X,τ_1,τ_2) is said to be "L –open" set if and only if there exists a τ_1-open set U such that $U \subseteq G \subseteq cl\tau_2(U)$,the family of all L-open subsets of X is denoted by L-O(X).The complement of an L-open set is called "L-closed" set,the family of all L-closed subsets of X is denoted by L-C(X).In a bitopological space (X,τ_1,τ_2) every τ_1-open set is an L-open set[5].The union of any family of L-open subsets of X is an L-open set, but the intersection of any two L-open subsets of X need not be L-open set[5].Al-Talkahny [6],introduces two new concepts "L-T_2 -spaces" and "L-continuous functions ". A bitopological space (X,τ_1,τ_2) is

Ibn Al-Haitham Journal for Pure and Applied Science
No. 1 Vol. 25 Year 2012
مجلة إبن الهيثم للعلوم الصرفة و التطبيقية
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said to be "L-T_2 -space" if and only if for each pair of distinct points x and y in X,there exists two disjoint L-open subsets G and H of X such that $x \in G$ and $y \in H$.Let (X, τ_1, τ_2), (Y, τ_1', τ_2') be any bitopological spaces and let $f : X \to Y$ be any function, then f is said to be "L-continuous" function if and only if the inverse image of any L-open subset of Y is an L-open subset of X.

2- L-pre - open and L-semi-P- open Sets

Definition 2.1

Let (X, τ_1, τ_2) be a bitopological space and let G be a subset of X. then G is said to be:

1- " L-pre-open" set if and only if there exists a τ_1-pre-open set U such that $U \subseteq G \subseteq cl\tau_2(U)$.the family of all L-pre-open sub sets of X is denoted by $L-PO(X)$.

2- " L-semi-P-open" set if and only if there exists a τ_1- semi-P-open setU such that $U \subseteq G \subseteq cl\tau_2(U)$.the family of all L- semi-P-open sub sets of X is denoted by $L-SPO(X)$.

Remark(2.2):

1- The complement of an L-pre-open subset of a bitopological space X is called an L-pre-closed set. The family of all L- pre-closed sub sets of X is denoted by $L-PC(X)$.

2- The complement of an L-semi-P-open sub set of a bitopological space X is called an L-semi-P-closed set. The family of all L- semi-P-closed sub sets of X is denoted by $L-SPC(X)$.

Remark (2.3):

In a bitopological space (X, τ_1, τ_2) :

1- Every L-open set is an L-pre-open set.
2- Every L-pre-open set is an L-semi-P-open set.
3- Every L-open set is an L-semi-P-open set.

The converse of each case of remark (2.3) is not true in general as the following example shows:

Example (2.4):

Let $X = \{a, b, c, d\}$

$\tau_1 = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$

$\tau_2 = D$ =the discrete topology

Then $L-O(X) = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$

$L-po(X) = L-O(X) \cup \{\{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, d\}, \{a, c, d\}\}$

Ibn Al-Haitham Journal for Pure and Applied Science

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$\boldsymbol{L-SPO(X)=L-PO(X)\cup\{\{a,d\},\{b,c,d\}\}}$

Note that, $\{b\}$ is an L-pre-open set, but it is not L- open. And $\{a,d\}$ is an L-semi-P-open set but it is neither L-pre-open nor L-open.

Remark(2.5)

In a bitopological space (X,τ_1,τ_2) :

1- Every τ_1-pre-open set is an L-pre-open set.
2- Every τ_1-semi- p-open set is an L-semi-p-open set.

The opposite direction of each case in remark (2.5) is not true in general, as the following example shows:

Example (2.6):

Let $X=\{a,b,c,d\}$ $\quad \tau_1=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$

$\tau_2=I$ =the indiscrete topology

$$\tau_1-po(X)=\{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,b,d\},\{a,c,d\}.\{a,b,c\}\}$$

$$\tau_1-SPO(X)=\tau_1-PO(X)\cup\{\{a,d\},\{b,c,d\}\}$$

$$L-PO(X)=\tau_1-PO(X)\cup\{\{a,d\},\{b,d\},\{c,d\},\{b,c,d\}\}$$

$$L-SPO(X)=L-PO(X)$$

Note that, $\{a,d\}$ is an L-pre-open set, but it is not τ_1-pre- open. And $\{b,d\}$ is an L-semi-P-open set but it is not τ_1 –semi-P-open.

Proposition (2.7):

The union of any family of L -pre-open (L-semi-P-open) subsets of a bitopological space (X,τ_1,τ_2) is an L -pre-open (L-semi-P-open) respectively.

Proof:

Let $\{G_\alpha:\alpha\in\Lambda\}$ be a family of L -pre-open (L-semi-P-open) subsets of X , then for each G_α there exists a τ_1-pre-open(τ_1-semi- p-open) set U_α in X such that $U_\alpha\subseteq G_\alpha\subseteq cl\tau_2(U_\alpha)$.So $\bigcup_{\alpha\in\Lambda}U_\alpha\subseteq\bigcup_{\alpha\in\Lambda}G_\alpha\subseteq\bigcup_{\alpha\in\Lambda}cl\tau_2(U_\alpha)=cl\tau_2\left(\bigcup_{\alpha\in\Lambda}U_\alpha\right)$.But $\bigcup_{\alpha\in\Lambda}U_\alpha$ is a τ_1-pre-open(τ_1-semi- p-open).Hence $\bigcup_{\alpha\in\Lambda}U_\alpha$ is an L -pre-open (L-semi-P-open) respectively.

Remark (2.8):

The intersection of any two L -pre-open (L-semi-P-open) sets need not be L -pre-open (L-semi-P-open) respectively.

For example

$X = \{1,2,3,4\}$

Let $\tau_1 = \{X, \phi, \{1\}, \{4\}, \{1,4\}\}$

$\tau_2 = \{X, \phi, \{1,2\}\}$

Note that $\{1,3\},\{3,4\}$ are two L -pre-open (L-semi-P-open) sets, but $\{1,3\} \cap \{3,4\} = \{3\}$ is neither L -pre-open nor L-semi-P-open.

Definition (2.9):

Let (X,τ_1,τ_2) be abitopological space and let $x \in X$,a subset M of X is said to be :

1- An "L-pre-neighbourhood" of x if and only if there exists an L-pre-open set G such that $x \in G \subseteq M$

2- An "L-semi-p- neighbourhood" of x if and only if there exists an L-semi-p-open set G such that $x \in G \subseteq M$

Definition (2.10):

Let (X,τ_1,τ_2) be abitopological space and let A be a subset of X, then:

1- The intersection of all L-pre-closed subset of X containing A is called "L-pre-closure of A" and is dented by L-pcl(A).

2- The intersection of all L-semi-p-closed subset of X containing A is called "L-semi-p-closure of A" and is dented by L-spcl(A).

Theorem (2.11):

Let (X,τ_1,τ_2) be abitopological space and let A be a subset of X.A point x in X is an L-pre-closure (L-semi-p-closure) point of A if and only if every L-pre-neighbourhood (L-semi-p- neighbourhood) of x intersects A.

Proof:

The "only if" part

Assum that x is an L-pre-closure (L-semi-p-closure) of A , then $x \in \Im = \cap \{F \subseteq X : A \subseteq F \quad and \quad F \, is \, an \, L - pre - closed \, (L - semi - p - closed)\}$.Suppose that there exists an L-pre-neighbourhood (L-semi-p- neighbourhood) M of x such that $M \cap A = \phi$, that is, there exists an L-pre-open(L-semi-p -open) set G such that $x \in G \subseteq M$,then such that $A \subseteq M^c \subseteq G^c$,but G^c is an L-pre-closed (L-semi-p-closed) with $x \notin G^c$. Therefore $x \notin \Im$ which is a contradiction hence every L-pre-neighborhood (L-semi-p- neighborhood) of x must intersect A.

The "if" part

Assume that every L-pre-neighborhood (L-semi-p- neighborhood) of x intersects A, and suppose that x is not L -pre-closure (L-semi-p-closure) point of A, then $x \notin \Im$,that is, there exists an L-pre-closed (L-semi-p -closed) subset F of X with $A \subseteq F$ such that $x \notin F$,it follows that $x \in F^C$ which is an L-pre-open(L-semi-p -open) set. Now there is an L-pre-neighborhood (L-semi-p- neighborhood) F^C of x with $A \cap F^c = \phi$ .that implies to contradiction with our assumption. Hence x must be an L-pre-(L-semi-p-) closure point of A

Theorem (2.12):

Let (X, τ_1, τ_2) be a bi topological space. A subset A of X is an L-pre-(L-semi-p-) closed if and only if $A = L - Pcl\,(A)(L - SPcl\,(A))$

Proof:

The "only if" part

Suppose that $A \in L - PC(X)(L - SPC(X))$ and $A \neq L - Pcl(A)(L - SPcl(A))$ .Since $A \subseteq L - Pcl(A)(L - SPcl(A))$, so $L - Pcl(A)(L - SPcl(A)) \not\subset A$,that is, there exists an element $r \in L - Pcl(A)(L - SPcl(A))$ and $r \notin A$,it follows that $r \in A^c$ which is an L-pre-(L-semi-p-) open set. Then by theorem (3.33) $A \cap A^c \neq \phi$ which is a contradiction with the fact $A \cap A^c = \phi$ .Hence $A = L - Pcl(A)(L - SPcl(A))$

The "if" part

Assume that $A = L - Pcl(A)(L - SPcl(A))$, but $L - Pcl(A)(L - SPcl(A))$ is an L-pre-(L-semi-p-) closed subset of X by definition (3.32). So A is an L-pre-(L-semi-p-) closed set.

Definition (2.13):

A bi topological space (X, τ_1, τ_2) is said to be :

1- "$L - pre - T_2$ space" if and only if for each pair of distinct points x and y,there are two disjoint L-pre-open subsets U and V of X such that $x \in U$ and $y \in V$.
2- "$L - semi - p - T_2$ space" if and only if for each pair of distinct points x and y,there are two disjoint L-semi-p-open subsets U and V of X such that $x \in U$ and $y \in V$.

Proposition (2.14)

1- $Every\ L - T_2 - space\ is\ an\ L - pre - T_2$.
2- $Every\ L - pre - T_2 - space\ is\ an\ L - semi - p - T_2$.
3- $Every\ L - T_2 - space\ is\ an\ L - semi - p - T_2$.

Proof:

follows from remark (2.3).

Remark (2.15):

The opposite direction of each case proposition (2.6) is not true in general. As the following two examples show:

$X = \{1,2,3\}$

$\tau_1 = \{X, \phi, \{1,2\}\}$

$\tau_2 = \{X, \phi, \{1\}, \{3\}, \{1,3\}\}$

$L-O(X) = \{X, \phi, \{1,2\}\}$

$L-PO(X) = L-O(X) \cup \{\{1\}, \{2\}, \{2,3\}, \{1,3\}\}$

Clear that X is $L-pre-T_2$ space, but it is not $L-T_2-space$.

$X = \{1,2,3,4\}$

$\tau_1 = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$

$\tau_2 = D$

$L-O(X) = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$

$L-PO(X) = \{X, \phi, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}, \{1,2,4\}\}$

$L-SPO(X) = L-PO(X) \cup \{\{2,3,4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,3,4\}\}$

Clear that X is $L-semi-p-T_2$ –space, but it is neither $L-pre-T_2$ –space nor $L-T_2$ $-space$.

Definition (2.16):

Let $f: (X, \tau_1, \tau_2) \to (Y, \rho_1, \rho_2)$ be any function, then f is said to be:

1- "L-pre-irresolute" function if and only if the inverse image of an L-pre-open subset of Y is an L-pre-open subset of X

2- "L-semi-p-irresolute" function if and only if the inverse image of an L-semi-p-open subset of Y is an L-semi-p-open subset of X.

It is clear that their is no relation among the concepts of L-continuous, L-pre-irresolute and L-semi-p-irresolute function. See the following examples:

Example(2.17):

$X = \{1,2,3\}$ $\quad \tau_1 = \{X, \emptyset, \{1\}\}$

$\tau_2 = I$ $\quad L-O(X) = \{X, \emptyset, \{1\}, \{1,2\}, \{1,3\}\}$

$\tau_1\_PO(X) = \{X, \emptyset, \{1\}, \{1,2\}, \{1,3\}\}$

$L-PO(X) = \tau_1\_PO(X) = L-SPO(X)$

$Y = \{a,b,c\}$ $\quad \tau'_1 = \{Y, \emptyset, \{b,c\}\}$

$\tau'_2 = \{Y, \emptyset, \{b\}, \{a\}, \{a,b\}\}$

$L-O(Y) = \{Y, \emptyset, \{b,c\}\}$

$\tau'_1\text{-}PO(Y) = \{Y, \emptyset, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, \{c\}\}$

$L-PO(Y) = \tau'_1\text{-}PO(Y) = L-SPO(Y)$

$$Let\ \ f{:}Y \to X\ \ such\ that\ \ f(a) = f(c) = 1\ \ and\ \ f(b) = 2$$

It is clear that f is L-pre-(L-semi-p-)irresolute function but it is not L-continuous function.

Eample(2.18):

$X = \{1,2,3\}$ $\tau_1 = \{X, \emptyset, \{1\}, \{2\}, \{1,2\}\}$

$\tau_2 = I$ $L-O(X) = \{X, \emptyset, \{1\}, \{1,2\}, \{1,3\}, \{2\}, \{2,3\}\}$

$L-O(X) = L-PO(X) = L-SPO(X)$

$Y = \{a,b,c\}$ $\tau'_1 = \{Y, \emptyset, \{a\}\}$

$\tau'_2 = I$

$L-O(Y) = \{Y, \emptyset, \{a\}, \{a,b\}, \{a,c\}\}$

$L-PO(Y) = L-O(Y) = L-SPO(Y)$

$$Let\ \ f{:}X \to Y\ \ such\ that\ \ f(1) = a, f(2) = b\ \ and\ \ f(3) = c$$

It is clear that f is L-pre-(L-semi-p-)irresolute function and L-continuous function.

Theorem(2.19):

A bi topological space (X, τ_1, τ_2) is $\mathrm{L-pre-(L-semi-p-)T_2-}$ space if and only if for each pair of distinct points x, y in X there exists L-pre-(L-semi-p-)irresolute function f from (X, τ_1, τ_2) into (Y, ρ_1, ρ_2) which is L-pre-(L-semi-p-) T_2-space such that $f(x) \neq f(y)$.

Proof:

"first direction"

Suppos that (X, τ_1, τ_2) is $\mathrm{L-pre-(L-semi-p)-T_2}$ space.If we take the identity function $i{:}(X, \tau_1, \tau_2) \to (X, \tau_1, \tau_2)$ clear that i is L-pre (L-semi-p)-irresolute function .Now let $x \neq y$ in X so $i(x) = x$, $i(y) = y$,it follows that $i(x) \neq i(y)$.

"second direction"

Let $f{:}(X, \tau_1, \tau_2) \to (Y, \rho_1, \rho_2)$ be an L-pre-(L-semi-p-) irresolute function and $(Y, \tau_1, \tau_2)\ is\ an\ L-pre\ (L-semi-p)-T_2$ space and let $x \neq y\ \ in\ X$,then by ypothesis

$f(x) \neq f(y) in\ Y$. So there are L-pre(L-semi-p)-open sets U, V *such that* $f(x) \in U, f(y) \in V$ *and* $U \cap V = \emptyset$ *that is* $x \in f^{-1}(U), y \in f^{-1}(V)$ *and* $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\emptyset) = \emptyset$ *where* $f^{-1}(U), f^{-1}(V)$ *are* $L - pre(L - semi - p) -$ *open sets in* X. *Hence* X *is* $L - pre(L - semi - p) - T_2$ *space.*

Definition(2 .20):

A function $f: (X, \tau_1, \tau_2) \to (Y, \rho_1, \rho_2)$ is called:

1. " L-pre-open" function if and only if $f(U) \in L - PO(Y)$ *for each* $U \in L - PO(X)$.
2. "L-semi-p-open" function if and only if $f(U) \in L - SPO(Y)$ *for each* $U \in L - SPO(X)$.
3. "L-pre-closed" function if and only if $f(F) \in L - PC(Y)$ *for each* $F \in L - PC(X)$.
4. "L-semi-p-closed" function if and only if $f(F) \in L - SPC(Y)$ *for each* $F \in L - SPC(X)$.

proposition(2 .21):

if $f: (X, \tau_1, \tau_2) \to (Y, \rho_1, \rho_2)$ is bijectiv, $\mathrm{L - pre(L - semi - p) - open}$ and $\mathrm{L - pre(L - semi - p) - irresolute}$ function and (X, τ_1, τ_2) is $\mathrm{L - pre(L - semi - p)} - T_2$ *space* , *then* (Y, ρ_1, ρ_2) *is* $L - pre(L - semi - p) - T_2$ *space.*

Proof:

Suppose that $y_1 \neq y_2$ *in* Y. Since f is onto, then there exist x_1, x_2 *in* X such that $y_1 = f(x_1), y_2 = f(x_2)$ and since f *is* $(1-1)$, *then* $x_1 \neq x_2$ *in* X *wich is* $L - pre(L - semi - p) - T_2 - space$. Therefore there exist $\mathrm{L - pre(L - semi - p)} -$ open sets U, V such that $x_1 \in U, x_2 \in V$ *and* $U \cap V = \emptyset$. It follows that $y_1 \in f(U), y_2 \in f(V)$ *where* $f(U), f(V)$ *are* $L - pre(L - semi - p) -$ *open sets in* Y *and* $f(U) \cap f(V) = \emptyset$.

Hence (Y, ρ_1, ρ_2) *is* $L - pre(L - semi - p) - T_2 - space$.

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