مقدرات ثومبسن ذو الاختبار الأولي المعدلة لمعالم أنموذج الانحدار الخطي البسيط

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الخلاصة

يتعلق موضوع هذا البحث بمقدرات ثومبسن التي تُعرف بمقدرات الاختبار الاولي المقلصه ذي المرحلة الواحدة مع بعض التعديلات على صيغته العامة عن طريق عامل تقلص موزون $\psi(\cdot)$. اقترح هذا النوع من المقدرات لتقدير المعالم (θ) لانموذج الانحدار الخطي البسيط عند توافر تقديرات مسبقه حول هذه المعالم بشكل θ_0. ان قيمة θ_0 يُشار لها في الادبيات الاحصائيه على شكل تقدير نقطي (point guess) حول المعلمة θ.

اشتقت معادلات التحيز، ومتوسط مربعات الخطأ والكفاية النسبية للمقدرات المقترحة. أُعطيت النتائج العددية الخاصة بالمعادلات اعلاه والمتعلقة بالمقدرات المقترحة عندما تكون هذه المقدرات، ومقدرات الاختبار الاولي بمستوى معنوية α. أجريت مقارنات بين المقدرات المقترحة مع المقدرات الاعتيادية والمشابهة لبيان افضلية المقدرات المقترحة من حيث الكفاية النسبيه ومتوسط مربعات الخطأ.

الكلمات المفتاحية : الانحدار الخطي البسيط، طريقة المربعات الصغرى، مقدر التقلص، مقدر الاختيار الاولي، التقدير الاولي، التحيز، متوسط مربعات الخطأ والكفاية النسبية.

Modified Thompson –Type Testimators for the Parameters of Simple Linear Regression Model

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Abstract

This paper is concerned with Thompson-Type estimators, which is known as preliminary test shrinkage estimator with some modification on its form via shrinkage weight factor $\psi(\cdot)$.

This type of estimators have been considered for estimating the parameters θ of simple linear regression model, when a prior estimate of the parameter value (θ) is available, say θ_0. This θ_0 has been referred in statistical literatures as guess point about the parameter θ.

The expressions for Bias, Mean Squared Error (MSE) and Relative Efficiency of the proposed estimators are obtained. Numerical results are provided when the proposed estimators are testimators of level of significance α.

Comparisons with the usual (L.S.M) and existing estimators were made to show the usefulness of the proposed estimators in the sense of Relative Efficiency and Mean Squared Error.

Key Words: Simple linear regression, least square method, Shrinkage estimator, preliminary test estimator, prior estimator, Bias, Mean Square Error and Relative Efficiency.

Introduction

Some time we may have a prior estimate value (point guess) of the parameter to be estimated. If this value is in the vicinity of the true value, the shrinkage technique is useful to get an improved estimator. Thompson in [14], Mehta and Srinivasan in [8], Singh at el in [12] and others suggested shrunken estimators for different distributions when a prior estimate or guess point is available. They showed that these estimators perform better in the term of Mean Square Error when a guess value θ_0 close to the true value θ.

Consider the following simple linear regression model:

$$y_i = \gamma + \beta(x_i - \bar{x}) + e_i,\quad i = 1,2,\ldots,n \qquad \ldots(1)$$

where x_i is the independent variable and y_i is the response variable, $\gamma = \zeta + \beta\bar{x}$, $\bar{x}$ is the mean of x_i, e_i is the random error which is distribute as normal distribution with zero Mean and Variance σ^2 and $Cov(e_i,e_j) = 0$ and $y_i \sim N[\gamma + \beta(x_i - \bar{x}), \sigma^2(\frac{1}{n} + \frac{(x_i - \bar{x})}{SS_x})]$, see [5], [6].

Thompson-Type estimator in [14] is considered for estimating the parameter θ (θ may refer to γ or β) of previous model when a guess point θ_0 is available about θ due the past experience or similar cases.

From the empirical studies it has been established that the shrunken estimators performs better than the usual estimator when our guess point be very close to the true value of the parameter. Therefore to make sure whether θ is closed to θ_0 or not, we may test $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$, so we denote by R to the critical region for above test.

Thompson suggested shrinking the least square estimator $\hat{\theta}$ of θ towards the prior guess point θ_0 and proposed the estimator $\tilde{\theta} = \psi(\hat{\theta})\hat{\theta} + (1-\psi(\hat{\theta}))\theta_0$, where $(1-\psi(\hat{\theta}))$ represents the experimenters belief in the guess point θ_0. He was found the estimator $\tilde{\theta}$ is more efficient than $\hat{\theta}$ if the true value θ is close to θ_0 (H_0 accepted) but may be less efficient otherwise, therefore to resolve the uncertainty that a guess point value is approximately the true value or not, a preliminary test of significance may be employed. So he take the least square estimator $\hat{\theta}$ when θ is far a way from θ_0 (H_0 rejected) after he made the preliminary test.

Thus, the preliminary test shruken estimator has the following form

$$\tilde{\theta} = \begin{cases} \psi(\hat{\theta})\hat{\theta} + (1-\psi(\hat{\theta}))\theta_0 & , \text{if } \hat{\theta} \in R \\ \hat{\theta} & , \text{if } \hat{\theta} \notin R \end{cases} \quad \ldots(2)$$

where R is the preliminary test region for acceptance the null hypothesis H_0 as we mentioned above, $\hat{\theta}$ is the least square estimator of θ, $\psi(\hat{\theta})$ is a shrinkage weight factor such that $0 \le \psi(\hat{\theta}) \le 1$ which may be a function of $\hat{\theta}$ or may be a constant (ad hoc basis).

Several authors had studied a preliminary test shrunken estimator which is defined in (2) for special population by choosing different weight factors $\psi(\hat{\theta})$. See for example [1], [2], [3], [4], [7], [10], [11] and [13].

The aim of this paper is to modify the preliminary test shrunken estimator which is defined in (2) for estimating the parameters (θ) of the proposed simple linear regression model (1).

Therefore, the form of the proposed preliminary test shrunken estimator is as below:-

$$\tilde{\theta}_{PT} = \begin{cases} \psi_1(\hat{\theta})\hat{\theta} + (1-\psi_1(\hat{\theta})) & , \text{if } \hat{\theta} \in R \\ \psi_2(\hat{\theta})\hat{\theta} + (1-\psi_2(\hat{\theta})) & , \text{if } \hat{\theta} \notin R \end{cases} \quad \ldots(3)$$

where $\psi_i(\hat{\theta})$, $i = 1,2$ is a shrinkage weight factor such that $0 \le \psi_i(\hat{\theta}) \le 1$.

The expressions for Bias, Mean Square Error and Relative Efficiency of the estimator $\tilde{\theta}_{PT}$ above are derived. Numerical results of these expressions were made to show the validity and the usefulness of the proposed estimator when it is compared with the least square and existing estimators.

Preliminary Test Single Stage Shrunken Estimator $\tilde{\beta}_{PT}$

In this section recall the estimator which is defined in (3) for estimating the parameter β of assuming model as below

$$\tilde{\beta}_{PT} = \begin{cases} \psi_1(\hat{\beta})\hat{\beta} + (1-\psi_1(\hat{\beta}))\beta_0 & , \text{if } \hat{\beta} \in R_1 \\ \psi_2(\hat{\beta})\hat{\beta} + (1-\psi_2(\hat{\beta}))\beta_0 & , \text{if } \hat{\beta} \notin R_1 \end{cases} \quad \ldots(4)$$

where β_0 is a prior guess value of β, $\hat{\beta}$ is a unbiased estimator (L.S.M.) of β and R_1 is a preliminary test region of acceptance of size α for testing the hypothesis $H_0:\beta = \beta_0$ against the hypothesis $H_1 : \beta \neq \beta_0$.

i.e.

$$R_1 = \left[\beta_0 - t_{\frac{\alpha}{2},n-2}\sqrt{\frac{\sigma^2}{SS_x}}, \beta_0 + t_{\frac{\alpha}{2},n-2}\sqrt{\frac{\sigma^2}{SS_x}}\right], \text{ see [5]} \quad \ldots(5)$$

where $\hat{\beta} = \dfrac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})}$, $\hat{\beta} \sim N(\beta, \dfrac{\sigma^2}{SS_x})$ and $SS_x = \sum_{i=1}^{n}(x_i-\bar{x})^2$, see [6] .. (6)

while $t_{\alpha/2,\, n-2}$ is the $100(\alpha/2)$ percentile of t-distribution with $(n – 2)$ degree of freedom.

Now, put forward $\psi_1(\hat{\theta}) = 0$ and $\psi_2(\hat{\theta}) = k = e^{-10/n}$.

The expressions for Bias and Mean Square Error (MSE) of $\tilde{\beta}$ are respectively given by

$$\text{Bias}(\tilde{\beta}_{PT}|\beta, R_1) = E(\tilde{\beta}_{PT} - \beta)$$

$$= \frac{\sigma}{\sqrt{SS_x}}\{-\lambda_1[1-k+kJ_0(a^*,b^*)] - kJ_1(a^*,b^*)\} \quad \ldots(7)$$

where $J_\ell(a^*,b^*) = \dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{a^*}^{b^*} t^l e^{-t^2/2}dt, l = 0,1,2$ ...(8)

and $t = \dfrac{\sqrt{SS_x}(\hat{\beta}-\beta)}{\sigma}$, $\lambda_1 = \dfrac{\sqrt{SS_x}(\beta-\beta_0)}{\sigma}$, $a^* = -\lambda_1 - t_{\alpha/2,n-2}$, $b^* = -\lambda_1 + t_{\alpha/2,n-2}$ ...(9)

we denote to the Bias ratio of $\tilde{\beta}_{PT}$ as $B(\tilde{\beta}_{PT})$ and defined as below

$$B(\tilde{\beta}_{PT}) = \frac{\text{Bias}(\tilde{\beta}_{PT}|\beta, R_1)}{\sigma/\sqrt{SS_x}} \quad \ldots(10)$$

and

$$MSE(\tilde{\beta}_{PT}|\beta,R_1)=E(\tilde{\beta}_{PT}-\beta)^2$$

$$=\frac{\sigma^2}{SS_x}\{\ k^2(1+\lambda_1^2)-\lambda^2(2k-1)-k^2[J_2(a^*,b^*)+2\lambda_1J_1(a^*b^*)+J_0(a^*,b^*)\lambda_1^2]+$$

$$2k\lambda_1[J_1(a^*,b^*)+\lambda_1J_0(a^*,b^*)]\ \} \qquad ...(11)$$

The Efficiency of the proposed estimator $\tilde{\beta}_{PT}$ relative to $\hat{\beta}$ is defined as

$$R.Eff(\tilde{\beta}_{PT}|\beta,R)=\frac{MSE(\hat{\beta})}{MSE(\tilde{\beta}_{PT}|\beta,R_1)} \qquad ...(12)$$

See [3], [4] and [7].

Preliminary Test Single Stage Shrunken Estimator $\tilde{\gamma}$

Let $y_1, y_2, \ldots, y_n$ distribute as normal distribution with mean μ and known variance σ^2, where $\hat{\gamma}=\bar{y}$.

In this section, we want to estimate the parameter γ using the following preliminary test Shrunken estimator:

$$\tilde{\gamma}_{PT}=\begin{cases}\psi_3(\hat{\gamma})\hat{\gamma}+(1-\psi_3(\hat{\gamma}))\gamma_0 & ,\ \text{if } \hat{\gamma}\in R_2\\ \psi_4(\hat{\gamma})\hat{\gamma}+(1-\psi_4(\hat{\gamma}))\gamma_0 & ,\ \text{if } \hat{\gamma}\notin R_2\end{cases} \qquad ...(13)$$

where $\psi_i(\hat{\gamma})$, i = 3,4 are shrinkage weight factors such that $0\le\psi_i(\hat{\gamma})\le 1$ and $\hat{\gamma}$ is an unbiased estimator (L.S.M) of γ as well as R_2 is the pretest region for acceptance of testing the hypothesis $H_{00}:\gamma=\gamma_0$ vs. the hypothesis $H_{11}:\ \gamma\neq\gamma_0$ with level of significance α.

i.e. $R\ =[\gamma-Z\sqrt{\frac{\sigma}{n}},\gamma+Z\sqrt{\frac{\sigma}{n}}]$, see [5] $\qquad ...(14)$

where $Z_{\alpha/2}$ is the $100(\alpha/2)$ percentile point of the standard normal distribution.

In the estimator $\tilde{\gamma}_{PT}$ which is defined in (11), we assume that $\psi_3(\hat{\gamma})\ =\ 0$ and $\psi_4(\hat{\gamma})=h=e^{-Z_{\alpha/2}}$.

The expressions for Bias and Mean Square Error (MSE) of $\tilde{\gamma}_{PT}$ are respectively given as below:-

$$Bias(\tilde{\gamma}_{PT}|\gamma,R_2)=E(\tilde{\gamma}_{PT}-\gamma)$$

$$=\frac{\sigma}{\sqrt{n}}\{-\lambda_2[1-h+hJ_0(a_1,b_1)]-hJ_1(a_1,b_1)\} \qquad \ldots(15)$$

where $J_\ell(a_1,b_1)=\frac{1}{\sqrt{2\pi}}\int_{a_1}^{b_1} Z^\ell e^{-Z^2/2}dZ,\ \ell=0,1,2$ …(16)

and

$$Z=\frac{\sqrt{n}(\hat{\gamma}-\gamma)}{\sigma},\lambda_2=\frac{\sqrt{n}(\gamma-\gamma_0)}{\sigma},a_1=-\lambda_2-Z_{\alpha/2},b_1=-\lambda_2+Z_{\alpha/2} \qquad \ldots(17)$$

we denote to the Bias ratio of $\tilde{\gamma}_{PT}$ as $B(\tilde{\gamma}_{PT})$ which is defined as

$$B(\tilde{\gamma}_{PT})=\frac{Bias(\tilde{\gamma}_{PT}|\gamma,R_2)}{\sigma/\sqrt{n}} \qquad \ldots(18)$$

and

$$MSE(\tilde{\gamma}_{PT}|\gamma,R_2)=E(\tilde{\gamma}_{PT}-\gamma)^2$$

$$=\frac{\sigma^2}{n}\{h^2(1+\lambda_2^2)-\lambda_2^2(2h-1)-h^2[J_2(a_1,b_1)+2\lambda_2J_1(a_1,b_1)+\lambda_2^2J_0(a_1,b_1)]-2h\lambda_2[J_1(a_1,b_1)+\lambda_2J_0(a_1,b_1)]\} \qquad \ldots(19)$$

The Efficiency of the proposed estimator $\tilde{\gamma}_{PT}$ relative to estimator $\hat{\gamma}$ is defined as

$$R.Eff(\tilde{\gamma}_{PT}|\gamma,R_2)=\frac{MSE(\hat{\gamma})}{MSE(\tilde{\gamma}_{PT}|\gamma,R_2)} \qquad \ldots(20)$$

Numerical Results

1. The computation of Relative Efficiency [R.Eff(·)] and Bias Ratio [B(·)] were used for the estimator $\tilde{\beta}_{PT}$, these computations were performed for α = 0.01, 0.05, 0.1, λ_1 = 0.0(0.1)2 and n = 8, 10, 12, 20. Some of these computations are displayed in the attached table 1 which leads to the following results.

i. The Relative Efficiency [R.Eff(·)] of $\tilde{\beta}_{PT}$ are adversely proportional with small value of α and those of n and k.

ii. R.Eff($\tilde{\beta}_{PT}$) has a maximum value when $\beta = \beta_0$ ($\lambda_1 = 0$).

iii. The Bias Ratio [B(·)] of $\tilde{\beta}_{PT}$ are reasonably small when $\beta = \beta_0$ and vice – versa otherwise.

iv. The Bias Ratio [B(·)] of $\tilde{\beta}_{PT}$ are increasing function with icreases value of sample size (n).

v. The Effective Interval [The value of λ_1 which make the R.Eff(·) of $\tilde{\beta}_{PT}$ greater than 1] is [0,1].

vi. The proposed estimator $\tilde{\beta}_{PT}$ dominate the usual estimator $\hat{\beta}$ with large sample size n.

i.e.; $\lim_{n\to\infty}[MSE(\tilde{\beta}_{PT}|\beta, R_1) - MSE(\hat{\beta})] \le 0.$

vii. $\tilde{\beta}_{PT}$ is consistent estimator

i.e.; $\lim_{n\to\infty} MSE(\tilde{\beta}_{PT}|\beta, R_1) = 0.$

viii. The considered estimator $\tilde{\beta}_{PT}$ is better than the usual estimator and also than the estimator introduced by [1] and [2] in the sense of Mean Squared Error.

2. The computation of Relative Efficiency [R.Eff(·)] and Bias Ratio [B(·)] of the proposed estimator $\tilde{\gamma}_{PT}$ were made on different constants involved in it, some of these computations are given in annexed table (2) for samples of these constant e.g. α = 0.01, 0.05, 0.1, n = 8, 10, 12, 20 and λ_2 = 0.0(0.1)2. The following results from the mentioned table were made

i. The Relative Efficiency [R.Eff(·)] of $\tilde{\gamma}_{PT}$ has a maximum value when γ very close to γ_0 (λ_2 = 0) and decreases with increases value of λ_2 and h.

ii. R.Eff($\tilde{\gamma}_{PT}$) increasing function with small value of α [level of significance of acceptance region R_2].

iii. The Bias Ratio [B(·)] of $\tilde{\gamma}_{PT}$ are reasonably small when γ close to γ_0 (λ_2 = 0) and increases otherwise

iv. B($\tilde{\gamma}$) are increases when α increases.

v. The Effective Interval [The value of λ_2 which make the R,Eff(·) of $\tilde{\gamma}_{PT}$ greater than 1] is [0,1].

vi. The considered estimator $\tilde{\gamma}_{PT}$ is consistent estimator and dominate the usual estimator $\hat{\gamma}$.

vii. The considered estimator $\tilde{\gamma}_{PT}$ is better than the estimator $\hat{\gamma}$ (least square method) and some existing estimator e.g. [1] and [2] in terms of higher Efficiency especially at $\gamma \simeq \gamma_0$.

Conclusions

From the above discussions it is obvious that by using guess point value one can improve the usual estimator. It can be noted that if the guess point θ_0 is very close to the true value of the parameter θ (i.e.; λ_i is approximate close to one), the proposed estimators perform better than the usual estimator. If one has no confidence in the guessed value then proposed preliminary test Shrunken estimators can be suggested. We can safely use the proposed estimators for small sample size at usual level of significance α and moderate value of shrunken weight factor $\psi(\hat{\theta})$.

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Table (1) Shown the R.Eff. (·) and B(·)of $\hat{\beta}$ w.r.t. α, n and λ_1

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R.Eff(·)	3689.8	97.349	24.831	11.079	6.2411	3.9977	2.7781	2.0424	1.5648	1.2374	1.003
B(·)	(1.7707 e^{-15})	(0.099901)	(0.19979)	(0.29966)	(0.39948)	(0.49924)	(0.59892)	(0.69848)	(0.7979)	(0.8971)	(0.897
R.Eff(·)	287.39	73.968	22.931	10.672	6.1085	3.9449	2.7558	2.0343	1.5644	1.2417	1.010
B(·)	(2.9328 e^{-})	(0.099156)	(0.19827)	(0.29722)	(0.39593)	(0.49431)	(0.59223)	(0.68958)	(0.786)	(0.8819)	(0.976
R.Eff(·)	66.572	39.599	17.898	9.3655	5.6251	3.7232	2.6398	1.9688	1.5262	1.2199	0.999
B(·)	(6.6945 e^{-10})	(0.097423)	(0.19972)	(0.29164)	(0.38797)	(0.48349)	(0.57796)	(0.67111)	(0.7626)	(0.8523)	(0.939
R.Eff(·)	108.11	52.823	20.867	10.411	6.1335	4.0245	2.8422	2.1166	1.6406	1.3121	1.076
B(·)	(5.6271 e^{-5})	(0.096686)	(0.19326)	(0.28949)	(0.3852)	(0.48022)	(0.57437)	(0.66746)	(0.759)	(0.8497)	(0.938
R.Eff(·)	30.176	23.453	14.08	8.4753	5.4637	3.7677	2.7455	2.0906	1.649	1.3387	1.13
B(·)	(0.0001948)	(0.092123)	(0.18411)	(0.27545)	(0.36582)	(0.4549)	(0.54238)	(0.62795)	(0.71131)	(0.7922)	(0.870

R.Eff(·) B(·)	12.33 (0.00011205)	11.021 (0.086432)	8.3709 (0.17248)	5.9936 (0.25754)	4.3075 (0.34113)	3.1811 (0.42277)	2.4262 (0.50201)	1.9085 (0.57841)	1.5433 (0.656)	1.2786 (0.7211)	1.082 (0.786
R.Eff(·) B(·)	42.278 (0.00020265)	30.698 (0.091496)	16.867 (0.18294)	9.6606 (0.27387)	6.0679 (0.36405)	4.1232 (0.45324)	2.9769 (0.54122)	2.252 (0.6278)	1.7671 (0.7128)	1.4279 (0.79603)	1.181 (0.877
R.Eff(·) B(·)	15.085 (0.00024397)	13.378 (0.084433)	10 (0.16871	7.061 (0.25219)	5.0266 (0.33449)	3.6898 (0.41527)	2.8035 (0.4942)	2.1997 (0.57098)	1.775 (0.6454)	1.4685 (0.71711)	1.240 (0.786
R.Eff(·) B(·)	6.9216 (0.0007499)	6.5433 (0.07533)	5.6271 (0.15084)	4.5754 (0.22522)	3.6438 (0.29797)	2.9063 (0.36858)	2.3478 (0.4366)	1.9298 (0.50167)	1.616 (0.5634)	1.3781 (0.62162)	1.195 (0.676

Table (2) Shown the R.Eff. (·) and B(·) of $\tilde{\psi}$ w.r.t. α and λ_2

	0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	
R.Eff(·) B(·)	2080.9 ($4.0707\ e^{-20}$)	96.418 (0.09936)	24.981 (0.19868)	11.184 (0.29793)	6.3112 (0.39706)	4.048 (0.49604)	2.8167 (0.59482)	2.0736 (0.69335)	1.591 (0.7916)	1.2601 (0.88951)	1.0234 (0.98704)	0 (
R.Eff(·) B(·)	180.59 ($8.8577\ e^{-19}$)	67.159 (0.096048)	23.31 (0.19197)	11.182 (0.28765)	6.4832 (0.38297)	4.2189 (0.4778)	2.964 (0.57205)	2.1986 (0.6656)	1.6982 (0.75838)	1.3534 (0.8503)	1.1059 (0.9413)	0 (
R.Eff(·) B(·)	61.117 ($1.0505\ e^{-18}$)	40.007 (0.091493)	19.674 (0.18281)	10.677 (0.27378)	6.5289 (0.36424)	4.3687 (0.45403)	0.1218 (0.54302)	2.3429 (0.63109)	1.8259 (0.71814)	1.4658 (0.8041)	1.2053 (0.889)	0 (