Solving System of Linear Fredholm Integral Equations of Second Kind Using Open Newton-Cotes Formulas

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Abstract

In this paper, the linear system of Fredholm integral equations is solving using Open Newton-Cotes formula, which we use five different types of Open Newton-Cotes formula to solve this system.

Compare the results of suggested method with the results of another method (closed Newton-Cotes formula)

Finally, at the end of each method, algorithms and programs developed and written in MATLAB (version 7.0) and we give some numerical examples, illustrate suggested method.

Keyword: Open Newton – Cotes formula, Closed Newton-Cotes formula, System of linear Fredholm integral equation.

Introduction

The problem of Newton-Cotes formula arises when the integration cannot be carried out exactly or when the function is known only at a finite number of data. Furthermore, Newton-Cotes rules are primary tool used by engineers and scientists to obtain approximate answers for definite integrals that cannot be solved analytically. [1]

This paper is organized as follows: in Section 2, we introduce a brief introduction to the Open Newton-Cotes and some basic definitions for integral equation. In Section 3, we construct our methods to approximate the solution of linear system of Fredholm integral equations. Numerical examples are given in Section 4.

1- Review and Background

1.1 Some definitions of integral equation

Definition 1-1: [2]

Integral equation is an equation in which the unknown function appears under an integral sign.

A general form of **linear integral equations** may be written as follows:

$$h(x)u(x) = f(x) + \lambda \int_{a}^{b(x)} k(x,t)u(t)dt \qquad a \le x \le b \qquad \text{……………….(1)}$$

Where $h(x)$ and $f(x)$are given function of x, $k(x,t)$is a function of two variables x and t called kernel of integral equation which are also known, while $u(x)$ is to be determined and λ is a scalar parameter [in this paper we take $\lambda = 1$].

Definition 1-2: [2]

If the function h(x) =1, then the linear integral equation (1) is said to be an **equation of the second kind** (i.e.)

$$u(x) = f(x) + \int_{a}^{b(x)} k(x,t)u(t)dt \qquad a \le x \le b \qquad \text{……………………..(2)}$$

Definition 1-3: [2]

The integral equation (1) is called **Fredholm integral equation (FIE)** if $b(x) = b$, where b is constant such that $b \ge a$. Therefore, the integral equations

$$f(x) = \int_{a}^{b} k(x,t)u(t)dt \qquad a \le x \le b$$

$$u(x) = f(x) + \int_{a}^{b} k(x,t)u(t)dt \qquad a \le x \le b$$

Represent the one-dimensional Fredholm integral equation of the first and second kind respectively.

1.2 Open Newton-Cotes formula

In numerical analysis, the Newton-Cotes (N-C) formulas are a group of formulas of numerical integration based on evaluating the integrand at equally- spaced points. They are named after Isaac Newton and Roger Cotes [3]. There are two types of N-C formulas:

- The (closed) type which uses the function value at all point in the domain.

- The (open) type which does not use the function value at the initial and end point of the domain .

Numerical integration formulas of the form $\int_{a}^{b} f(x)dx = \int_{x_0}^{x_n} f(x)dx \approx \sum_{i=0}^{n} w_i f(x_i) + E(f)$

where $E(f)$ is the error, $x_i = x_0 + i * h \quad for \quad i = 0,1,..., n$, $x_0 = a, \ x_n = b \ and \ h = \frac{b-a}{n}$ and $w_i \quad i = 0,1,..., n$ are the **weights**, is called **closed Newton-Cotes formulas**

Some of the common **closed N-C** formulas are as follows:

- Trapezoidal rule , Simpson 1/3 rule and Simpson 3/8 rule

Numerical integration formulas of the form $\int_a^b f(x)dx = \int_{x_{-1}}^{x_{n+1}} f(x)dx \approx \sum_{i=0}^{n} w_i f(x_i) + E(f)$

where $E(f)$ is the error, $x_i = x_0 + (i+1)*h \quad for \ i = -1,0,\ldots,n+1$, $x_{-1} = a$, $x_{n+1} = b$, $h = \frac{b-a}{n+2}$ and $w_i \quad i = 0,1,\ldots,n$ are the **weights**, is called **open Newton-Cotes formulas.**

Some of the common **open N-C** formulas with their error terms are a follows: [1]

- n=0 Midpoint rule $\int_{x_{-1}}^{x_1} f(x)dx = 2hf(x_0) + \frac{h^3}{3} f''(\zeta) \qquad where\ \zeta \in (x_{-1}, x_1)$

- n=1 $\int_{x_{-1}}^{x_2} f(x)dx = \frac{3h}{2}[f(x_0) + f(x_1)] + \frac{3h^3}{4} f''(\zeta) \qquad where\ \zeta \in (x_{-1}, x_2)$

- n=2 $\int_{x_{-1}}^{x_3} f(x)dx = \frac{4h}{3}[2f(x_0) - f(x_1) + 2f(x_2)] + \frac{14h^5}{45} f^{(4)}(\zeta) \qquad where\ \zeta \in (x_{-1}, x_3)$

- n=3 $\int_{x_{-1}}^{x_4} f(x)dx = \frac{5h}{24}[11f(x_0) + f(x_1) + f(x_2) + 11f(x_3)] + \frac{95h^5}{144} f^{(4)}(\zeta) \quad where\ \zeta \in (x_{-1}, x_4)$

- n=4 $\int_{x_{-1}}^{x_5} f(x)dx = \frac{3h}{10}[11f(x_0) - 14f(x_1) + 26f(x_2) - 14f(x_3) + 11f(x_4)] + \frac{4h^7}{1404} f^{(6)}(\zeta) \quad where\ \zeta \in (x_{-1}, x_5)$

2- Solution of a system of linear Fredholm integral equations of the second kind Using Open N-C formulas

In this section, we use the common formula of Open N-C to solve the system of linear Fredholm integral equations of the second kind.

Consider the system of linear Fredholm integral equations of the second kind

$$u_r(x) = f_r(x) + \sum_{s=1}^{n} \int_a^b k_{rs}(x,t) u_s(x) dt \qquad , r = 1,2,\ldots, n \qquad \ldots\ldots(3)$$

where $n \in N$, f_r , k_{rs} , $r,s = 1,2,\ldots,n$ are assumed to be continuous function.

Suppose that the interval [a, b] is divided into n+2 equal subintervals of length $h = \frac{b-a}{n+2}$, such that $a = x_{-1}, b = x_{n+1}$ with $x_i = \alpha + i*h \quad , i = 0,1,\ldots,n$ this implies that $x_0 = a + h$ and $x_n = b - h$ and $u_r(x_i)$ for $i = 0,1,\ldots, n$, $r = 1,2,\ldots, n$ can be determined by:

$$u_r(x_i) = f_r(x_i) + \sum_{s=1}^{n} \int_a^b k_{rs}(x_i,t) u_s(t) dt \qquad , i = 0,1,\cdots,n \quad , r = 1,2,\cdots,n \qquad \ldots\ldots\ldots\ldots(4)$$

Thus, we are approximating each integral term by the open N-C formulas.

2.1 Using Open N-C: with n=0 (Midpoint Rule)

We replace the integral term that appeared in the right hand side of the above equation by the composite midpoint rule which illustrate in the following theorem: [4]

Theorem: Let $f \in C^2[a,b]$. With $h=(b-a)/(2m+2)$ and $x_j = \alpha + (j+1)h$, $j=-1,0,\ldots,2m+1$

, the midpoint rule for n=2m subintervals is:

$$\int_a^b f(x)dx = 2h\sum_{j=0}^{m} f(x_{2j}) + \frac{b-a}{6} h^{20} f''(\mu) \text{ for some } \mu \in (a,b).$$

If the number of subinterval is even we apply open N-C with **(n=1)** rule. Therefore $u_{r_{2i}} \quad ,r=1,2,\cdots,n \quad ,i=0,1,\cdots,m \quad where \quad m=n/2$ are obtained by solving the equation:

$$u_{r_i} = f_{r_i} + 2h\sum_{j=0}^{n/2}\left[k_{rs}(x_{2i},x_{2j})\right]u_{s_{2j}} \quad ,i=0,1,\cdots,n/2 \quad ,r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad \ldots\ldots\ldots\ldots(5)$$

where $u_{r_{2i}}$ denote the numerical solution at x_{2i} , $i=0,1,\ldots,m$, m=n/2,

Transform all the terms involving the solution $u_{r_{2i}}$, $r=1,2,\cdots,n$, $i=0,1,\cdots,m=n/2$ to left side of the equation (5) and $f_{r_{2i}}$ to the right side.

$$\begin{bmatrix}
1-2hk_{11}(x_0,x_0) & -2hk_{11}(x_0,x_1)\cdots & -2hk_{11}(x_0,x_m) & \cdots-2hk_{1n}(x_0,x_0) & -2hk_{1n}(x_0,x_1) & \cdots -2hk_{1n}(x_0,x_m)\\
-2hk_{11}(x_1,x_0) & 1-2hk_{11}(x_1,x_1)\cdots & -2hk_{11}(x_1,x_m) & \cdots-2hk_{1n}(x_1,x_0) & -2hk_{1n}(x_1,x_1) & \cdots -2hk_{1n}(x_1,x_m)\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
-2hk_{11}(x_m,x_0) & -2hk_{11}(x_m,x_1)\cdots & 1-2hk_{11}(x_m,x_m) & \cdots-2hk_{1n}(x_m,x_0) & -2hk_{1n}(x_m,x_1) & \cdots -2hk_{1n}(x_m,x_m)\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
-2hk_{n1}(x_0,x_0) & -2hk_{n1}(x_0,x_1)\cdots & -2hk_{n1}(x_0,x_m) & \cdots 1-2hk_{nn}(x_0,x_0) & -2hk_{nn}(x_0,x_1) & \cdots -2hk_{nn}(x_0,x_m)\\
-2hk_{n1}(x_1,x_0) & -2hk_{n1}(x_1,x_1)\cdots & -2hk_{n1}(x_1,x_m) & \cdots-2hk_{nn}(x_1,x_0) & 1-2hk_{nn}(x_1,x_1) & \cdots -2hk_{nn}(x_1,x_m)\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
-2hk_{n1}(x_m,x_0) & -2hk_{n1}(x_m,x_1)\cdots & -2hk_{n1}(x_m,x_m) & \cdots-2hk_{nn}(x_m,x_0) & -2hk_{nn}(x_m,x_1) & \cdots 1-2hk_{nn}(x_m,x_m)
\end{bmatrix}
\begin{bmatrix} u_{11}\\ u_{12}\\ \vdots\\ u_{1n}\\ \vdots\\ u_{m1}\\ u_{m2}\\ \vdots\\ u_{mn}\end{bmatrix}
=
\begin{bmatrix} f_{11}\\ f_{12}\\ \vdots\\ f_{1n}\\ \vdots\\ f_{m1}\\ f_{m2}\\ \vdots\\ f_{mn}\end{bmatrix}\ldots(6)$$

Remark: equation (6) has unique solution if the determent of the matrix K not equal to zero

Also, if the number of subintervals is odd, we get combination between open N-C: n=1 and open N-C: n=0: Midpoint rules.

Therefore $u_{r_i} \quad ,r=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+2)-m$ where $m=(n+1)/2$ are obtained by solving the equations:

$$u_{r_i} = f_{r_i} + \frac{3h}{2}k_{rs_{i1}}u_{s_1} + \frac{3h}{2}k_{rs_{i2}}u_{s_2} + 2h\sum_{j=3}^{(n+2)-m} k_{rs_{ij}}u_{s_j}$$

$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+2)-m \quad where \quad m=(n+1)/2 \quad \ldots\ldots(7)$$

Transform all the terms involving the solution $u_{r_i} \quad ,r=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+2)-m$ where $m=(n+1)/2$ to left side of the equation (7) and f_{r_i} to the right side.

Finally, each of system in equations (6) and (7) can be written in matrix form as $KU = F$ where K is the matrix of the coefficients, U is the matrix of solution and F is the matrix of non-homogeneous part. To find the approximate solution u_{r_i} $, r = 1,2,\cdots,n$ we find $U = K^{-1}F$.

The Algorithm of Numerical Solution of a System of linear Fredholm Integral Equations Using Open N-C: {n=0} (SONCn0)

Step 1: compute $h = \dfrac{b-a}{n+2}$, $n \in N$

Step 2:

- Compute $u_{r_{2i}}$ $, r = 1,2,\cdots,n$ $, i = 0,1,\cdots, m = n/2$ **,** using equation (6) when the number of subintervals is even.
- Compute u_{r_i} $, r = 1,2,\cdots,n$ $, i = 1,2,\cdots,(n+2)-m$ where $m = (n+1)/2$ **,** using equation (7) when the number of subintervals is odd.

Step 3: solve the resulting system by multiplication it with K^{-1}**.**

2.2 Using Open N-C: n=1 Rule

By the same steps of condition on equation (4), use the open N-C where n=1 formula to approximate each integral term in equation (4). If the number of subintervals is (a multiple of three).

Therefore u_{r_i} $, r = 1,2,\cdots,n$ $, i = 1,2,\cdots,(n+2)-m$ where $m = (n+2)/3$ are obtained by solving the equations:

we apply open N-C with **(n=1)**. Therefore u_{r_i} $, r = 1,2,\cdots,n$ $, i = 1,2,\cdots,(n+2)-m$, where $m = (n+2)/3$ are obtained by solving the equations: where $m = (n+2)/3$

$$u_{r_i} = f_{r_i} + \frac{3h}{2}\sum_{j=1}^{(n+2)-m}\left[k_{rs}(x_i, x_j)\right]u_{s_j} \quad , i = 1,2,\cdots,(n+2)-m \quad , r = 1,2,\cdots,n \quad , s = 1,2,\cdots,n \qquad \ldots..(8)$$

where u_{r_i} denote the numerical solution at x_i $, i = 1,2,\ldots,(n+2)-m$, *where* $m = (n+2)/3$

Transform all the terms involving the solution u_{r_i} $, r = 1,2,\cdots,n$, i=1,2,..,(n+2)-m to left side of the equation (8) and fr_i to the right side.

$$\begin{bmatrix}
1-\frac{3}{2}hk_{11}(x_1,x_1) & -\frac{3}{2}hk_{11}(x_1,x_2)\cdots & -\frac{3}{2}hk_{11}(x_1,x_m)\cdots & -\frac{3}{2}hk_{1n}(x_1,x_1) & -\frac{3}{2}hk_{1n}(x_1,x_2) & \cdots -\frac{3}{2}hk_{1n}(x_1,x_m) \\
-\frac{3}{2}hk_{11}(x_2,x_1) & 1-\frac{3}{2}hk_{11}(x_2,x_2)\cdots & -\frac{3}{2}hk_{11}(x_2,x_m)\cdots & -\frac{3}{2}hk_{1n}(x_2,x_1) & -\frac{3}{2}hk_{1n}(x_2,x_2) & \cdots -\frac{3}{2}hk_{1n}(x_2,x_m) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\frac{3}{2}hk_{11}(x_m,x_1) & -\frac{3}{2}hk_{11}(x_m,x_2)\cdots & 1-\frac{3}{2}hk_{11}(x_m,x_m)\cdots & -\frac{3}{2}hk_{1n}(x_m,x_1) & -\frac{3}{2}hk_{1n}(x_m,x_2) & \cdots -\frac{3}{2}hk_{1n}(x_m,x_m) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\frac{3}{2}hk_{n1}(x_1,x_1) & -\frac{3}{2}hk_{n1}(x_1,x_2)\cdots & -\frac{3}{2}hk_{n1}(x_1,x_m)\cdots & 1-\frac{3}{2}hk_{nn}(x_1,x_1) & -\frac{3}{2}hk_{nn}(x_1,x_2) & \cdots -\frac{3}{2}hk_{nn}(x_1,x_m) \\
-\frac{3}{2}hk_{n1}(x_2,x_1) & -\frac{3}{2}hk_{n1}(x_2,x_2)\cdots & -\frac{3}{2}hk_{n1}(x_2,x_m)\cdots & -\frac{3}{2}hk_{nn}(x_2,x_1) & 1-\frac{3}{2}hk_{nn}(x_2,x_2) & \cdots -\frac{3}{2}hk_{nn}(x_2,x_m) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\frac{3}{2}hk_{n1}(x_m,x_1) & -\frac{3}{2}hk_{n1}(x_m,x_2)\cdots & -\frac{3}{2}hk_{n1}(x_m,x_m)\cdots & -\frac{3}{2}hk_{nn}(x_m,x_1) & -\frac{3}{2}hk_{nn}(x_m,x_2) & \cdots 1-\frac{3}{2}hk_{nn}(x_m,x_m)
\end{bmatrix}
\begin{bmatrix} u_{11} \\ u_{12} \\ \vdots \\ u_{1n} \\ \vdots \\ u_{m1} \\ u_{m2} \\ \vdots \\ u_{mn} \end{bmatrix}
=
\begin{bmatrix} f_{11} \\ f_{12} \\ \vdots \\ f_{1n} \\ \vdots \\ f_{m1} \\ f_{m2} \\ \vdots \\ f_{mn} \end{bmatrix} \quad \ldots\ldots..(9)$$

Also, if the number of subintervals is (a multiple of three +1), we get combination between open N-C : n=0 Midpoint and open N-C: n=1 rules.

Therefore u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=(n+1)/3$ are obtained by solving the equation:

$$u_{r_i} = f_{r_i} + 2hk_{rs_{i1}}u_{s_1} + \frac{3h}{2}\sum_{j=2}^{(n+2)-m} k_{rs_{ij}}u_{s_j} \quad \ldots\ldots.(10)$$

$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+2)-m \quad where \;\; m=(n+1)/3$$

Transform all the terms involving the solution u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$, where $m=(n+1)/3$ to left side of the equation (10) and f_{r_i} to the right side.

if the number of subintervals is (a multiple of three +2), we get combination between open N-C : n=0 (Midpoint method) and open N-C : n=1 rules.

Therefore u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$, where $m=n/3$ are obtained by solving the equation:

$$u_{r_i} = f_{r_i} + 2hk_{rs_{i1}}u_{s_1} + 2hk_{rs_{i2}}u_{s_2} + \frac{3h}{2}\sum_{j=3}^{(n+2)-m} k_{rs_{ij}}u_{s_j} \quad \ldots\ldots\ldots.(11)$$

$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where \;\; m=n/3$$

Transform all the terms involving the solution u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=n/3$ to left side of the equation (11) and f_{r_i} to the right side.

Finally, each system in equations (9), (10) and (11) can be written in a matrix form as $KU = F$ where K is the matrix of the coefficients, U is the matrix of solution and F is the matrix of non-homogeneous part. To find the approximate solution u_{r_i} $, r=1,2,\cdots,n$ we find

$$U = K^{-1}F$$

The Algorithm of Open N-C {n=1} (SONCn1)

Step 1: compute $h = \dfrac{b-a}{n+2}$, $n \in N$

Step 2:

- Compute u_{r_i} $, r = 1,2,\cdots,n$, $i=1,2,\ldots,(n+2)-m$ where $m = (n+2)/3$, using equation (9) when the number of subintervals is (a multiple of 3).
- Compute u_{r_i} $, r = 1,2,\cdots,n$ $, i = 1,2,\cdots,(n+1)-m$ where $m = (n+1)/3$, using equation (10) when the number of subintervals is (a multiple of 3 +1).
- Compute u_{r_i} $, r = 1,2,\cdots,n$ $, i = 1,2,\cdots,(n+1)-m$ where $m = n/3$, using equation (11) when the number of subintervals is (a multiple of 3 +2).

Step 3: solve the resulting system by multiplication it with K^{-1}.

2.3 Using Open N-C: n=2 Rule

The open N-C: n=2 formula can be used to approximate equation (4) such that:

If the number of subintervals is (a multiple of four), we apply the open N-C: n=2 formula to each integral term in equation (4) as the form:

$$u_{r_i} = f_{r_i} + \frac{4h}{3}\left[2k_{rs_{i1}}u_{s_1} - k_{rs_{i2}}u_{s_2} + 2k_{rs_{i3}}u_{s_3} + 2k_{rs_{i4}}u_{s_4} + \cdots - k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}} + 2k_{rs_{i((n+2)-m)}}u_{s_{(n+2)-m}}\right]$$

$$for \quad r = 1,2,\cdots,n \quad , s = 1,2,\cdots,n \quad , i = 1,2,\cdots,(n+2)-m \quad where \ m = (n+2)/4 \qquad \ldots\ldots(12)$$

Transform all the terms involving the solution u_{r_i} $, r = 1,2,\cdots,n$, $i=1,2,\ldots,(n+2)-m$ where $m = (n+2)/4$ to the left side of the equation (12) and f_{r_i} to the right side.

Also, if the number of subintervals (n+2) is (a multiple of four +1), we get combination between open N-C: n=0, open N-C: n=1 and open N-C: n=2 rules and u_{r_i} $, r = 1,2,\cdots,m$ $, i = 1,2,\cdots,(n+1)-m$ where $m = (n+1)/4$ are obtained by solving the system of equations:

$$u_{r_i} = f_{r_i} + 2hk_{rs_{i1}}u_{s_1} + \frac{3h}{2}k_{rs_{i2}}u_{s_2} + \frac{3h}{2}k_{rs_{i3}}u_{s_3} + \frac{4h}{3}[2k_{rs_{i4}}u_{s_4} - k_{rs_{i5}}u_{s_5} \cdots - k_{rs_{i(n-m)}}u_{s_{n-m}} + 2k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$

$$for \quad r = 1,2,\cdots,n \quad , s = 1,2,\cdots,n \quad , i = 1,2,\cdots,(n+1)-m \quad where \quad m = (n+1)/4 \qquad \ldots(13)$$

Transform all the terms involving the solution u_{r_i} $, r = 1,2,\cdots,n$, $i=1,2,\ldots,(n+1)-m$, where $m = (n+1)/4$ to the left side of the equation (13) and f_{r_i} to the right side.

If the number of subintervals (n+2) is (a multiple of four +2), we get combination between open N-C: n=0 and open N-C: n=2 rules and u_{r_i} $, r = 1,2,\cdots,m$ $, i = 1,2,\cdots,(n+1)-m$, where $m = n/4$ are obtained by solving the system of equations.

$$u_{r_i} = f_{r_i} + 2hk_{rs_{i1}}u_{s_1} + \frac{4h}{3}[2k_{rs_{i2}}u_{s_2} - k_{rs_{i3}}u_{s_3} + 2k_{rs_{i4}}u_{s_4} \cdots - k_{rs_{i(n-m)}}u_{s_{n-m}} + 2k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$

$$for \quad r = 1,2,\cdots,n \quad , s = 1,2,\cdots,n \quad , i = 1,2,\cdots,(n+1)-m \quad where \ m = n/4 \qquad \ldots\ldots\ldots\ldots(14)$$

Transform all the terms involving the solution u_{r_i} $, r = 1,2,\cdots,n$, $i=1,2,\ldots,(n+1)-m$ where $m = n/4$ to the left side of the equation (14) and f_{r_i} to the right side.

]Also, if the number of subintervals (n+2) is (a multiple of four +3) we get combination between open N-C: n=1 and open N-C: n=2 rules and u_{r_i} $,r=1,2,\cdots,m$ $,i=1,2\cdots,(n+1)-m,$ where $m=(n-1)/4$ are obtained by solving the system of equations.

$$u_{r_i}=f_{r_i}+\frac{3h}{2}hk_{rs_{i1}}u_{s_1}+\frac{3h}{2}k_{rs_{i2}}u_{s_2}+\frac{4h}{3}[2k_{rs_{i3}}u_{s_3}-k_{rs_{i4}}u_{s_4}\cdots-k_{rs_{i(n-m)}}u_{s_{n-m}}+2k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$
$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where \;\; m=(n-1)/4 \qquad \text{……..(15)}$$

Transform all the terms involving the solution u_{r_i} $,r=1,2,\cdots,n,$ i=1,2,..,(n+1)-m where $m=(n-1)/4$ to the left side of the equation (15) and f_{r_i} to the right side.

Finally, each system in equations (12), (13), (14) and (15) can be written in a matrix form as $KU=F$, where K is the matrix of the coefficients, U is the matrix of solution and F is the matrix of non-homogeneous part. To find the approximate solution u_{r_i} , $r=1,2,\cdots,n,$ we find $U=K^{-1}F$

The Algorithm of Open N-C:{n=2} (SONCn2)

Step 1: compute $h=\frac{b-a}{n+2}$, $n\in N$

Step 2:

❖ Compute u_{r_i} $,r=1,2,\cdots,n,$ i=1,2,..,(n+2)-m where $m=(n+2)/4$, using equation (12) when the number of subintervals is (a multiple of 4).
❖ Compute u_{r_i} $,r=1,2,\cdots,n$ $,i=1,2,\cdots,(n+1)-m$ where $m=(n+1)/4$, using equation (13) when the number of subintervals is (a multiple of 4 +1).
❖ Compute u_{r_i} $,r=1,2,\cdots,n$ $,i=1,2,\cdots,(n+1)-m$ where $m=n/4$, using equation (14) when the number of subintervals is (a multiple of 4 +2).
❖ Compute u_{r_i} $,r=1,2,\cdots,n$ $,i=1,2,\cdots,(n+1)-m$ where $m=(n-1)/4$, using equation (15) when the number of subintervals is (a multiple of 4 +3).

Step 3: solve the resulting system by multiplication it with K^{-1}.

2.4 Using Open N-C: n=3 Rule

The open N-C: n=3 formula can be used to approximate equation (4) such that:

If the number of subintervals is (a multiple of five) we apply the open N-C: n=3 formula to each integral term in equation (4) as the form:

$$u_{r_i}=f_{r_i}+\frac{5h}{24}\left[11k_{rs_{i1}}u_{s_1}+k_{rs_{i2}}u_{s_2}+k_{rs_{i3}}u_{s_3}+11k_{rs_{i4}}u_{s_4}+\cdots+k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}+11k_{rs_{i((n+2)-m)}}u_{s_{(n+2)-m}}\right] \qquad \text{……….(16)}$$
$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+2)-m \quad where \;\; m=(n+2)/5$$

Transform all the terms involving the solution u_{r_i} $,r=1,2,\cdots,n,$ i=1,2,..,(n+2)-m, where $m=(n+2)/5$ to the left side of the equation (16) and f_{r_i} to the right side.

Also, if the number of subintervals (n+2) is (a multiple of five +1) we get combination between open N-C: n=1 and open N-C: n=3 rules and u_{r_i} $,r=1,2,\cdots,m$ $,i=1,2\cdots,(n+1)-m,$ where $m=(n+1)/5$ are obtained by solving the system of equations.

$$u_{r_i}=f_{r_i}+\frac{3h}{2}hk_{rs_{i1}}u_{s_1}+\frac{3h}{2}k_{rs_{i2}}u_{s_2}+\frac{3h}{2}k_{rs_{i3}}u_{s_3}+\frac{3h}{2}k_{rs_{i4}}u_{s_4}+\frac{5h}{24}[11k_{rs_{i5}}u_{s_5}+\cdots+k_{rs_{i(n-m)}}u_{s_{n-m}}+11k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$

$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where\ m=(n+1)/5 \qquad ...(17)$$

Transform all the terms involving the solution u_{r_i} $,r=1,2,\cdots,n$, i=1,2,..,(n+1)-m, where $m=(n+1)/5$ to the left side of the equation (17) and f_{r_i} to the right side.

If the number of subintervals (n+2) is (a multiple of five +2) we get combination between open N-C: n=0 and open N-C: n=3 rules and u_{r_i} $,r=1,2,\cdots,m$ $,i=1,2\cdots,(n+1)-m$, where $m=n/5$ are obtained by solving the system of equations.

$$u_{r_i}=f_{r_i}+2hk_{rs_{i1}}u_{s_1}+\frac{5h}{24}[11k_{rs_{i2}}u_{s_2}+k_{rs_{i3}}u_{s_3}+k_{rs_{i4}}u_{s_4}\cdots+k_{rs_{i(n-m)}}u_{s_{n-m}}+11k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$

$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where\ \ m=n/5 \qquad ....(18)$$

Transform all the terms involving the solution u_{r_i} $,r=1,2,\cdots,n$, i=1,2,..,(n+1)-m where $m=n/5$ to the left side of the equation (18) and f_{r_i} to the right side.

Also, if the number of subintervals (n+2) is (a multiple of five +3) we get combination between open N-C: n=1 and open N-C: n=3 rules and u_{r_i} $,r=1,2,\cdots,m$ $,i=1,2\cdots,(n+1)-m,$ where $m=(n-1)/5$ are obtained by solving the system of equations.

$$u_{r_i}=f_{r_i}+\frac{3h}{2}hk_{rs_{i1}}u_{s_1}+\frac{3h}{2}k_{rs_{i2}}u_{s_2}+\frac{5h}{24}[11k_{rs_{i3}}u_{s_3}+k_{rs_{i4}}u_{s_4}\cdots+k_{rs_{i(n-m)}}u_{s_{n-m}}+11k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$

$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where\ \ m=(n-1)/5 \qquad ....(19)$$

Transform all the terms involving the solution u_{r_i} $,r=1,2,\cdots,n$, i=1,2,..,(n+1)-m, where $m=(n-1)/5$ to the left side of the equation (19) and f_{r_i} to the right side.

And, if the number of subintervals (n+2) is (a multiple of five +4) we get combination between open N-C: n=2 and open N-C: n=3 rules and u_{r_i} $,r=1,2,\cdots,m$ $,i=1,2\cdots,(n+1)-m,$ where $m=(n-2)/5$ are obtained by solving the system of equations.

$$u_{r_i}=f_{r_i}+\frac{4h}{3}[2hk_{rs_{i1}}u_{s_1}-k_{rs_{i2}}u_{s_2}+2k_{rs_{i3}}u_{s_3}]+\frac{5h}{24}[11k_{rs_{i4}}u_{s_4}+\cdots+k_{rs_{i(n-m)}}u_{s_{n-m}}+11k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$

$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where\ \ m=(n-2)/5 \qquad ...(20)$$

Transform all the terms involving the solution u_{r_i} $,r=1,2,\cdots,n$, i=1,2,..,(n+1)-m, where $m=(n-2)/5$ to the left side of the equation (20) and f_{r_i} to the right side.

Finally, each system in equations (16), (17), (18), (19) and (20) can be written in a matrix form as $KU=F$ where K is the matrix of the coefficients, U is the matrix of solution and F is the matrix of non-homogeneous part. To find the approximate solution u_{r_i} $, r=1,2,\cdots,n$ we find $U = K^{-1}F$

The Algorithm of Open N-C {n=3} (SONCn3)

Step 1: compute $h = \dfrac{b-a}{n+2}$, $n \in N$

Step 2:

- Compute u_{r_i} $, r=1,2,\cdots,n$, i=1,2,..,(n+2)-m where $m=(n+2)/5$, using equation (16) when the number of subintervals is (a multiple of 5).
- Compute u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=(n+1)/5$, using equation (17) when the number of subintervals is (a multiple of 5 +1).
- Compute u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=n/5$, using equation (18) when the number of subintervals is (a multiple of 5 +2).
- Compute u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=(n-1)/5$, using equation (19) when the number of subintervals is (a multiple of 5 +3).
- Compute u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=(n-2)/5$, using equation (20) when the number of subintervals is (a multiple of 5 +4).

Step 3: solve the resulting system by multiplication it with K^{-1}.

2.4 Using Open N-C: n=4 Rule

The open N-C: n=4 formula can be used to approximate equation (4) such that:

If the number of subintervals is (a multiple of six) we apply the open N-C: n=4 formula to each integral term in equation (4) as the form:

$$u_{r_i} = f_{r_i} + \frac{3h}{10}\left[11k_{rs_{i1}}u_{s_1} - 14k_{rs_{i2}}u_{s_2} + 26k_{rs_{i3}}u_{s_3} - 14k_{rs_{i4}}u_{s_4} + 11k_{rs_{i5}}u_{s_5} + \cdots - 14k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}} + 11k_{rs_{i((n+2)-m)}}u_{s_{(n+2)-m}}\right] \quad ....(21)$$

$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+2)-m \quad where\ m=(n+2)/6$$

Transform all the terms involving the solution u_{r_i} $, r=1,2,\cdots,n$, i=1,2,..,(n+2)-m, where $m=(n+2)/6$ to the left side of the equation (21) and f_{r_i} to the right side.

Also, if the number of subintervals (n+2) is (a multiple of six +1) we get combination between open N-C: n=0, open N-C: n=3, and open N-C: n=4 rules and u_{r_i} $, r=1,2,\cdots,m$ $, i=1,2\cdots,(n+1)-m,$ where $m=(n+1)/6$ are obtained by solving the system of equations.

$$u_{r_i} = f_{r_i} + 2hk_{rs_{i1}}u_{s_1} + \frac{5h}{24}[11k_{rs_{i2}}u_{s_2} + k_{rs_{i3}}u_{s_3} + k_{rs_{i4}}u_{s_4} + 11k_{rs_{i5}}u_{s_5}] + \frac{3h}{10}[11k_{rs_{i6}}u_{s_6} - \cdots - 14k_{rs_{i(n-m)}}u_{s_{n-m}} + 11k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$

$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where\ m=(n+1)/5 \quad ....(22)$$

Transform all the terms involving the solution u_{r_i} $, r=1,2,\cdots,n$, i=1,2,..,(n+1)-m, where $m=(n+1)/6$ to the left side of the equation (22) and f_{r_i} to the right side.

If the number of subintervals (n+2) is (a multiple of six +2) we get combination between open N-C: n=0 and open N-C: n=4 rules and u_{r_i} $,r=1,2,\cdots,m$ $,i=1,2\cdots,(n+1)-m$ where $m=n/6$ are obtained by solving the system of equations.

$$u_{r_i}=f_{r_i}+2hk_{rs_{i1}}u_{s_1}+\frac{3h}{10}[11k_{rs_{i2}}u_{s_2}-14k_{rs_{i3}}u_{s_3}+26k_{rs_{i4}}u_{s_4}\cdots-14k_{rs_{i(n-m)}}u_{s_{n-m}}+11k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$
$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where \ m=n/6 \qquad .....(23)$$

Transform all the terms involving the solution u_{r_i} $,r=1,2,\cdots,n$, i =1,2,..,(n+1)-m, where $m=n/6$ to the left side of the equation (23) and f_{r_i} to the right side.

Also, if the number of subintervals (n+2) is (a multiple of six +3) we get combination between open N-C: n=1 and open N-C: n=4 rules and u_{r_i} $,r=1,2,\cdots,m$ $,i=1,2\cdots,(n+1)-m,$ where $m=(n-1)/6$ are obtained by solving the system of equations.

$$u_{r_i}=f_{r_i}+\frac{3h}{2}hk_{rs_{i1}}u_{s_1}+\frac{3h}{2}k_{rs_{i2}}u_{s_2}+\frac{3h}{10}[11k_{rs_{i3}}u_{s_3}-14k_{rs_{i4}}u_{s_4}\cdots-14k_{rs_{i(n-m)}}u_{s_{n-m}}+11k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$
$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where \ m=(n-1)/6 \qquad ....(24)$$

Transform all the terms involving the solution u_{r_i} $,r=1,2,\cdots,n$, i =1,2,..,(n+1)-m where $m=(n-1)/6$ to the left side of the equation (24) and f_{r_i} to the right side.

If the number of subintervals (n+2) is (a multiple of six +4) we get combination between open N-C: n=2 and open N-C: n=4 rules and u_{r_i} $,r=1,2,\cdots,m$ $,i=1,2\cdots,(n+1)-m,$ where $m=(n-2)/6$ are obtained by solving the system of equations.

$$u_{r_i}=f_{r_i}+\frac{4h}{3}[2hk_{rs_{i1}}u_{s_1}-k_{rs_{i2}}u_{s_2}+2k_{rs_{i3}}u_{s_3}]+\frac{3h}{10}[11k_{rs_{i4}}u_{s_4}-\cdots-14k_{rs_{i(n-m)}}u_{s_{n-m}}+11k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$
$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where \ m=(n-2)/6 \qquad ....(25)$$

Transform all the terms involving the solution u_{r_i} $,r=1,2,\cdots,n$, i =1,2,..,(n+1)-m, where $m=(n-2)/6$ to the left side of the equation (25) and fr_i to the right side.

Also, if the number of subintervals (n+2) is (a multiple of six +5) we get combination between open N-C: n=3 and open N-C: n=4 rules and u_{r_i} $,r=1,2,\cdots,m$ $,i=1,2\cdots,(n+1)-m,$ where $m=(n-3)/6$ are obtained by solving the system of equations.

$$u_{r_i}=f_{r_i}+\frac{5h}{24}[11hk_{rs_{i1}}u_{s_1}+k_{rs_{i2}}u_{s_2}+k_{rs_{i3}}u_{s_3}+11k_{rs_{i4}}u_{s_4}]+\frac{3h}{10}[11k_{rs_{i5}}u_{s_5}-\cdots-14k_{rs_{i(n-m)}}u_{s_{n-m}}+11k_{rs_{i((n+1)-m)}}u_{s_{(n+1)-m}}]$$
$$for \quad r=1,2,\cdots,n \quad ,s=1,2,\cdots,n \quad ,i=1,2,\cdots,(n+1)-m \quad where \ m=(n-3)/6 \qquad ....(26)$$

Transform all the terms involving the solution u_{r_i} $,r=1,2,\cdots,n$, i =1,2,..,(n+1)-m, where $m=(n-3)/6$ to the left side of the equation (26) and f_{r_i} to the right side.

Finally, each system in equations (21), (22), (23), (24), (25) and (26) can be written in a matrix form as $KU=F$ where K is the matrix of the coefficients, U is the matrix of solution and F is the matrix of non-homogeneous part. To find the approximate solution u_{r_i} $, r=1,2,\cdots,n$ we find $U = K^{-1}F$

The Algorithm of Open N-C {n=4} (SONCn4)

Step 1: compute $h = \dfrac{b-a}{n+2}$, $n \in N$

Step 2:

- Compute u_{r_i} $, r=1,2,\cdots,n$, i=1,2,..,(n+2)-m where $m=(n+2)/6$, using equation (21) when the number of subintervals is (a multiple of 6).
- Compute u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=(n+1)/6$, using equation (22) when the number of subintervals is (a multiple of 6 +1).
- Compute u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=n/5$, using equation (23) when the number of subintervals is (a multiple of 6 +2).
- Compute u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=(n-1)/6$, using equation (24) when the number of subintervals is (a multiple of 6 +3).
- Compute u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=(n-2)/6$, using equation (25) when the number of subintervals is (a multiple of 6 +4).
- Compute u_{r_i} $, r=1,2,\cdots,n$ $, i=1,2,\cdots,(n+1)-m$ where $m=(n-2)/6$, using equation (26) when the number of subintervals is (a multiple of 6 +5).

Step 3: solve the resulting system by multiplication it with K^{-1}.

3- Numerical Examples

In this section, we test some of the numerical examples performed to solving this linear system of Fredholm integral equations. The exact solution is used only to show the accuracy of the numerical solution which obtained with our method.

Example (1): Consider the problem:

$$u_1(x) = \frac{x}{18} + \frac{17}{36} + \int_0^1 \frac{(x+t)}{3}\big(u_1(t) + u_2(t)\big)dt$$

$$u_2(x) = x^2 - \frac{19}{12}x + 1 + \int_0^1 xt\big(u_1(t) + u_2(t)\big)dt$$

which is a system of two linear FIE's, with exact solution is [5]: $u_1(x) = x+1$ $, u_2(x) = x^2 + 1$

Tables (1) and (2) present a comparison between the exact and numerical solution of four types of Open Newton – Cotes and two types of Closed Newton – Cotes for u_1 and u_2 respectively depending on least square error and running time with h=1/16.

Example (2): Consider the problem:

$$u_1(x) = x^2 - \frac{x}{3} - \frac{29}{30} + \int_0^1 (x+t)\,u_1(t)dt + \int_0^1 t\,u_2(t)dt + \int_0^1 t^2\,u_3(t)\,dt$$

$$u_2(x) = \frac{2}{3}x^2 + \frac{5}{3}x - \frac{7}{6} + \int_0^1 (x^2)\,u_1(t)dt + \int_0^1 (2t)\,u_2(t)dt + \int_0^1 (-x)\,u_3(t)dt$$

$$u_3(x) = -x^2 - \frac{7}{12}x - \frac{11}{6} + \int_0^1 (x+1)\,u_1(t)dt + \int_0^1 3\,u_2(t)dt + \int_0^1 (t\,x)\,u_3(t)dt$$

which is a system of three linear FIE's, with exact solution is $u_1(x)=x^2$, $u_2(x)=x+x^2$, $u_3(x)=1-x^2$

Tables (3)- (5) present a comparison between the exact and numerical solution of four types of Open Newton – Cotes and three types of Closed Newton – Cotes for u_1, u_2 and u_3 respectively depending on least square error and running time with h= 0.1

Example (3): Consider the problem:

$$u_1(x) = \frac{5}{6}x^2 - \frac{25}{12}x + 1 + \int_0^1 x(1+t)\,u_1(t)dt + \int_0^1 x^2 t\,u_2(t)dt$$

$$u_2(x) = x^4 - \frac{1}{5}x^2 - \frac{7}{12}x + \int_0^1 xt\,u_1(t)dt + \int_0^1 (x^2 - xt)\,u_2(t)dt$$

which is a system of two linear FIE's, with exact solution is [6]: $u_1(x) = x^2 + 1$, $u_2(x) = x^4$

Tables (6) and (7) present a comparison between the exact and numerical solution of four types of Open Newton – Cotes and two types of Closed Newton – Cotes for u_1 and u_2 respectively depending on least square error and running time with h=1/18

Conclusion

In this paper we suggest open N-C formula to solve system of linear Fredholm integral equations of the second kind and we obtain the following results:

1-The results obtained using open N-C formulas are more accurate than the results obtained using closed N-C formulas in general.
2-Open N-C formulas are more efficient than closed N-C formulas since the open N-C formulas have most results than closed N-C with fewer nodes in the open N-C formulas.
3-The results obtained in open N-C formula when n=4 or multiple of four is most of the results in a short time in other open N-C formulas.

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Table (1) A comparison between the exact and numerical solution of 4 types of Open N– C and 2 types of Closed N– C for u_1

x	Exact u_1	Open N-C n=0	Open N-C n=1	Open N-C n=2	Open N-C n=3	Open N-C n=4	Closed N-C Trap.	Closed N-C Simp. 3/8
0.62500	1.0625000	1.05935926	1.05802317	1.06250000	1.06135355	1.0625000	1.06408126	1.06254649
0.12500	1.1250000			1.12500000	1.12376887	1.1250000	1.12668959	1.12505018
0.37500	1.3750000		1.36899583	1.37500000		1.3750000	1.31451459	1.31256126
0.50000	1.5000000		1.49338489		1.49826084	1.5000000	1.50233959	1.50007234
0.75000	1.7500000		1.74216302		1.74792216	1.7500000	1.75277293	1.75008711
0.87500	1.8750000		1.86655208		1.87275281	1.8750000	1.87798960	1.87509449
	L.S.E.	3.789e-005	7.662e-005	1.972e-031	5.437e-006	1.972e-031	1.028e-005	1.038e-008
	Time	0.07800	0.12500	0.203000	0.172000	0.18800	0.31200	0.297000

Table (2) A comparison between the exact and numerical solution of 4 types of Open N– C and 2 types of Closed N– C for u_2

x	Exact u_2	Open N-C n=0	Open N-C n=1	Open N-C n=2	Open N-C n=3	Open N-C n=4	Closed N-C Trap.	Closed N-C Simp. 3/8
0.62500	1.0039062	1.00335774	1.00312412	1.00390625	1.00370716	1.00390625	1.00418242	1.00391427
0.12500	1.0156250			1.01562500	1.01522683	1.0156250	1.01617734	1.01564105
0.37500	1.1406250		1.13593222	1.14062500		1.1406250	1.14228204	1.14067315
0.50000	1.2500000		1.24374297		1.24840733	1.2500000	1.25220939	1.25006420
0.75000	1.5625000		1.55311445		1.56011100	1.5625000	1.56581408	1.56259630
0.87500	1.7656250		1.75467520	1.76562500	1.76283783	1.7656250	1.76949143	1.76573735
	L.S.E.	6.769e-005	1.376e-004	4.930e-032	8.917e-006	0	1.952e-005	1.648e-008
	Time	0.07800	0.12500	0.203000	0.172000	0.18800	0.31200	0.297000

Table (3) A comparison between the exact and numerical solution of 4 types of Open N–C and 2 types of Closed N– C for u_1

x	Exact u_1	Open N-C n=0	Open N-C n=1	Open N-C n=2	Open N-C n=3	Open N-C n=4	Closed N-C Trap.	Closed N-C Simp. 3/8
0.1	0.010000	0.05805579	0.07564343	0.01634641	0.00939200	0.00985664	-0.0123549	0.00853472
0.3	0.090000	0.15296328	0.17608013	0.09823966	0.08918933	0.08980885	0.06074898	0.08810184
0.5	0.250000	0.32787077	0.35651683	0.26013291		0.24976106	0.21385289	0.24766898
0.7	0.490000	0.58277826		0.50202615	0.48878400	0.48971327	0.44695676	0.48723610
0.9	0.810000	0.91768575	0.95739024	0.82391940	0.80858133	0.80966549	0.76006071	0.80680323
L.S.E.		0.01159622	0.02172388	1.937 e-004	2.012 e-006	1.118 e-007	0.00285	1.16e-005
Time		0.078000	0.110000	0.110000	0.1710000	0.15600	1.188	1.172

Table (4) A comparison between the exact and numerical solution of 4 types of Open N–C and 2 types of Closed N– C for u_2

x	Exact u_2	Open N-C n=0	Open N-C n=1	Open N-C n=2	Open N-C n=3	Open N-C n=4	Closed N-C Trap.	Closed N-C Simp. 3/8
0.1	0.11000	0.06875521	0.05300473	0.10543206	0.11036142	0.11008521	0.12946933	0.11102031
0.3	0.39000	0.34870102	0.33304139	0.38528876	0.39043235	0.39010194	0.40942113	0.39105674
0.5	0.75000	0.71460984	0.70125273	0.74590276		0.75009955	0.76661449	0.75092003
0.7	1.19000	1.16648165		1.18727406	1.19033102	1.19007805	1.20104942	1.19061016
0.9	1.71000	1.70431646	1.70219946	1.70940265	1.71015875	1.71003743	1.71272591	1.71012715
L.S.E.		3.230 e-005	6.084 e-005	3.568 e-007	2.520 e-008	1.401 e-009	6.102e-006	3.214e-008
Time		0.078000	0.110000	0.110000	0.1710000	0.15600	1.188	1.172

Table (5) A comparison between the exact and numerical solution of 4 types of Open N– C and 2 types of Closed N– C for u_3

x	Exact u_3	Open N-C n=0	Open N-C n=1	Open N-C n=2	Open N-C n=3	Open N-C n=4	Closed N-C Trap.	Closed N-C Simp. 3/8
0.1	0.99000	0.99000000	0.97099871	0.98878499	0.98984800	0.98996416	0.99687951	0.99026659
0.3	0.91000	0.91000000	0.91867773	0.91131067	0.90954400	0.90989248	0.90749750	0.90969268
0.5	0.75000	0.75000000	0.78635675	0.75383634		0.74982080	0.73811548	0.74911876
0.7	0.51000	0.51000000		0.51636201	0.50893600	0.50974912	0.51000000	0.50854484
0.9	0.19000	0.19000000	0.28171479	0.19888768	0.18863200	0.18967744	0.19000000	0.18797093
L.S.E.		0.00450909	0.00841160	7.899 e-005	1.871 e-006	1.040 e-007	0.0012	5.36e-006
Time		0.07800	0.1100	0.11000	0.17100	0.15600	1.188	1.172

Table (6) A comparison between the exact and numerical solution of 4 types of Open N– C and 2 types of Closed N– C for u_1

x	Exact u_1	Open N-C n=0	Open N-C n=1	Open N-C n=2	Open N-C n=3	Open N-C n=4	Closed N-C Trap.	Closed N-C Simp. 3/8
0.05556	1.0030864	1.00056628	0.99935102	1.00299173	1.00286469	1.00308641	1.00439048	1.00308849
0.22222	1.0493827		1.03397239	1.04899455	1.04847428	1.04938271	1.05476199	1.04939139
0.38889	1.1512345	1.13248750	1.12344622	1.15053877	1.14960709	1.15123456	1.16093361	1.15125043
0.55556	1.3086419		1.26777251		1.30626312	1.30864197	1.32290534	1.30866561
0.72222	1.5216049	1.48473476	1.46695127	1.52025143		1.52160493	1.54067718	1.52163693
0.88889	1.7901234		1.72098248	1.78841988	1.78614486	1.79012345	1.81424913	1.79016439
L.S.E.		0.00250056	0.00549471	3.324e-006	1.812e-005	4.146e-029	7.634e-004	2.229e-009
Time		0.11000	0.157000	0.188000	0.203000	0.281000	0.359000	0.391000

Table (7) A comparison between the exact and numerical solution of 4 types of Open N– C and 2 types of Closed N– C for u_2

x	Exact u_2	Open N-C n=0	Open N-C n=1	Open N-C n=2	Open N-C n=3	Open N-C n=4	Closed N-C Trap.	Closed N-C Simp. 3/8
0.05556	0.0000095	−0.0005813	−0.00086507	−0.00001319	−0.00004540	0.00000952	0.00031686	0.00000979
0.22222	0.0024386		−0.00164220	0.00233587	0.00219055	0.00243865	0.00387053	0.00243999
0.38889	0.0228718	0.01736184	0.01471108	0.02267126	0.02238811	0.02287189	0.02573211	0.02287473
0.55556	0.0952598		0.08214541		0.09449787	0.09525986	0.09985221	0.09526463
0.72222	0.2720717	0.25928723	0.25312991	0.27162193		0.27207171	0.27869997	0.27207881
0.88889	0.6242950		0.59865224	0.62369401	0.62284907	0.62429507	0.63326303	0.62430494
L.S.E.		3.588e-004	7.879e-004	4.296e-007	2.485e-006	8.985e-030	1.144e-004	1.425e-010
Time		0.094000	0.157000	0.188000	0.203000	0.281000	0.359000	0.391000

حل منظومة معادلات فريدهوم التكاملية الخطية من النوع الثاني باستعمال صيغ نيوتن – كوتس المفتوحة

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الخلاصة

يتضمن هذا البحث حل منظومة من معادلات فريدهوم التكاملية الخطية باستعمال صيغ نيوتن –كوتس المفتوحة، اذ استعملنا خمس صيغ مختلفة من صيغ نيوتن –كوتس المفتوحة لحل هذا النظام .

كذلك قارنا نتائج الطريقة المقترحة في هذا البحث مع نتائج طرائق أخرى مثل طرائق نيوتن –كوتس المغلقة .

أخيرا في نهاية كل طريقة ذكرنا الخوارزمية وبرنامج للطريقة المقترحة للحل وبلغة (MATLAB (version 7.0)) أيضا وضحنا الطريقة المقترحة من خلال الأمثلة العددية .

الكلمات المفتاحية : صيغ نيوتن –كوتس المفتوحة ، صيغ نيوتن –كوتس المغلقة ، منظومة معادلات فريدهولم التكاملية الخطية.