المجموعات – α – شبه المنتظمة المغلقة

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الخلاصة

لقد قمنا في هذا البحث بتقديم ودراسة نوع جديد من المجموعات المغلقة في الفضاءات التبولوجية يدعى بالمجموعات – α – شبه المنتظمة المغلقة، اذ ان هذا النوع من المجموعات المغلقة تحوي مجموعات شبه مغلقة – α – وتكون محتواه في المجموعات قبل شبه المغلقة. وكما قدمنا ودرسنا نوعا جديدا من الدوال المستمرة والمترددة تدعى دالة من النمط – α – شبه المنتظمة المستمرة ودالة من النمط – α – شبه المنتظمة المترددة. كما وجدنا ان الاستمرارية من النمط – α – شبه المنتظمة تكون واقعة تماماً بين الاستمرارية من النمط شبه – α – والاستمرارية من النمط قبل الشبه.

الكلمات المفتاحية : المجموعة من النمط –α– شبه المنتظمة المغلقة ، الدالة من النمط –α– شبه المنتظمة المستمرة ، الدالة من النمط –α– شبه المنتظمة المترددة.

α - Semi-Regular Closed Sets

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Abstract

In this paper, a new class of sets, namely α- semi-regular closed sets is introduced and studied for topological spaces. This class properly contains the class of semi-α-closed sets and is property contained in the class of pre-semi-closed sets. Also, we introduce and study αsr-continuity and αsr-irresoleteness. We showed that αsr-continuity falls strictly in between semi-α- continuity and pre-semi-continuity.
Key words: α- semi-regular closed set, α- semi-regular continuous, α- semi-regular irresolute.

Introduction

Najasted [1] and Levine [2] introduced α-open sets and generalized closed sets, Kummar introduced α-generalized regular closed set and pre-semi closed set, see [3] and [4]. Alot of work was done in the field of generalized closed sets. In this paper we employ a new technique to obtain a new class of sets, called α-semi-regular closed sets. This class is obtained by semi-α-closed set and regular open set. It is shown that the class of α-semi-regular closed sets properly contains the class of semi-α-closed sets and is properly contained in the class of pre-semi-closed sets. We also introduce and study two classes of maps, namely, α-semi-regular continuity and α-semi-regular irresoluteness, α-semi-regular continuity falls strictly in between semi-α-continuity and pre-semi-continuity.

1- Preliminaries

Throughout this paper (X,τ) and (Y,τ') represent non-empty topological spaces. For a subset A of a space (X,τ), cl(A) and int(A) represent the closure of A and the interior of A respectively.

1.1 Definition:

A subset A of a space (X,τ) is called

(1) an α-open set [1], [5] if $A \subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A))) \subseteq A$.

(2) a semi-α-open set [6], [7] if $A \subseteq cl(int(cl(int(A))))$ and semi-α-closed if $int(cl(int(cl(A)))) \subseteq A$.

(3) a semi-preopen set [8], [9] if $A \subseteq cl(int(cl(A)))$ and semi-preclosed if $int(cl(int(A))) \subseteq A$.

(4) a regular open set [10], [11] if $A = int(cl(A))$ and regular closed if $A = cl(int(A))$.

(5) a generalized closed set (briefly g-closed) [2], [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ). The complement of a g-closed set is called a g-open set.

(6) an α-generalized closed set (briefly αg-closed) [13] if α $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

(7) a generalized α-closed set (briefly gα-closed) [14] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X,τ).

(8) a generalized α*-closed set (briefly gα*-closed) [14] if $\alpha cl(A) \subseteq int(U)$ whenever $A \subseteq U$ and U is α-open in (X,τ).

(9) an α**-generalized closed set (briefly α**g-closed) [14] if α cl(A) $\subseteq$ int(cl(U)) whenever A $\subseteq$ U and U is open in (X,τ).

(10) a generalized α**-closed set (briefly gα**-closed) [14] if αcl(A) $\subseteq$ int(cl(U)) whenever A $\subseteq$ U and U is α-open in (X,τ).

(11) a regular generalized closed set (briefly rg-closed) [15] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in (X,τ).

(12) an α-generalized regular closed set (briefly αgr-closed) [3] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in (X,τ).

(13) a generalized semi-preclosed set (briefly gsp-closed) [16] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).

(14) a pre-semi-closed set [4] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X,τ).

The semi-α-closure (resp. α-closure, semi-pre-closure) of A in (X,τ) is the intersection of all semi-α-closed (resp. α-closure, semi-pre-closure) sets of (X,τ) that contain A and is denoted by $S_\alpha$cl(A) (resp. αcl(A), spcl(A)).

1.2 Proposition:

(1) Every α-closed set is semi-α-closed set, not conversely, [6].

(2) Every closed set is α-closed set, so it is semi-α-closed set, not conversely, [6].

(3) Every closed (resp. α-closed, g-closed, gα-closed) set is an αgr-closed set, [3].

(4) Every gα*-closed (resp. α**g-closed, gα**-closed set is an αgr-closed set, [3].

(5) Every pre-semi-closed set ia gsp-closed set [4].

(6) Every semi-α-closed set is semi-pre-closed set

(the proof follows directly from the definitions).

1.3 Remark: [6]

Let X be a topological space, A and B be two subsets of X, then

(1) A is semi-α-closed set if and only if A = $S_\alpha$cl(A).

(2) A $\subseteq$ $S_\alpha$cl(A) $\subseteq$ αcl(A) $\subseteq$ cl(A).

(3) $S_\alpha$cl(A) $\subseteq$ $S_\alpha$cl(B), whenever A $\subseteq$ B.

1.4 Definition:

A function f:(X,τ) $\longrightarrow$ (Y,τ') is said to be:

(1) semi-α-continuous [6], [7] if f^{-1}(V) is a semi-α-closed set in (X,τ) for every closed set V of (Y,τ').

(2) g-continuous [17] if f^{-1}(V) is a g-closed set in (X,τ) for every closed set V of (Y,τ').

(3) αg-continuous [18] if f^{-1}(V) is an αg-closed set in (X,τ) for every closed set V of (Y,τ').

(4) gα -continuous [14] if f^{-1}(V) is a gα-closed set in (X,τ) for every closed set V of (Y,τ').

(5) αgr-continuous [3] if f^{-1}(V) is an αgr-closed set in (X,τ) for every closed set V of (Y,τ').

(6) pre-semi-continuous [4] if f^{-1}(V) is a pre-semi-closed set in (X,τ) for every closed set V of (Y,τ').

(7) gsp-continuous [16] if f^{-1}(V) is a gsp-closed set in (X,τ) for every closed set V of (Y,τ').

(8) semi-α-irresolute [6] if f^{-1}(V) is a semi-α-closed set in (X,τ) for every semi-α-closed set V of (Y,τ').

(9) αgr-irresolute [3] if f^{-1}(V) is an αgr-closed set in (X,τ) for every αgr-closed set V of (Y,τ').

(10) regular irresolute [19] if f^{-1}(V) is a regular open set in (X,τ) for every regular open set V of (Y,τ').

(11) semi-α*-closed [6] if f(U) is a semi-α-closed set in (Y,τ') for every semi-α-closed set U in (X,τ).

1.5 Proposition:

(1) Every αg-continuous map is αgr-continuous map [3].

(2) Every g-continuous (resp.gα-continuous) map is an αgr-continuous map [3].

(3) Every pre-semi-continuous map is gsp-continuous map [4].

(4) Every αgr-irresolut map is αgr-continuous map [3].
(5) Every continuous and open map is semi-α-irresolute map [20].

2- α-Semi-Regular Closed Sets

In this section we introduce the class of α-semi-regular closed sets and study some of it's basic properties.

2.1 Definition:

A subset A of (X,τ) is called α-semi-regular closed set (briefly αsr-closed) if $S_\alpha cl(A)\subseteq U$ whenever $A \subseteq U$ and U is regular open in (X,τ).
$\alpha SRC(X)$ denotes the collection of all αsr-closed subset of (X,τ)

2.2.Proposition:

Every semi-α-closed set is an αsr-closed set.

Proof: Let A be a semi-α-closed set, let U be a regular open set of (X,τ) such that $A \subseteq U$. Since $S_\alpha cl(A) = A$ for any semi-α-closed set (by part 1 of remark 1.3), then $S_\alpha cl(A) \subseteq U$. Therefore A is also an αsr-closed set.

The following example shows that the converse of the above proposition is not true in general.

2.3 Example:

Let $X=\{a,b,c\}$ and $\tau=\{X,\phi,\{a\},\{c\},\{a,c\}\}$. Let $A=\{a,c\}$, X is the only regular open set containing A. It is clear A is an αsr-closed set. But A is not semi-α-closed set since $S_\alpha cl(\{a,c\}) = X \neq \{a,c\}$.

Thus the class of αsr-closed set properly contains the class of semi-α-closed sets.

2.4 Proposition:

Every αgr-closed set is an αsr-closed set.

Proof: Let A be an αgr-closed set, let U be a regular open set of (X,τ) such that $A \subseteq U$. Since A is αgr-closed set and $S_\alpha cl(A) \subseteq \alpha cl(A)$ (by part (2) of remark 1.3), then $S_\alpha cl(A) \subseteq U$. Therefore A is also an αsr-closed set.

The following example shows that the αsr-closed set need not to be an αgr-closed set.

2.5 Example:

Let $X=\{a,b,c\}$ and $\tau=\{X,\phi,\{a\},\{b\},\{a,b\}\}$. Let $A=\{b\}$, let $\{b\}$ is the regular open set containing A. Trivially A is an αsr-closed set since $S_\alpha cl(A)=\{b\} \subseteq \{b\}$. But A is not αgr-closed set since $\alpha cl(A)=\{b,c\} \not\subseteq \{b\}$.

2.6 Corollary:

Every closed (resp. α-closed, g-closed, gα-closed) set is an αsr-closed set.

Proof: Since every αgr-closed set is an αsr-closed set, then in vitue of proposition 1.2 part (2) the proof is over.

The following example shows that the reveres implications in the above corollary are not true in general.

2.7 Example:

Let X, τ and A be as in example 2.3. A is neither closed (since $cl(A) = X \neq A$) nor α-closed (since $\alpha cl(A) = X \neq A$) and also it is neither g-closed (since $A=\{a,c\} \subseteq \{a,c\}$ whenever $\{a,c\} \in \tau$, but $cl(A) = X \not\subseteq \{a,c\}$) nor g$\alpha$-closed (since $A=\{a,c\} \subseteq \{a,c\}$ whenever $\{a,c\} \in \alpha O(X)$, but $\alpha cl(A) = X \not\subseteq \{a,c\}$).

2.8 Corollary:

Every gα*-closed (resp. α**g-closed, gα**-closed) set is an αsr-closed set.

Proof: Since every αgr-closed set is an αsr-closed set, part (4) of proposition 1.2 is applicable.

The following example shows that an αsr-closed set needs not to be a gα*-closed set.

2.9 Example:

Let $X=\mathbb{R}$ and $\tau = \tau_U$, let $A=(a,b)$ is αsr-closed set but not a gα*-closed set, since $(a,b) \subseteq (a,b)$ and (a,b) is α-open set in $(\mathbb{R},\tau_U)$, but $\alpha cl(A) = [a,b] \not\subseteq (a,b)$.

2.10 Proposition:

Every r-g- closed is an αsr-closed set.

Proof: Let A be a regular generalized closed set of (X,τ). Let U be a regular open set of (X,τ) such that $A \subseteq U$. Then $cl(A) \subseteq U$ since A is r-g closed set. Since every closed set is semi-α-closed set, then $S_\alpha cl \subseteq cl(A)$ (part 2 of remark 1.3). Thus $S_\alpha cl(A) \subseteq U$, therefore A is an αsr-closed set.

The converse of above proposition is not always true as the following example shows.

2.11 Example:

Let X and τ be as in example 2.3, let $A=\{c\}$ and $U=\{c\}$ is regular open set containing A. It is clear A is an αsr-closed set since $S_\alpha cl(A) = \{c\} \subseteq \{c\}$. But is not r-g closed set since $cl(A) = \{b,c\} \not\subseteq \{c\}$.

2.12 Proposition:

Every αg-closed set is an αsr-closed set

Proof: Let A be an αg-closed set, let U be a regular open set of (X,τ) such that $A \subseteq U$. Since A is αg-closed and every regular open set is an open set, then $\alpha cl(A) \subseteq U$. But $S\alpha cl(A) \subseteq \alpha cl(A)$ since every α-closed set is semi-α-closed set. Therefore A is also an αsr-closed set.

The converse in the above proposition is not true as it can be seen from the following example.

2.13 Example:

In example 2.3 $\alpha cl(A) = X \not\subseteq \{a,c\}$. Thus A is not αg-closed set, but it is αsr-closed set.

2.14 Proposition:

Let A be an αsr-closed set of (X,τ). Then $S_\alpha cl(A)$-A does contain any non-empty regular closed set.

Proof: Let F be any regular closed set of (X,τ) such that $F \subseteq S_\alpha cl(A) - A$. Then $F \subseteq X - A$ implies that $A \subseteq X - F$. Since A is αsr-closed and $X - F$ is a regular open set of (X,τ), then $S_\alpha cl(A) \subseteq X - F$, so $F \subseteq X - S_\alpha cl(A)$. Therefore $F \subseteq S\alpha cl(A) \cap (X - S_\alpha cl(A)) = \phi$. Hence $S_\alpha cl(A) - A$ does not contain any non-empty regular closed set.

2.15 Proposition:

Every αsr-closed set is a pre-semi-closed set.

Proof: Let A be an αsr-closed set of (X,τ), let U be a regular open set of (X,τ) such that $A \subseteq U$. Then $S_\alpha cl(A) \subseteq U$ since A is αsr-closed set. Since every semi-α-closed set is semi-pre-closed set (by part 6 of proposition 1.2), then $spcl(A) \subseteq S_\alpha cl(A)$ and every regular open set is g-open set. Thus A is pre-semi-closed set. Thus the class of αsr-closed set properly contained in the class of pre-semi-closed sets.

2.16 Corollary:

Every αsr-closed set is gsp-closed set.

Proof: Follows the above proposition and part (5) of proposition 1.2.

2.17 Corollary:

Every αgr-closed set is pre-semi-closed set.

Proof: Follows from the fact every αgr-closed set is αsr-closed and proposition 2.15.

2.18 Proposition:

If A is regular open and αsr-closed set then A is semi-α-closed set.

Proof: It is clear.

2.19 Proposition:

Let A be an αsr-closed subset of (X,τ). If $B \subseteq X$ such that $A \subseteq B \subseteq S_\alpha cl(A)$, then B is αsr-closed set.

Proof: Let U be a regular open set of (X,τ) such that $B \subseteq U$. Then $A \subseteq U$, since A is αsr-closed set, $S_\alpha cl(A) \subseteq U$. Now, $S_\alpha cl(B) \subseteq S_\alpha cl(S_\alpha cl(A)) = S_\alpha cl(A) \subseteq U$. Therefore B is also an αsr-closed set.

Fig. (1) shows the relations among the different types of weakly closed sets that were studied in this section.

3- α-Semi Regular Continuous Maps and α-Semi-Regular-Irresolute Maps

3.1 Definition:

A function $f:(X,\tau) \longrightarrow (Y,\tau')$ is called an α-semi-regular continuous map (briefly αsr-continuous if $f^{-1}(V)$ is an αsr-closed set of (X,τ) for every closed set V of (Y,τ').

3.2 Proposition:

Every semi-α-continuous map is αsr-continuous.

Proof: Follows from proposition 2.2.

We show that the class of αsr-continuous maps properly contains the class of αgr-continuous maps.

3.3 Proposition:

Let $f:(X,\tau) \longrightarrow (Y,\tau')$ be an αgr-continuous map. Then f is an αsr-continuous map.

Proof: Let V be a closed set of (Y,τ'). Since f is an αgr-continuous map, then $f^{-1}(V)$ is an αgr-closed set of (X,τ). By proposition 2.4 $f^{-1}(V)$ is an αsr-closed set of (X,τ). Thus f is an αsr-continuous map.

The implications in proposition 3.3 is not reversible. Follows from the following example.

3.4 Example:

Let $X = \{a,b,c\} = Y$, $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $\tau' = \{Y,\phi,\{a,c\}\}$. Define $f:(X,\tau) \longrightarrow (Y,\tau')$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$, $\{b\}$ is a closed set of (Y,τ') but $f^{-1}(\{b\}) = \{b\}$ is not αgr-closed set of (X,τ). So f is not αgr-continuous map. However f is an αsr-continuous map.

3.5 Corollary:

Every αg-continuous map is αsr-continuous.

Proof: Follow from part (1) of proposition 1.5 and proposition 3.3.

The converse of the above corollary is not true in general as we see in the following example.

3.6 Example:

Let X, Y, τ and the definition of f as in example 3.4, let $\tau' = \{Y,\phi,\{a\},\{b,c\}\}$. f is not αg-continuous map since $\{b,c\}$ is a closed set of (Y,τ') but $f^{-1}(\{b,c\}) = \{a,b\}$ is not αg-closed set of (X,τ). However f is an αsr-continuous map.

3.7 Corollary:

Every g-continuous (resp. gα-continuous) is an αsr-continuous.

Proof: Follows from part (2) of proposition 1.5 and proposition 3.3.

The converse of the above corollary is not true in general as we see in the following example.

3.8 Example:

See example 3.4 f is αsr-continuous map but not g-continuous map.

3.9 Corollary:

Every αgr-irresolute map is an αsr-continuous.

Proof: Necessity follows from part (4) of proposition 1.5 and proposition 3.3.

The converse of the above corollary is not true in general as we see in the following example

3.10 Example:

Let X = {a,b,c} = Y, τ = {X,ϕ,{a},{b},{a,b}} and τ'=I. Define f:X $\longrightarrow$ Y by f(a) = c, f(b) = b and f(c) = a, {b} is an αsr-closed set of (Y,τ') but f^{-1}({b}) = {b} is not αgr-closed set of (X,τ). So f is not αgr-irresolute map. However f is an αsr-continuous map.

3.11 Theorem:

Let f:(X,τ) $\longrightarrow$ (Y,τ') be an αsr-continuous map. Then f is a pre-semi-continuous map.

Proof : Let V be a closed set of (Y,τ'). Since f is αsr-continuous map, then f^{-1}(V) is an αsr-closed set of (X,τ). By proposition (2.15) f^{-1}(V) is a pre-semi-closed set of (X,τ). Thus f is a pre-semi-continuous map.

3.12 Corollary:

Every αsr-continuous map is gsp-continuous.

Proof: Follows from the above proposition and part (3) of proposition 1.5.

3.13 Definition:

A function f:(X,τ) $\longrightarrow$ (Y,τ') is called an α-semi-regular irresolute (briefly αsr-irresolute) if f^{-1}(V) is an αsr-closed set of (X,τ) for every αsr- closed set of (Y,τ').

3.14 Proposition:

Let f:(X,τ) $\longrightarrow$ (Y,τ') be an αsr-irresolute map. Then f is an αsr-continuous map.

Proof: Let V be a closed set of (Y,τ'). By corollary 2.6 V is an αsr-closed set of (Y,τ'). Since f is an αsr-irresolute map, f^{-1}(V) is an αsr-closed set of (X,τ). Therefore f is an αsr-continuous map.

Thus the class of αsr-continuous maps property continuous the class of αsr-irresolute map.

3.15 Corollary:

Every αsr-irresolute map is a pre-semi-continuous.

Proof: Follows from the above proposition and proposition 3.11.

3.16 Corollary:

Every αsr-irresolute is a gsp- continuous.

Proof: Follows from proposition 3.14 and corollary 3.12.

3.17 Theorem:

Let f:(X,τ) $\longrightarrow$ (Y,τ') be a regular irresolute and semi-α-irresolute map. Then f is αsr-irresolute map.

Proof: Let A be an αsr-closed set of (Y,τ'), then there exists a regular open set U of Y such that $S_\alpha cl(A) \subseteq U$ whenever $A \subseteq U$. By taking the inverse image we get $f^{-1}(S_\alpha cl(A)) \subseteq f^{-1}(U)$. Since f is regular irresolute map, then f^{-1}(U) is regular open subset of X. Since f is semi-α-irresolute map, then $f^{-1}(S_\alpha cl(A))$ is semi-α-closed subset of X. This implies $S_\alpha cl(f^{-1}(S_\alpha cl(A)))= f^{-1}(S_\alpha cl(A))$ (by part (1) of remark 1.3), then $S_\alpha clf^{-1}(A) \subseteq S_\alpha cl(f^{-1}(S_\alpha cl(A))$. Thus $S_\alpha clf^{-1}(A) \subseteq f^{-1}(U)$. Therefore f^{-1}(A) is αsr-closed set in X. Therefore f is αsr-irresolute map.

3.18 Corollary:

Every continuous, open and regular irresolute map is αsr-irresolute.

Proof: It is clear by part (5) of proposition 1.5 and the above theorem.

3.19 Definition:

Let f:(X,τ) $\longrightarrow$ (Y,τ') be a function, then f is said to be:

(1)α-semi-regular closed (briefly αsr-closed) if f(A) is an αsr-closed set of (Y,τ') for every closed set A of (X,τ).

(2) α*-semi-regular closed (briefly α*sr-closed) if f(A) is an αsr-closed set of (Y,τ') for every αsr-closed set A of (X,τ).

3.20 Remark:

It is clear that every closed function is α-semi-closed function, but the converse is not true in general as the following example shows:

3.21 Example:

Let X={a,b,c,d}, τ = {X,ϕ,{a},{b},{a,b},{a,b,c}}. Define f:(X,τ) ⟶ (X,τ) by f(a) = a, f(b) = b, f(c) = f(d) = d we observe f is α-semi-regular closed function which is not closed function since {a,c,d} is closed set in X, but f({a,c,d}) = {a,d} is not closed set in X. Hence f is α-semi-regular closed function, which is not closed function.

Finally, we prove the following theorem.

3.22 Theorem:

Let f:(X,τ) ⟶ (Y,τ') be a regular irresolute and semi-α*-closed map. Then f is α*-semi-regular closed map.

Proof: Let A be an αsr-closed set of (X,τ), let U be a regular open set of (Y,τ') such that f(A) ⊆ U. Since f is regular irresolute, then $f^{-1}(U)$ is a regular open set of (X,τ). Since A ⊆ $f^{-1}(U)$ and A is an αsr-closed, then $S_\alpha cl(A) \subseteq f^{-1}(U)$. This implies $f(S_\alpha cl(A)) \subseteq U$. Since f is semi-α*-closed map, then $f(S_\alpha cl(A)) = S_\alpha cl\ (f(S_\alpha cl(A)))$. So $S_\alpha cl(f(A)) \subseteq S_\alpha cl(f(S_\alpha cl(A))) = f(S_\alpha cl(A))) \subseteq U$. Therefore f(A) is an αsr-closed set of (Y,τ').

3.23 Corollary:

Let f:(X,τ) ⟶ (Y,τ') be a regular irresolute and semi-α*-closed map. Then f(A) is a pre-semi-closed set of (Y,τ') for every αsr-closed set of (X,τ).

Proof: It is clear.

Fig. (2) explains the relationships among the different types of weakly continuous function.

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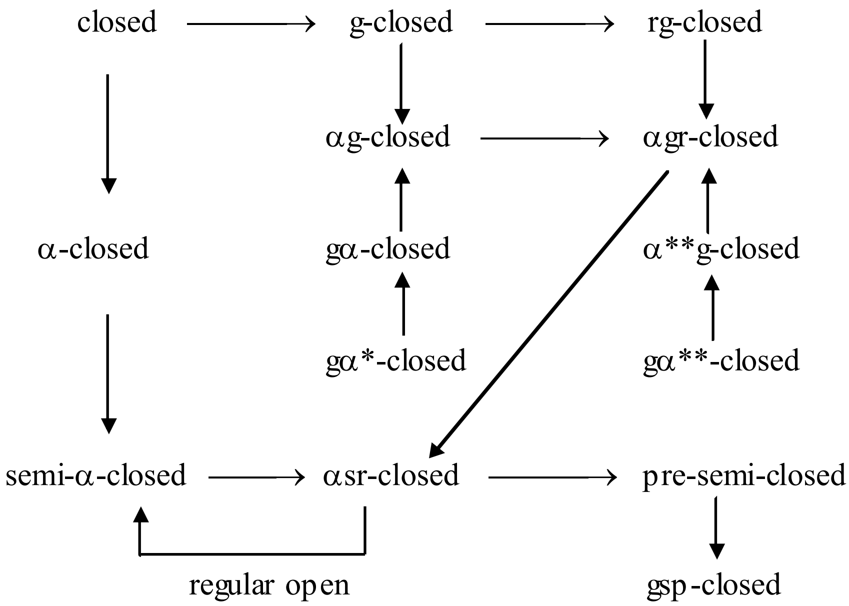

Fig. (1) the relations among the different types of weakly closed sets

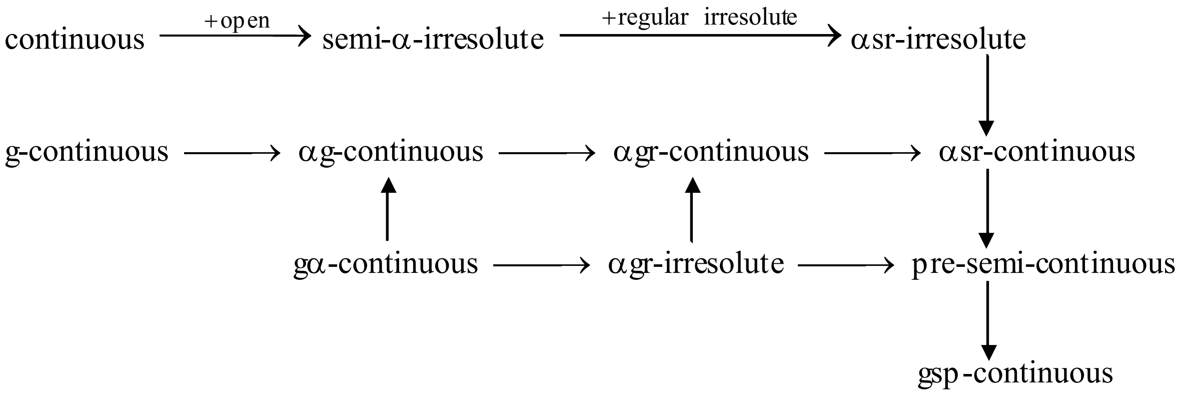

Fig. (2) the relationships among the different types of weakly continuous function.