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Chained fuzzy modules

S. B.Semeein
Department of Mathematics, College of Education Ibn-Al-Haitham,University of Baghdad

Abstract

Let R be a commutative ring with unity. In this paper we introduce the notion of chained fuzzy modules as a generalization of chained modules. We investigate several characterizations and properties of this concept

Introduction

In this paper we introduce the concept of chained fuzzy modules as a generalization of the concept (chained modules) in ordinary algebra .This paper consists of three sections

In section one, we recall some basic definitions and results which we needed later.

In section two, we give some results about chained fuzzy modules such as it's relationship with it levels.

Section three is devoted for studying the direct sum of chained fuzzy modules.

Finally , we study the homomorphic image and inverse of chained fuzzy modules.

1. Preliminaries

The following definitions and results are needed later.

1.1 Definition, [1]

Let M be a nonempty set and I be the closed interval [0,1] of the real line (numbers). A fuzzy set A in M (a fuzzy subset A of M) is a function from M into I.

1.2 Definition, [2]

Let x_t:M $\longrightarrow$ [0,1] be a fuzzy set in M, where x$\in$M, t$\in$[0,1] defined by:

$$X_t(y)=\begin{cases} t & \text{if } x=y \\ 0 & \text{if } x\neq y \end{cases}$$

for all y$\in$M, x_t is called a fuzzy singleton or fuzzy point in M, if x=0 and t=1, then

$$0_1(y)=\begin{cases} 1 & \text{if } y=0 \\ 0 & \text{if } y\neq 0 \end{cases}$$

We shall call such fuzzy singleton the fuzzy zero singleton.

1.3 Definition , [2]

Let A and B be two fuzzy sets in M , then

1. A = B if and only if A(X) = B(X), for all x $\in$ M.
2. A$\subseteq$B if and only if A(X) $\leq$ B(X), for all x$\in$M.
3. (A$\cap$B)(x)=min{A(x),B(x)} for all x$\in$M.
4. (A$\cup$B)(x)=max{A(x),B(x)} for all x$\in$M.

1.4 Definition, [3]

Let A be a fuzzy set in M and t$\in$[0,1]. The set A_t={x$\in$M,A(x)$\geq$t} is called level subset of A.

1.5 Remark (1)

The following properties of level subsets hold for each t$\in$[0,1]

1. $(A\cap B)_t=A_t \cap B_t$
2. A=B if and only if $A_t=B_t$, for all t$\in$[0,1].

Where A and B are fuzzy sets.

Now, we can give the definition of image and inverse image of a fuzzy set.

1.6 Definition, [4]

Let f be a mapping from a set M into a set N, A be a fuzzy set in M and B be a fuzzy set in N. The image defined by:

$$f(A)=\begin{cases}\sup\{A(z) \mid z\in f^{-1}(y)\} & \text{if } f^{-1}(y)\neq\phi, \\ & \text{for all } y\in N \\ 0 & \text{otherwise}\end{cases}$$

where $f^{-1}(y)=\{x:f(x)=y\}$

and the inverse image of B, denoted by $f^{-1}(B)$, is the fuzzy set in M defined by:

$f^{-1}(B)(x)=B(f(x))$, for all $x\in M$.

1.7 Definition, [5]

Let f be a mapping from a set M into a set M'. A fuzzy subset A of M is called f-invariant if A(x)=A(y) whenever f(x)=f(y), where x, y $\in$M.

The following lemma is needed in section three.

1.8 Lemma, [5]

If f is a function defined on a set M, A_1 and A_2 are fuzzy subset of M, B_1 and B_2 are fuzzy subset of f(M). Then the following are true:

1. $A_1=f^{-1}(f(A))$, whenever A_1 is f-invariant.
2. $f(f^{-1}(B_1))=B_1$
3. if $A_1\subseteq A_2$, then $f(A_1)\subseteq f(A_2)$
4. if $B_1\subseteq B_2$, then $f^{-1}(B_1)\subseteq f^{-1}(B_2)$.

1.9 Definition, [6]

Let $(R,+,\cdot)$ be a ring and let X be a fuzzy set in R. Then X is called a fuzzy ring in ring $(R,+,\cdot)$ if and only if, for each x, y $\in$ R

1. $X(x+y)\geq\min\{X(x), X(y)\}$
2. $X(x) = X(-x)$
3. $X(xy)\geq\min\{X(x), X(y)\}$.

1.10 Definition [7]

A fuzzy subset X of a ring R is called a fuzzy ideal of R, if for each x, y $\in$ R

1. $X(x-y)\geq\min\{X(x), X(y)\}$
2. $X(xy)\geq\max\{X(x), X(y)\}$.

1.11 Definition [2]

Let M be an R-module. A fuzzy set X of M is called a fuzzy module of M if

1. $X(x-y)\geq\min\{X(x), X(y)\}$, for all x, y $\in$M.
2. $X(rx)\geq X(x)$, for all $x\in M$ and $r\in R$.
3. $X(0)=1$.

1.12 Definition [6]

Let X and A be two fuzzy modules of an R-module M. A is called a fuzzy submodule of X if $A\subseteq X$.

1.13 Proposition [7]

Let A be a fuzzy set of M. Then the level subset A_t, $t\in(0,1]$ is a submodule of M if and only if A is a fuzzy submodule of X where X is a fuzzy module of M such that $A(x)\leq X(x)$, $\forall x\in M$.

1.14 Definition [8]

A fuzzy module X of an R-module M is called fuzzy simple if and only if X has no fuzzy proper submodules.

1.15 Definition [8]

A fuzzy module X of an R-module M is called fuzzy cyclic module, if there exists $x_t\subseteq X$ such that each $y_k\subseteq X$ written as $y_k=r_\ell x_t$ for some fuzzy singleton r_ℓ of R where k, ℓ, t∈[0,1]. In this case, we shall write X=(x_t) to denoted the fuzzy cyclic module generated by x_t.

1.16 Definition [6]

Let X and Y be two fuzzy modules of R-modules M_1 and M_2 respectively, f:X⟶Y is called a fuzzy homomorphism if f:M_1⟶M_2 is R-homomorphism and y(f(x))=X(x) for each x∈M_1.

1.17 Remark [9]

1- Let M and M' be two R-modules, f:M⟶M' be an epimorphism. If A is a fuzzy submodule of M, then f(A) is a fuzzy submodule of M'.

2- Let M and M' be two R-modules, f:M⟶M' be a homomorphism. If B is a fuzzy submodule of M', then f^{-1}(B) is a fuzzy submodule of M.

1.18 Definition [2]

Suppose A and B be two fuzzy modules of R-module M. We define (A:B) by:-
(A:B)={r_t:r_t is a fuzzy singleton of R such that $r_tB \subseteq A$} and (A:B)(r)=sup{t∈[0,1]| $r_tB \subseteq A$, for all r∈R}. If B=(b_k), (A:(b_k))={r_t| $r_tb_k \subseteq A$, r_t is a fuzzy singleton of R}.

1.19 Definition [10]

Let X and Y be two fuzzy modules of M_1, M_2 respectively. Define
$X \oplus Y: M_1 \oplus M_2 \longrightarrow [0,1]$ by
$(X \oplus Y)(a,b)=\min\{X(a),Y(b)\}$ for all (a,b)∈ $M_1 \oplus M_2$.
X⊕Y is called a fuzzy external direct sum of X and Y.

1.20 Proposition [10]

Let X and Y are fuzzy modules of M_1 and M_2 respectively, then X⊕Y is a fuzzy module of $M_1 \oplus M_2$.

1.21 Proposition [10]

Let A and B be two fuzzy submodules of a fuzzy module X such that X=A⊕B, then $X_S=A_S \oplus B_S$, for all s ∈(0,1].

2. Chained Fuzzy Module

In this section we introduce the concept of chained fuzzy module . some basic results of this concept are considerate

2.1 Definition, [11]

An R-module M is called chained module if for each submodules A, B of M, either A ⊆ B or B ⊆ A.

We fuzzify this definition as follows:

2.2 Definition

Let X be a fuzzy module of an R-module M then X is called a chained fuzzy module if for each fuzzy submodules of X either A ⊆ B or B ⊆ A.

To prove our next theorem, first we prove the following lemma:

2.3 Lemma

Let A and B be two fuzzy subset of R then A ⊆ B if and only if $A_t \subseteq B_t$, for each t∈[0,1].

Proof: It is easy so it is omitted.

The following theorem characterizes chained fuzzy module in terms of it is level module.

2.4 Theorem

A fuzzy module X of an R-module M is a chained if and only if X_t is a chained module, ∀ t∈(0,1].

Proof: If X is chained fuzzy module. To prove X_t is chained module ∀ t∈(0,1].
Let I, J be submodules of X_t. Define :

$$A(x)=\begin{cases} t & x \in I \\ 0 & x \notin I \end{cases}, \quad B(x)=\begin{cases} t & x \in J \\ 0 & x \notin J \end{cases}$$

A, B are fuzzy submodules of X. But A_t=I, B_t=J since X is chained fuzzy module, then either $A \subseteq B$ or $B \subseteq A$. Hence $A_t \subseteq B_t$ or $B_t \subseteq A_t$ (by lemma (2.3)). Thus $I \subseteq J$ or $J \subseteq I$.
Conversely, if X_t is chained module, to prove X is a chained fuzzy module, let A, B fuzzy submodules in X. Then A_t, B_t are submodules in X_t, for all $t\in(0,1]$ since X_t is chained R-module then $A_t \subseteq B_t$ or $B_t \subseteq A_t$ which implies $A \subseteq B$ or $B \subseteq A$ (lemma (2.3)).

2.5 Examples

1. Let X(x)=1 for all $x\in Z_8$ $X_t=Z_8$ for all $t\in[0,1]$. But Z_8 is chained. Hence by theorem (2.4) X is a chained fuzzy module.

2. Every fuzzy simple module is a chained fuzzy module.

2.6 Remark

If $Y \leq X$ and X is a chained fuzzy module then Y is chained fuzzy module.

Proof: Let A, B be two fuzzy submodules of Y then A, B are fuzzy submodules of X, since X is chained fuzzy module. Then $A\subseteq B$ or $B\subseteq A$ which implies Y is a chained fuzzy module.

2.7 Definition, [10]

A fuzzy module X is called uniform fuzzy module if $A\cap B\neq 0_1$ for any nontrivial fuzzy submodules A and B of X.

2.8 Proposition

A fuzzy module X of an R-module M is uniform if and only if X_t is a uniform module, $\forall$ $t\in(0,1]$.

Proof: If X is uniform fuzzy module, to prove X_t is uniform module $\forall$ $t\in(0,1]$. Let I, J be submodules of X_t. Define

$$A(x)=\begin{cases} t & x\in I \\ 0 & x\notin I \end{cases},\quad B(x)=\begin{cases} t & x\in J \\ 0 & x\notin J \end{cases}$$

A, B are fuzzy submodules of X. But A_t=I, B_t=J since X is uniform fuzzy module, then $A\cap B\neq 0_1$. Hence $(A\cap B)_t\neq 0_1$ which implies $A_t\cap B_t\neq 0_1$ (by remark 1.5). This $I\cap J\neq 0_1$.
Conversely, if X_t is uniform module, to prove X is a uniform fuzzy module, let A, B fuzzy submodules in X. Then A_t, B_t are submodules in X_t, for all $t\in(0,1]$, since X_t is uniform R-module then $A_t\cap B_t\neq 0_1$ which implies $(A\cap B)_t\neq 0_1$ (by remark 1.5). Thus $A\cap B\neq 0_1$.

Now, we shall show the relationship between uniform fuzzy module and chained fuzzy module as the following proposition:

2.9 Proposition

Every chained fuzzy module is a uniform fuzzy module.

Proof: Let X be a chained fuzzy module of an R-module M then $A\subseteq B$ or $B\subseteq A$
if $A\subseteq B$ then $A\cap B= A$
if $B\subseteq A$ then $A\cap B=B$
which implies $A\cap B=0_1$.

2.10 Remark

The converse of proposition (2.9) is not true for the following example shows:

2.11 Example

Let M=Z as a Z-module
$X:M\longrightarrow[0,1]$ such that X(x)=1 $\forall x\in M$ X_t=z for all $t\in[0,1]$. But Z is uniform. Hence by proposition (2.9) X is a uniform fuzzy module. But X is not chained fuzzy module since $\exists$ A, B fuzzy submodules of X defined by

$$A(x)=\begin{cases} 1 & x\in(2) \\ \frac{1}{4} & x\notin(2) \end{cases},\quad B(x)=\begin{cases} 1 & x\in(5) \\ \frac{1}{4} & x\notin(5) \end{cases}$$ and $A \not\subseteq B$ and $B \not\subseteq A$.

Recall that if A and B are two submodules of an R-module M, then A and B are called comparable if $A\subseteq B$ and $B\subseteq A$.

We shall fuzzify this concept as follows:

2.12 Definition

Let A, B be two fuzzy submodules of a fuzzy module X of an R-module M, then A and B are called comparable if $A\subseteq B$ and $B\subseteq A$.

2.13 Proposition

A fuzzy module X of an R-module M is chained iff every two cyclic fuzzy submodules of X are comparable.

Proof: Let A and B be fuzzy submodules of X. Suppose $A \not\subseteq B$, we show $B\subseteq A$ since $A \not\subseteq B$, there exists $x_t\in A$ and $x_t\notin B$ $<x_t>\subseteq A$ and $<x_t> \not\subseteq B$. Let $y_k\in B$, then $<y_k>\subseteq B$, $<x_t>$, $<y_k>$ are cyclic fuzzy submodules of X, then either $<x_t> \subseteq <y_k>$ or $<y_k> \subseteq <x_t>$.

If $<x_t> \subseteq <y_k>$ implies $<x_t> \subseteq B$ (since $<y_k>\subseteq B$). Thus $x_t\in B$ is a contradiction.

If $<y_k> \subseteq <x_t>$ implies that $<y_k> \subseteq A$ (since $<x_t> \subseteq A$). Thus $B\subseteq A$ so X is chained.

The converse is obvious.

2.14 Remark

A chained fuzzy module is indecomposable.

Proof: Suppose X is decomposable, then $X=A\oplus B$ for some fuzzy submodule A and B of X. Thus $A\cap B=O_1$ is a contradiction (proposition 2.9).

Now, we introduce the notion of chained fuzzy ring. First we have the following definition.

2.15 Definition, [11]

A ring R is called chained if and only if for each fuzzy ideals I, J of R either $I\subseteq J$ or $J\subseteq I$.

2.16 Definition

A fuzzy ring X of a ring R is called chained if and only if for each fuzzy ideals I, J of X either $I\subseteq J$ or $J\subseteq I$.

2.17 Remark

A fuzzy ring X is chained if and only if X_t is chained ring $\forall\ t\in(0,1]$.

Proof: It is easy so it is omitted.

2.18 Definition, [8]

A fuzzy module X of an R-module M is called multiplication fuzzy module if for each nonempty fuzzy submodule A of X, there exists a fuzzy ideal I of R such that A=IX.

2.19 Proposition

Let X be a multiplication module of an R-module M if R is a chained ring then X is a chained fuzzy module.

Proof: Let A and B be fuzzy submodules of X. Then there exists fuzzy ideals I and J of R such that A=IX and B=JX, since I_t and J_t ideals of R and R is chained, therefore $I_t \subseteq J_t$ or $J_t\subseteq I_t$. Thus $I\subseteq J$ or $J\subseteq I$ (by remark 2.3) implies that $IX\subseteq JX$ or $JX\subseteq IX$. Thus $A\subseteq B$ or $B\subseteq A$.

2.20 Definition

Let X be a chained fuzzy module of an R-module M and let

$V(X)=\{(O_1:X_t) \mid X_t\in X\}$.

2.21 Definition, [10]

Let X be a non empty fuzzy module of R-module M. The fuzzy annihilator of A denoted by (F-annA) is defined by $\{X_t: x\in R, X_tA\subseteq O_1\}, t\in[0,1]$, where A is a proper fuzzy submodule of X.

Note that:

$(F\text{-ann}A)(a)=\sup\{t: t\in[0,1], a_tA\subseteq O_1\}$, for all $a\in R$; that is $F\text{-ann}A=(O_1:A)$.

2.22 Definition, [10]

A fuzzy module X is called faithful if $F\text{-ann}X=O_1$.

2.23 Remark

If X is chained faithful fuzzy module then $\bigcap_{O_1\neq X_t\in X} (O_1:X_t)=O_1$, $X_t\in X$.

Proof: If $\bigcap_{O_1 \neq X_t \in X} (O_1:X_t) \neq O_1$ then there is $r_\ell \in R$, $r_\ell \neq O_1$ such that $r_\ell x_t = O_1$, $\forall x_t \in X$ then $r_\ell X = O_1$ a contradiction.

2.24 Definition, [13]

A fuzzy ideal A of a ring R is called fuzzy prime ideal, if A is non-constant and for any fuzzy ideals B and C of R such that $B \circ C \subseteq A$, then either $B \subseteq A$ or $C \subseteq A$.

Equivalently, A fuzzy ideal A of a ring R is called fuzzy prime ideal if A is a non-constant and for all a_ℓ, b_h fuzzy singletons of R such that $a_\ell b_h \subseteq A$ implies that either $a_\ell \subseteq A$ or $b_h \subseteq A$, ℓ, $h \in [0,1]$.

2.25 Remark

If X is a chained fuzzy module of an R-module M then,

1. V(X) is a linearly ordered set of fuzzy ideals of R.
2. $P = \bigcup_{O_1 \neq X_t \in X} (O_1:x_t)$ is a fuzzy prime ideal of R.

Proof: (1) Let A, B $\in$ V(X) then $A=(O_1:x_t)$ and $B=(O_1:y_t)$ for some $x_t \neq O_1$, $y_t = O_1$ and x_t, $y_t \in X$ since X is chained fuzzy module then X_t is chained module (by theorem (2.4)) implies that $V(X_t)$ is a linearly ordered set of ideals of R (see [12,remark (1.9)). Thus V(X) is a linearly ordered set of fuzzy ideals of R.

(2) $P = \bigcup_{O_1 \neq X_t \in X} (O_1:x_t)$ is a fuzzy ideal of R. To show that P is a fuzzy prime ideal, let a_ℓ, $b_h \in R$ such that $a_\ell b_h \in P$, then there is $O_1 \neq x_t \in X$ such that $a_\ell b_h \in (O_1:x_t)$. Then $a_\ell b_h x_t = O_1$. This implies that $a_\ell \in (O_1: b_h x_t)$. Now if $b_h x_t = O_1$ then $b_h \in (O_1:x_t)$. Thus $b_h \in P$ and if $b_h x_t \neq O_1$ then $b_h x_t \neq O_1 \in X$, and hence $a_\ell \in P$.

3. Direct Sum of Chained Fuzzy Module

We turn attention to the direct sum of chained fuzzy modules.

3.1 Remark

If X and Y are two chained fuzzy modules of an R-module M_1 and M_2 respectively then $X \oplus Y$ is not necessary chained fuzzy module of $M_1 \oplus M_2$ as the following example shows:

3.2 Example

Let $X: Z_6 \longrightarrow \{0, \frac{1}{3}\}$ such that

$$X(a) = \begin{cases} \frac{1}{3} & \text{if } a \in <2> \\ 0 & \text{if } a \notin, <2> \end{cases}$$

Let $Y: Z_6 \longrightarrow \{0, \frac{1}{3}\}$ such that

$$Y(a) = \begin{cases} \frac{1}{3} & \text{if } a \in <3> \\ 0 & \text{if } a \notin, <3> \end{cases}$$

It is clear that X and Y are chained fuzzy modules of Z_6. Hence $X \oplus Y$ is not a chained fuzzy module of $Z_6 \oplus Z_6$. Since $\exists$ A,B fuzzy submodules of $X \oplus Y$, where

$$A(a,b) = \begin{cases} \frac{1}{3} & \text{if } (a,b) \in <2> \oplus <0> \\ 0 & \text{if } (a,b) \notin, <2> \oplus <0> \end{cases}$$

and

$$B(a,b) = \begin{cases} \frac{1}{3} & \text{if } (a,b) \in <0> \oplus <3> \\ 0 & \text{if } (a,b) \notin, <0> \oplus <3> \end{cases}$$

But A(2,0)= $\frac{1}{3}$, B(2,0)=0, that is A $\not\subseteq$ B. Alaso A(0,3)=0, B(0,3)= $\frac{1}{3}$, that is B $\not\subseteq$ A. Thus X⊕Y is not a chained fuzzy module of $Z_6 \oplus Z_6$.

3.3 Theorem

Let X and Y be a fuzzy modules of an R-modules M_1 and M_2 respectively, if X⊕Y is a chained fuzzy module of $M_1 \oplus M_2$ then X is a chained fuzzy module of M_1 and Y is a chained fuzzy module of M_2.

Proof: By similar proof of theorem (4.10) in [14].

Next, we shall indicate the behaviors of chained fuzzy modules under homomorphism.

3.4 Theorem

Let X and Y be a fuzzy modules of an R-modules M_1 and M_2 respectively. Let f:X⟶Y be a fuzzy epimorphism. If X is a chained fuzzy module, then Y is a chained fuzzy module.

Proof: Let A, B are fuzzy submodules in Y. Then $f^{-1}(A)$, $f^{-1}(B)$ are fuzzy submodules in X (remark (1.17),(2)), since X is chained fuzzy module, then either $f^{-1}(A) \subseteq f^{-1}(B)$ or $f^{-1}(B) \subseteq f^{-1}(A)$.

Now, if $f^{-1}(A) \subseteq f^{-1}(B)$, then $f(f^{-1}(A)) \subseteq f(f^{-1}(B))$ (by lemma (1.8),(2)).

Similarly, if $f^{-1}(B) \subseteq f^{-1}(A)$, then B⊆A. Therefore Y is a chained fuzzy module.

3.5 Proposition

Let X and Y be two fuzzy modules of an R-modules M_1 and M_2 respectively. Let f:X⟶Y be a fuzzy homomorphism and every submodule of Y is f-invariant. If Y is a chained fuzzy module, then X is a chained fuzzy module.

Proof: Let A, B are fuzzy submodules in X. Hence f(A), f(B) are fuzzy submodules in Y (remark (1.17),(1)), since Y is a chained fuzzy module then f(A)⊆ f(B) or f(B)⊆ f(A).

Now, if f(A)⊆ f(B), then $f^{-1}(f(A)) \subseteq f^{-1}(f(B))$ (by lemma (1.8),(4)). Hence A⊆B (by lemma (1.8),(1)).

Similarly, if f(B)⊆ f(A), then B⊆A. Therefore X is a chained fuzzy module.

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