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IJ-ω-Perfect Functions Between Bitopological Spaces

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Abstract

In this paper we introduce a lot of concepts in bitopological spaces which are ij-ω-converges to a subset, ij-ω-directed toward a set, ij-ω-closed functions, ij-ω-rigid set, ij-ω-continuous functions and the main concept in this paper is ij-ω-perfect functions between bitopological spaces. Several theorems and characterizations concerning these concepts are studied.

1. Introduction and Preliminaries

A set X with two topologies τ_1 and τ_2 is called bitopological space [1] and is denoted by (X, τ_1, τ_2). The closure and interior of A in (X, τ_i) is denoted by τ_i-cl(A) and τ_i-int(A), where i=1, 2. For other notions or notations not defined here we follow closely R. Engelking [2].

Definition: 1.1. [3]. A filter $\Im$ on a set X is a nonempty collection of nonempty subsets of X with the properties:

(a) If $F_1, F_2 \in \Im$, then $F_1 \cap F_2 \in \Im$,

(b) If $F \in \Im$ and $F \subseteq F^* \subseteq X$, then $F^* \in \Im$.

Definition: 1.2. [3]. A filter base $\Im$ on a set X is a nonempty collection of nonempty subsets of X such that if $F_1, F_2 \in \Im$ then $F_3 \subseteq F_1 \cap F_2$ for some $F_3 \in \Im$.

Definition: 1.3. [3]. If $\Im$ and $\mathcal{G}$ are filter bases on X, we say that $\mathcal{G}$ is finer than $\Im$ (written as $\Im < \mathcal{G}$) if for each $F \in \Im$, there is $G \in \mathcal{G}$ such that $G \subseteq F$ and that $\Im$ meets $\mathcal{G}$ if $F \cap G \neq \phi$ for every $F \in \Im$ and $G \in \mathcal{G}$.

Definition: 1.4. [3]. A point x of a space X is called a condensation point of the set $A \subseteq X$ if every nbd of the point x contains an uncountable subset of this set.

Clearly the set of all condensation points of a set A is closed.

Definition: 1.5. [4]. A subset of a space X is called ω-closed if it contains all its condensation points. Also $cl^{\omega}(A)$ will denote the intersection of all ω-closed sets which contains A. i.e., $cl^{\omega}(A) = \cap\{F: F \text{ is } \omega\text{-closed and } A \subseteq F\}$, then A is ω-closed iff $A = cl^{\omega}(A)$.

2. IJ-ω-Perfect Functions between Bitopological Spaces

In this section a number of useful results about ij-ω-converges to a subset, ij-ω-directed toward a set, ij-ω-closed functions, ij-ω-rigid set, and ij-ω-continuous functions are derived and used to obtain characterization theorem for an ij-ω-perfect functions between bitopological spaces.

Definition: 2.1. A point x in bitopological space (X, τ_1, τ_2) is called an ij-ω-condensation point of a subset A of X iff for any τ_i-open nbd U of x, $(\tau_j\text{-}cl^{\omega}(U)) \cap A \neq \phi$. The set of all ij-ω-condensation points of A is called the ij-ω-closure of A and denoted by ij-ω-$cl^{\omega}(A)$. A set $A \subset X$ is called ij-ω-closed if A=ij-ω-$cl^{\omega}(A)$.

Definition: 2.2. A point x in bitopological space (X, τ_1, τ_2) is called an ij-ω-condensation point of a filter base $\Im$ on X if it is an ij-ω-condensation point of every number of $\Im$. The set of all ij-ω-condensation points of $\Im$ is called ij-ω-condensed of $\Im$ and is denoted by ij-ω-cod$\Im$.

Definition: 2.3. A filter base $\Im$ on bitopological space (X, τ_1, τ_2) is said to be ij-ω-converges to a subset A⊆X (written as $\Im \xrightarrow{ij-\omega} A$) if for every τ_i-open cover $\mathcal{A}$ of A, there is a finite subfamily $\mathcal{B} \subseteq \mathcal{A}$ and F∈$\Im$ such that $F \subseteq \cup\{\tau_i\text{-}cl^{\omega}(B) : B \in \mathcal{B}\}$. We say $\Im$ ij-ω-converges to a point x∈X (written as $\Im \xrightarrow{ij-\omega} x$) iff $\Im \xrightarrow{ij-\omega} \{x\}$ or equivalently, τ_i-$cl^{\omega}(U)$ of every τ_i-open nbd U of x contains some member of $\Im$.

Theorem: 2.4. A point x in bitopological space (X, τ_1, τ_2) is an ij-ω-condensation of a filter base $\Im$ on X if there exists a filter base $\Im^*$ finer than $\Im$ such that $\Im^* \xrightarrow{ij-\omega} x$.

Proof: (⇒) Let x be an ij-ω-condensation point of a filter base $\Im$ on X, then every τ_i-open nbd U of x, the τ_j-ω-closure of U contains a member of $\Im$ and thus contains a member of any filter base $\Im^*$ finer than $\Im$, so that $\Im^* \xrightarrow{ij-\omega} x$.

(⇐) Suppose that x is not an ij-ω-condensation point of a filter base $\Im$ on X, then there exists an τ_i-open nbd U of x such that τ_j-ω-closure of U contains no member of $\Im$. Denote by $\Im^*$ the family of sets $F^*=F\cap(X-\tau_j\text{-}cl^{\omega}(U))$ for F∈$\Im$, then the sets F* are nonempty. Also $\Im^*$ is a filter base and indeed it is finer than $\Im$, because given $F_1^*=F_1\cap(X-\tau_j\text{-}cl^{\omega}(U))$ and $F_2^*=F_2\cap(X-\tau_j\text{-}cl^{\omega}(U))$, there is an $F_3 \subseteq F_1\cap F_2$ and this gives $F_3^*=F_3\cap(X-\tau_j\text{-}cl^{\omega}(U)) \subseteq F_1\cap F_2\cap(X-\tau_j\text{-}cl^{\omega}(U))=F_1\cap(X-\tau_j\text{-}cl^{\omega}(U))\cap F_2\cap(X-\tau_j\text{-}cl^{\omega}(U))$, by construction $\Im^*$ not ij-ω-convergent to x. This is a contradiction, and thus x is an ij-ω-condensation point of a filter base $\Im$ on X.

Definition: 2.5. A filter base $\Im$ on bitopological space (X, τ_1, τ_2) is said to be ij-ω-directed toward a set A⊆X, written as $\Im \xrightarrow{ij-\omega-d} A$, iff every filter base $\mathcal{G}$ finer than $\Im$ has an ij-ω-condensation point in A. i.e., (ij-ω-cod$\mathcal{G}$)∩A≠ϕ. We write $\Im \xrightarrow{ij-\omega-d} x$ to mean $\Im \xrightarrow{ij-\omega-d} \{x\}$, where x∈X.

Theorem: 2.6. Let $\Im$ be a filter base on bitopological space (X, τ_1, τ_2) and a point x∈X, then $\Im \xrightarrow{ij-\omega} x$ iff $\Im \xrightarrow{ij-\omega-d} x$.

Proof: (⇐) If $\Im$ does not ij-ω-converge to x, then there exists a τ_i-open nbd U of x such that $F \not\subset \tau_j\text{-}cl^{\omega}(U)$, for all F∈$\Im$. Then $\mathcal{G}=\{(X\text{-}\tau_i\text{-}cl^{\omega}(U)\cap F : F\in\Im\}$ is a filter base on X finer than $\Im$, and clearly x∉ij-ω-cod$\mathcal{G}$. Thus $\Im$ cannot be ij-ω-directed towards x.

(⇒) Clear.

Definition: 2.7. A function f : (X, τ_1, τ_2)→(Y, S_1, S_2) is said to be ij-ω-perfect if for each filter base $\Im$ on f(X), ij-ω-directed towards some subset B of f(X), the filter base $f^{-1}(\Im)$ is ij-ω-directed towards $f^{-1}(B)$ in X.

In the following theorem we show that only points of Y could be sufficient for the subset B in definition (2.7) and hence ij-ω-direction can be replaced in view of theorem (2.4) by ij-ω-convergence.

Theorem: 2.8. Let f : (X, τ_1, τ_2)→(Y, S_1, S_2) be a function. Then the following are equivalent:

(a) f is ij-ω-perfect.

(b) For each filter base $\Im$ on f(X), which is ij-ω-convergent to a point y in Y, $f^{-1}(\Im) \xrightarrow{ij-\omega-d} f^{-1}(y)$.

(c) For any filter base $\Im$ on X, ij-ω-cod f($\Im$)⊂f(ij-ω-cod $\Im$).

Proof: (a)⇒(b) Follows from theorem (2.6).

(b)⇒(c) Let y ∈ij-ω-cod f($\Im$). Then by theorem (2.4), there is a filter base $\mathcal{G}$ on f(X) finer than f($\Im$) such that $\mathcal{G} \xrightarrow{ij-\omega} y$. Let $\mathcal{U}=\{f^{-1}(G)\cap F : G\in \mathcal{G}$ and $F\in\Im\}$. Then $\mathcal{U}$ is a filter base on X finer than $f^{-1}(\mathcal{G})$. Since $\mathcal{G} \xrightarrow{ij-\omega-d} y$, by theorem (2.6) and f is ij-ω-perfect, $f^{-1}(\mathcal{G}) \xrightarrow{ij-\omega-d} f^{-1}(y)$. $\mathcal{U}$ being finer than $f^{-1}(\mathcal{G})$, we have $f^{-1}(y)\cap$(ij-ω-cod $\mathcal{U}$)≠ϕ. It is then clear that $f^{-1}(y)\cap$(ij-ω-cod $\Im$)≠ϕ. Thus y ∈f(ij-ω-cod $\Im$).

(c)⇒(a) Let $\Im$ be a filter base on f(X) such that it is ij-ω-directed towards some subset B of f(X). Let $\mathcal{G}$ be a filter base on X finer than $f^{1}(\Im)$. Then f($\mathcal{G}$) is a filter base on f(X) finer than

$\Im$ and hence $B\cap$(ij-ω-cod f($\mathcal{G}$))$\neq\phi$. Thus by (c) $B\cap$f(ij-ω-cod$\mathcal{G}$)$\neq\phi$ so that $f^{-1}(B)\cap$(ij-ω-cod $\mathcal{G}$)$\neq\phi$. This shows that $f^{-1}(\Im)$ is ij-ω-directed towards $f^{-1}(B)$. Hence f is ij-ω-perfect.

Definition: 2.9. A function f : (X, τ_1, τ_2)$\to$(Y, S_1, S_2) is called ij-ω-closed if the image of each ij-ω-closed set in X is ij-ω-closed in Y.

Theorem: 2.10. A function f : (X, τ_1, τ_2)$\to$(Y, S_1, S_2) is ij-ω-closed if ij-ω-cl^{ω}f(A)$\subset$f(ij-ω-cl^{ω}(A)), for each $A\subset X$.

Proof: Straightforward.

Theorem: 2.11. The ij-ω-perfect function f : (X, τ_1, τ_2)$\to$(Y, S_1, S_2) is ij-ω-closed.

Proof: Follows from theorem (2.10) and theorem (2.8 (a)$\Rightarrow$(c)) by taking $\Im=\{A\}$.

Definition: 2.12. A subset A of bitopological space (X, τ_1, τ_2) is said to be ij-ω-rigid in X if for each filter base $\Im$ on X with (ij-ω-cod$\Im$)$\cap A=\phi$, there is $U\in\tau_i$ and $F\in\Im$ such that $A\subset U$ and τ_j-cl^{ω}(U))$\cap F=\phi$, or equivalent, iff for each filter base $\Im$ on X whenever $A\cap$(ij-ω-cod $\Im$)$=\phi$, then for some $F\in\Im$, $A\cap$(ij-ω-cl^{ω}(F))$=\phi$.

Theorem: 2.13. If a function f : (X, τ_1, τ_2)$\to$(Y, S_1, S_2) is ij-ω-closed such that for each $y\in Y$, $f^{-1}(y)$ is ij-ω-rigid in X, then f is ij-ω-perfect.

Proof: Let $\Im$ be a filter base on f(X) such that $\Im \xrightarrow{ij-\omega} y$ in Y, for some $y\in Y$. If $\mathcal{G}$ is a filter base on X finer than the filter base $f^{-1}(\Im)$, then f($\mathcal{G}$) is a filter base on Y, finer than $\Im$. Since $\Im \xrightarrow{ij-\omega-d} y$ by theorem (2.4), $y\in$ij-ω-cod f($\mathcal{G}$), i.e., $y\in\cap\{$ij-ω-cl^{ω}f(G) : $G\in\mathcal{G}\}$and hence $y\in\cap\{$f(ij-ω-cl^{ω}(G) : $G\in\mathcal{G}\}$ by theorem (2.10), since f is ij-ω-closed. Then $f^{-1}(y)\cap$ij-ω-cl^{ω}(G)$\neq\phi$, for all $G\in\mathcal{G}$. Hence for all $U\in\tau_i$ with $f^{-1}(y)\subset U$, τ_j-cl^{ω}(U)$\cap G\neq\phi$, for all $G\in\mathcal{G}$. Since $f^{-1}(y)$ is ij-ω-rigid, it then follows that $f^{-1}(y)\cap$(ij-ω-cod$\mathcal{G}$)$\neq\phi$. Thus $f^{-1}(\Im) \xrightarrow{ij-\omega-d} f^{-1}(y)$. Hence by theorem (2.8 (b)$\Rightarrow$(a)), f is ij-ω-perfect.

Definition: 2.14. A function f : (X, τ_1, τ_2)$\to$(Y, S_1, S_2) is called ij-ω-continuous if for any S_i-open nbd V of f(x), there exists a τ_i-open nbd U of x such that f(τ_j-cl^{ω}(U))$\subset S_j$-cl^{ω}(V).

Theorem: 2.15. If an ij-ω-continuous function f : (X, τ_1, τ_2)$\to$(Y, S_1, S_2) is ij-ω-perfect then f is ij-ω-closed and for each $y\in Y$, $f^{-1}(y)$ is ij-ω-rigid in X.

Proof: By theorem (2.11) f an ij-ω-perfect function is ij-ω-closed. To prove the other part, let $y\in Y$, and suppose $\Im$ is a filter base on X such that (ij-ω-cod $\Im$)$\cap f^{-1}(y)=\phi$. Then $y\notin$f(ij-ω-cod $\Im$). Since f is ij-ω-perfect, by theorem (2.8 (a)$\Rightarrow$(c)) $y\notin$ij-ω-cod f($\Im$). Thus there exists an $F\in\Im$ such that $y\notin$ij-ω-cl^{ω}f(F). There exists an S_i-open nbd V of y such that S_j-cl^{ω}(V)$\cap$f(F)$=\phi$. Since f is ij-ω-continuous, for each $x\in f^{-1}(y)$ we shall get a τ_i-open nbd U_x of x such that f(τ_j-cl^{ω}(U_x)$\subset S_j$-cl^{ω}(V)$\subset$Y-f(F). Then f(τ_j-cl^{ω}(U_x)$\cap$f(F)$=\phi$, so that τ_j-cl^{ω}(U_x)$\cap F=\phi$. Then $x\notin$ij-ω-cl^{ω}(F), for all $x\in f^{-1}(y)$, so that $f^{-1}(y)\cap$(ij-ω-cl^{ω}(F))$=\phi$, Hence $f^{-1}(y)$ is ij-ω-rigid in X.

From theorems (2.13) and (2.15) we obtain.

Corollary: 2.16. An ij-ω-continuous function f : (X, τ_1, τ_2)$\to$(Y, S_1, S_2) is ij-ω-perfect if f is ij-ω-closed and for each $y\in Y$, $f^{-1}(y)$ is ij-ω-rigid in X.

We show that the above theorem remains valid if ij-ω-closedness of f is replaced by a strictly weaker condition which we shall call weak ij-ω-closedness of f. Thus we define as follows.

Definition: 2.17. A function f : (X, τ_1, τ_2)$\to$(Y, S_1, S_2) is said to be weakly ij-ω-closed if for every $y\in$f(X) and every τ_i-open set U containing $f^{-1}(y)$ in X, there exists a S_i-open nbd V of y such that f^{-1}(S_j-cl^{ω}(V))$\subset\tau_j$-cl^{ω}(U).

Theorem: 2.18. The ij-ω-closed function f : (X, τ_1, τ_2)$\to$(Y, S_1, S_2) is weakly ij-ω-closed.

Proof: Let $y\in$f(X) and let U be a τ_i-open set containing $f^{-1}(y)$ in X. By theorem (2.10) and since f is ij-ω-closed, we have ij-ω-cl^{ω}f(X-τ_j-cl^{ω}(U))$\subset$f[τ_i-cl^{ω}(X-τ_j-cl^{ω}(U)]. Now since $y\notin$f[τ_i-cl^{ω}(X-τ_j-cl^{ω}(U)], $y\notin$ij-ω-cl^{ω}f(X-τ_j-cl^{ω}(U)) and thus there exists an S_i-open nbd V of y in Y such that S_j-cl^{ω}(V)$\cap$f(X-τ_j-cl^{ω}(U))$=\phi$ which implies that f^{-1}(S_j-cl^{ω}(V))$\cap$(X-τ_j-cl^{ω}(U))$=\phi$, i.e., f^{-1}(S_j-cl^{ω}(V))$\subset\tau_j$-cl^{ω}(U), and thus f is weakly ij-ω-closed.

The converse of the above theorem is not true, which is shown in the next example.

Example: 2.19. Let τ_1, τ_2, S_1 and S_2 be any topologies and f : (X, τ_1, τ_2)→(Y, S_1, S_2) be a constant function, then f is weakly ij-ω-closed for i, j=1 and 2 (i≠j). Now, let X=Y=IR. If S_1 or S_2 is the discrete topology on Y, then f : (X, τ_1, τ_2)→(Y, S_1, S_2) given by f(x)=0, for all x∈X, is neither 12-ω-closed nor 21-ω-closed, irrespectively of the topologies τ_1, τ_2 and S_2 (or S_1).

Theorem: 2.20. Let f : (X, τ_1, τ_2)→(Y, S_1, S_2) be ij-ω-continuous. Then f is ij-ω-perfect if

(a) f is weakly ij-ω-closed, and

(b) f^{-1}(y) is ij-ω-rigid, for each y∈Y.

Proof: Suppose f : (X, τ_1, τ_2)→(Y, S_1, S_2) is an ij-ω-continuous function satisfying the conditions (a) and (b). To prove that f is ij-ω-perfect we have to show in view of theorem (2.13) that f is ij-ω-closed. Let y∈ij-ω-cl^{ω}f(A), for some non-null subset A of X, but y∉f(ij-ω-cl^{ω}(A). Then $\mathcal{B}$={A} is a filter base on X and (ij-ω-cod$\mathcal{B}$)∩f^{-1}(y)=ϕ. By ij-ω-rigidity of f^{-1}(y), there is a τ_i-open set U containing f^{-1}(y) such that τ_j-cl^{ω}(U)∩A=ϕ. By weak ij-ω-closedness of f, there exists an S_i-open nbd V of y such that f^{-1}(S_j-cl^{ω}(V))⊂τ_j-cl^{ω}(U), which implies that f^{-1}(S_j-cl^{ω}(V))∩A=ϕ, i.e., (S_j-cl^{ω}(V))∩f(A)=ϕ, which is impossible since y∈ij-ω-cl^{ω}f(A). Hence y∈f(ij-ω-cl^{ω}(A)). So f is ij-ω-closed.

From theorems (2.18) and (2.20) we get.

Corollary: 2.21. Let f : (X, τ_1, τ_2)→(Y, S_1, S_2) be an ij-ω-continuous function. Then f is ij-ω-perfect if

(a) f is weakly ij-ω-closed, and

(b) f^{-1}(y) is ij-ω-rigid, for each y∈Y.

Definition: 2.22. A subset A in bitopological space (X, τ_1, τ_2) is called ij-ω-set in X if for each τ_i-open cover $\mathcal{A}$ of A, there is a finite subcollection $\mathcal{B}$ of $\mathcal{A}$ such that A⊂∪{τ_j-cl^{ω}(U) : B∈$\mathcal{B}$}.

Theorem: 2.23. A subset A of a bitopological space (X, τ_1, τ_2) is an ij-ω-set if for each filter base ℑ on A, (ij-ω-codℑ)∩A≠ϕ.

Proof: (⇒) Clear.

(⇐) Let $\mathcal{A}$ be a τ_i-open cover of $\mathcal{A}$ such that the τ_j-ω-closed of the union of any finite subcollection of $\mathcal{A}$ is not cover A. Then ℑ={A\\τ_j-$cl^{\omega}{}_X$($\cup_{\mathcal{B}}$ $U_{\mathcal{B}}$) : $\mathcal{B}$ is finite sub collection of $\mathcal{A}$} is a filter base on A and (ij-ω-codℑ)∩A=ϕ. This contradiction yields that A is ij-ω-set.

Theorem: 2.24. If f : (X, τ_1, τ_2)→(Y, S_1, S_2) is ij-ω-perfect and B⊂Y is an ij-ω-set in Y, then f^{-1}(B) is an ij-ω-set in X.

Proof: Let ℑ be a filter base on f^{-1}(B), then f(ℑ) is a filter base on B. Since B is an ij-ω-set in Y, B∩ij-ω-codf(ℑ)≠ϕ by theorem (2.23). By theorem (2.8 (a)⇒(c)), B∩f(ij-ω-cod(ℑ))≠ϕ, so that f^{-1}(B)∩ij-ω-cod(ℑ)≠ϕ. Hence by theorem (2.23), f^{-1}(B) is an ij-ω-set in X.

The converse of the above theorem is not true, is shown in the next example.

Example: 2.25. Let X=Y=IR, τ_1 and τ_2 be the cofinite and discrete topologies on X and S_1, S_2 respectively denote the indiscrete and usual topologies on Y. Suppose f : (X, τ_1, τ_2)→(Y, S_1, S_2) is the identity function. Each subset of either of (X, τ_1, τ_2) and (Y, S_1, S_2) is a 12-ω-set. Now, any non-void finite set A⊂X is 12-ω-closed in X, but f(A) (i.e., A) is not 12-ω-closed in Y (in fact, the only 12-ω-closed subsets of Y are Y and ϕ).

The theorem (2.24) and the above example suggest the definition of a strictly weaker version of ij-ω-perfect functions as given below.

Definition: 2.26. A function f : (X, τ_1, τ_2)→(Y, S_1, S_2) is said to be almost ij-ω-perfect if for each ij-ω-set K in Y, f^{-1}(K) is an ij-ω-set in X.

By analogy to theorem (2.13), a sufficient condition for a function to be almost ij-ω-perfect, is proved as follows.

Theorem: 2.27. Let f : (X, τ_1, τ_2)→(Y, S_1, S_2) be any function such that

(a) $f^{-1}(y)$ is ij-ω-rigid, for each $y \in Y$, and

(b) f is weakly ij-ω-closed.

Then f is almost ij-ω-perfect.

Proof: Let B be an ij-ω-set in Y and let ℑ b e a filter base on $f^{-1}(B)$. Now f(ℑ) is a filter base on B and so by theorem (2.23), (ij-ω-cod(ℑ))∩B≠ϕ. Let y∈[ij-ω-cod(ℑ)]∩B. Suppose that ℑ has no ij-ω-condensation point in $f^{-1}(B)$ so that (ij-ω-cod(ℑ))∩$f^{-1}(y)$=ϕ. Since $f^{-1}(y)$ is ij-ω-rigid, there exists an F∈ℑ and a τ_i-open set U containing $f^{-1}(y)$ such that F∩τ_i-$cl^{\omega}(U)$=ϕ. By weak ij-ω-closedness of f, there is a S_i-open nbd V of y such that $f^{-1}(S_j\text{-}cl^{\omega}(V)) \subset \tau_j\text{-}cl^{\omega}(U)$ which implies that $f^{-1}(S_j\text{-}cl^{\omega}(V)) \cap F=\phi$, i.e., S_j-$cl^{\omega}(V)$∩f(F)=ϕ, which is a contradiction. Thus by theorem (2.23), $f^{-1}(B)$ is an ij-ω-set in X and hence f is almost ij-ω-perfect.

We now give some applications of ij-ω-perfect functions. The following characterization theorem for an ij-ω-continuous function is recalled to this end.

Theorem: 2.28. A function f : (X, τ_1, τ_2)→(Y, S_1, S_2) is ij-ω-continuous if f(ij-ω-cl^{ω}(A))⊂ij-ω-cl^{ω}f(A), for each A⊂X.

Proof: (⇒) Suppose that x∈ij-ω-cl^{ω}(A) and V is S_i-open nbd of f(x). Since f is ij-ω-continuous, there exists a τ_i-open nbd U of x such that f(τ_j-cl^{ω}(U))⊂S_j-cl^{ω}(V). Since τ_j-cl^{ω}(U)∩A≠ϕ, then S_j-cl^{ω}(V)∩f(A)≠ϕ. So, f(x)∈ij-ω-cl^{ω}f(A). This shows that f(ij-ω-cl^{ω}(A))⊂ij-ω-cl^{ω}f(A).

(⇐) Clear.

Theorem: 2.29. Let f : (X, τ_1, τ_2)→(Y, S_1, R_2) be ij-ω-continuous and ij-ω-perfect. Then f^{-1} preserves ij-ω-rigidity.

Proof: Let B be an ij-ω-rigid set in Y and let ℑ be a filter base on X such that $f^{-1}(B)$∩(ij-ω-cod(ℑ))=ϕ. Since f is ij-ω-perfect and B∩f(ij-ω-cod(ℑ))=ϕ by theorem (2.8 (a)⇒(c)) we get B∩(ij-ω-cod f(ℑ))=ϕ. Now B being an ij-ω-rigid set in Y, there exists an F∈ℑ such that B∩ij-ω-cl^{ω}f(F)=ϕ. Since f is ij-ω-continuous, by theorem (2.28) it follows that B∩f(ij-ω-cl^{ω}(F))=ϕ. Thus $f^{-1}(B)$∩(ij-ω-cl^{ω}(F))=ϕ. This proves that $f^{-1}(B)$ is ij-ω-rigid.

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الدوال التامة من النمط-IJ-ω بين الفضاءات التبولوجية الثنائية

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قسم الرياضيات، كلية التربية - ابن الهيثم، جامعة بغداد

الخلاصة

في هذا البحث نحن قدمنا العديد من المفاهيم في الفضاءات التبولوجية الثنائية التي هية الاقتراب لمجموعة جزئية من النمط-ij-ω، الاتجاه المباشر لمجموعة من النمط-ij-ω، الدوال المغلقة من النمط,ij-ω، صلابة مجموعة من النمط-ij-ω، الدوال المستمرة من النمط-ij-ω، والمفهوم الرئيسي في هذا البجث هو الدوال التامة من النمط-ij-ω بين الفضاءات التبولوجية الثنائية. كذلك العديد من المبرهنات و المميزات المتعلقة بهذه المفاهيم درست.